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978-0-521-88262-0 — Stochastic Resonance
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Introduction and motivation

1.1 Background and motivation

We begin by briefly outlining the background and motivation for this book, before giving an overview of each chapter, and pointing out the most significant questions addressed.

Although the methodology used is firmly within the fields of signal processing and mathematical physics, the motivation is interdisciplinary in nature.

The initial open questions that inspired this direction were:

(i) How might neurons make use of a phenomenon known as *stochastic resonance*?
(ii) How might a path towards engineering applications inspired by these studies be initiated?

Stochastic resonance and sensory neural coding

Stochastic resonance (SR) is a counter-intuitive phenomenon where the presence of *noise* in a nonlinear system is essential for optimal system performance. It is not a technique. Instead, it is an effect that might be observed and potentially exploited or induced. It has been observed to occur in many systems, including in both neurons and electronic circuits.

A motivating idea is that since we know the brain is far better at many tasks compared to electronic and computing devices, then maybe we can learn something from the brain. If we can ultimately better understand the possible exploitation of SR in the brain and nervous system, we may also be able to improve aspects of electronic systems.

Although it is important to have an overall vision, in practical terms it is necessary to consider a concrete starting point. This book is particularly focused on an exciting new development in the field of SR, known as *suprathreshold stochastic resonance* (SSR) (Stocks 2000c). Suprathreshold stochastic resonance occurs in a parallel array of simple threshold devices. Each individual threshold device receives the same signal, but is subject to *independent* additive random noise. The output of each device is a binary signal, which is unity when the input is greater than the threshold value, and zero otherwise. The overall output of the *SSR model* is the *sum* of the individual binary signals. Originally, all threshold devices were considered to have the same threshold value.

Early studies into SSR considered the effect from the point of view of information transmission and the model in which SSR occurs as a communication channel. Furthermore, the model in which SSR occurs was originally inspired by questions of sensory neural coding in the presence of noise. Unlike previously studied forms of SR, either in neurons or simple threshold-based systems, SR can occur in such a model for signals that are not entirely or predominantly subthreshold, and for both very small and quite large amounts of noise. Although each threshold device in the SSR model is very simple in comparison with more biologically realistic neural models, such devices have actually been used to model neurons, under the name of the McCulloch–Pitts neural model (McCulloch and Pitts 1943).

The aim of this book is to comprehensively outline all known theoretical and numerical results on SSR and to extend this theory. It is anticipated that the landscape presented here will form a launching pad for future research into the specific role that SSR may play in real neural coding. Note that the goal is not to prove that living systems actively exploit SR or SSR – these are ongoing research areas in the domain of neurophysiology and biophysics. Rather, our starting point is the theoretical mathematical underpinning of SR-type effects in the very simple McCulloch–Pitts model.

This is in line with the time-honoured approach in physics and engineering; namely, to begin with the simplest possible model. As this book unfolds, it will be seen that the analysis of SR in arrays of such simplified neural models gives rise to rich complex phenomena and also to a number of surprises. Explanation, or indeed discovery, of these effects would not have been tractable if the starting point were an intricate neural model. Analysis of the simple model lays the foundation for adding further complexities in the future.

The intention is that the mathematical foundation provided by this book will assist future neurophysiologists in asking the right questions and in performing the right experiments when establishing if real neurons actively exploit SSR. In the meantime, this book also contributes to the application of SSR in artificial neural and electronic systems. To this end, the book culminates in a chapter on the application of SSR to electronic cochlear implants.

Low signal-to-noise ratio systems and sensor networks

Another motivation for this research is the important problem of overcoming the effects of noise in sensors and signal and data processing applications. For example, microelectronics technologies are shrinking, and are beginning to approach the nanoscale level (Martorell *et al.* 2005). At this scale, device behaviour can change and noise levels can approach signal levels. For such small signal-to-noise ratios (SNRs), it may be impossible for traditional noise reduction methods to operate, and it may be that optimal circuit design needs to make use of the effects of SR. Two experimental examples illustrating this possibility include the demonstrations of SR in (i) carbon nanotube transistors for the first time in 2003 (Lee *et al.* 2003, Lee *et al.* 2006), and (ii) a silicon nanoscale mechanical oscillator as reported in 2005 in the journal *Nature* (Badzey and Mohanty 2005, Bulsara 2005).

A second area of much current research is that of distributed sensor networks (Akyildiz *et al.* 2002, Pradhan *et al.* 2002, Chong and Kumar 2003, Iyengar and Brooks 2004, Martinez *et al.* 2004, Xiong *et al.* 2004). Of particular interest to this book is the problem where it is not necessarily a network of complete sensors that is distributed, but it is actually the data acquisition, or compression, that is distributed (Berger *et al.* 1996, Draper and Wornell 2004, Pradhan *et al.* 2002, Xiong *et al.* 2004, Pradhan and Ramchandran 2005). A key aspect of such a scenario is that data are acquired from a number of independently noisy sources that do not cooperate, and are then fused by some central processing unit. In the information theory and signal processing literature, this is referred to as *distributed source coding* or *distributed compression*.

Given that the SSR effect overcomes a serious limitation of all previously studied forms of SR, a complete theoretical investigation of its behaviour may lead to new design approaches to low SNR systems or data acquisition and compression in distributed sensor networks.

1.2 From stochastic resonance to stochastic signal quantization

During the course of writing this book, it became apparent that the SSR effect is equivalent to a noisy, or stochastic, *quantization* of a signal. Consequently, as well as describing its behaviour from the perspective of information transmission, it is equally valid to describe it from the perspective of *information compression* or, more specifically, *lossy compression*. Note that quantization of a signal is a form of lossy compression (Widrow *et al.* 1996, Berger and Gibson 1998, Gray and Neuhoff 1998).

The distinguishing feature of SSR, that sets it apart from standard forms of quantization, is that conventionally the rules that specify a quantizer's operation are considered to be fixed and deterministic. In contrast, when the SSR effect is viewed as quantization, the governing rules lead to a set of parameters that are independent random variables. Hence, we often refer to the SSR model's output as a *stochastic quantization*.

Given this perspective, there are three immediate questions that can be asked:

- Can we describe the SSR effect in terms of conventional quantization and compression theory?
- Given that a central SR question is that of finding the optimal noise conditions, what noise intensity optimizes the performance of the SSR model when it is described as a quantizer?
- How good is the SSR effect at quantization when compared with conventional quantizers?

The underlying theme of this book is to address these three questions.

1.3 Outline of book

This book consists of ten chapters, as follows:

- Chapter 1, the current chapter, provides the background and motivation for the work described in this book.
- Chapter 2 contains an overview of the historical landscape against which this book is set. It defines *stochastic resonance* as it is most widely understood, and gives a broad literature review of SR, with particular emphasis on aspects relevant to quantization. Chapter 2 is deliberately sparse in equations and devoid of quantitative results, but does provide qualitative illustrations of how SR works. It also provides some discussion that is somewhat peripheral to the main scope of this book, but that will prove useful for readers unfamiliar with, or confused about, SR.
- Chapter 3 contains the information-theoretic definitions required for the remainder of this book, and discusses the differences between *dithering* and stochastic resonance.
- Chapters 4 and 5 begin the main focus of the book, by defining the SSR model, giving a detailed literature review of all previous research on SSR, and replicating all the most significant theoretical results to date. These two chapters consider only the original concept of the SSR model as a communications channel. In particular, we examine how the mutual information between the input and output of the SSR model varies with noise intensity. A subset of these results pertains specifically to a large number of individual threshold devices in the SSR model. Chapter 5 is devoted to elaborating on results in this area, as well as developing new results in this area, while Chapter 4 focuses on more general behaviour.
- Chapters 6 and 7 contain work on the description of the SSR model as a quantizer. There are two main aspects to such a description. First, quantizers are specified by two operations: an encoding operation and a decoding operation. The encoding operation assigns ranges of values of the quantizer's input signal to one of a finite number of

output states. The decoding operation approximately reconstructs the original signal by assigning 'reproduction values' to each encoded state. In contrast to conventional quantizers, the SSR model's encoding is stochastic, as the output state for given input signal values is nondeterministic. However, it is possible to decode the SSR output in a similar manner to conventional quantizers, and we examine various ways to achieve this. The second aspect we consider is the performance of the decoded SSR model. Since the decoding is designed to approximate the original signal, performance is measured by the average properties of the error between this original signal and the quantizer's output approximation. Conventionally, mean square error distortion is used to measure this average error, and we examine in detail how this measure varies with noise intensity, the decoding scheme used, and the number of threshold devices in the SSR model. As with Chapters 4 and 5, Chapter 6 focuses on general behaviour, while Chapter 7 is devoted to discussion of the SSR model in the event of a large number of individual threshold devices.

- Chapter 8 expounds work that extends the SSR model beyond its original specification. We relax the constraint that all individual threshold devices must have identical threshold values, and allow each device to have an arbitrary threshold value. We then consider how to optimally choose the set of threshold values as the noise intensity changes. The most important result of this study is the numerical demonstration that the SSR model – where all threshold devices have the same threshold value – is optimal, for sufficiently large noise intensity.
- Motivated by recent neural-coding research, Chapter 9 further extends the SSR model, by including a constraint on the energy available to the system. The performance of a quantizer is characterized by two opposing factors: *rate* and *distortion*. Chapter 9 also explores the SSR model, and its extension to arbitrary thresholds, from the point of view of rate-distortion theory.
- Chapter 10 concludes the technical content of this book, by pointing to a concrete application of SSR theory, in the area of cochlear implants.
- Finally, Chapter 11 contains some speculations on the future of stochastic resonance and suprathreshold stochastic resonance.

2

Stochastic resonance: its definition, history, and debates

Stochastic resonance (SR), being an interdisciplinary and evolving subject, has seen many debates. Indeed, the term SR itself has been difficult to comprehensively define to everyone's satisfaction. In this chapter we look at the problem of defining stochastic resonance, as well as exploring its history. Given that the bulk of this book is focused on suprathreshold stochastic resonance (SSR), we give particular emphasis to forms of stochastic resonance where *thresholding* of random signals occurs. An important example where thresholding occurs is in the generation of action potentials by spiking neurons. In addition, we outline and comment on some of the confusions and controversies surrounding stochastic resonance and what can be achieved by exploiting the effect. This chapter is intentionally qualitative. Illustrative examples of stochastic resonance in threshold systems are given, but fuller mathematical and numerical details are left for subsequent chapters.

2.1 Introducing stochastic resonance

Stochastic resonance, although a term originally used in a very specific context, is now broadly applied to describe any phenomenon where the presence of internal noise or external input noise in a nonlinear system provides a better system response to a certain input signal than in the absence of noise. The key term here is *nonlinear*. Stochastic resonance cannot occur in a linear system – linear in this sense means that the output of the system is a linear transformation of the input of the system. A wide variety of performance measures have been used – we shall discuss some of these later.

The term *stochastic resonance* was first used in the context of noise enhanced signal processing in 1980 by Roberto Benzi, at the NATO International School of Climatology. Since then it has been used – according to the ISI Web of Knowledge database – in around 2000 publications, over a period of a quarter of a century. The frequency of publication, by year, of these papers is shown in Fig. 2.1. This figure

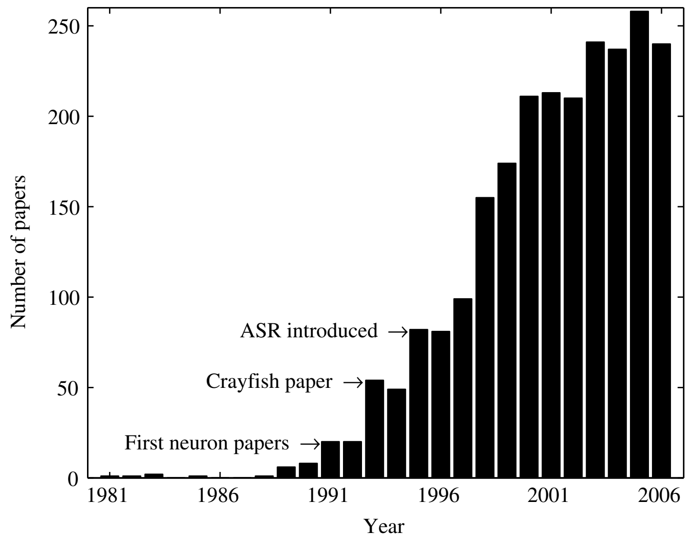

Fig. 2.1. Frequency of stochastic resonance papers by year – between 1980 and 2006 – according to the ISI Web of Knowledge database. There are several epochs in which large increases in the frequency of SR papers occurred. The first of these is between 1989 and 1992, when the most significant events were the first papers examining SR in neural models (Bulsara *et al.* 1991, Bulsara and Moss 1991, Longtin *et al.* 1991), and the description of SR by linear response theory (Dykman *et al.* 1990a). The second epoch is between about 1993 and 1996, when the most significant events were the observation of SR in physiological experiments on neurons (Douglass *et al.* 1993, Levin and Miller 1996), the popularization of array enhanced SR (Lindner *et al.* 1995, Lindner *et al.* 1996), and of aperiodic stochastic resonance (ASR) (Collins *et al.* 1995a). Around 1997, a steady increase in SR papers occurred, as investigations of SR in neurons and ASR became widespread.

illustrates how the use of the term *stochastic resonance* expanded rapidly in the 1990s, and is continuing to expand in the 2000s.

The 'resonance' part of 'stochastic resonance' was originally used because the signature feature of SR is that a plot of output signal-to-noise ratio (SNR) has a single maximum for some nonzero input noise intensity. Such a plot, as shown in Fig. 2.2, has a similar appearance to frequency dependent systems that have a maximum SNR, or output response, for some *resonant frequency*. However, in the case of SR, the resonance is 'noise induced', rather than at a particular frequency – see Section 2.3 for further discussion.

SR has been the subject of many reviews, including full technical journal articles (Jung 1993, Moss *et al.* 1994, Dykman *et al.* 1995, Gammaitoni *et al.* 1998, Wiesenfeld and Jaramillo 1998, Luchinsky *et al.* 1999a, Luchinsky *et al.* 1999b, Anishchenko *et al.* 1999, Hänggi 2002, Harmer *et al.* 2002, Wellens *et al.* 2004,

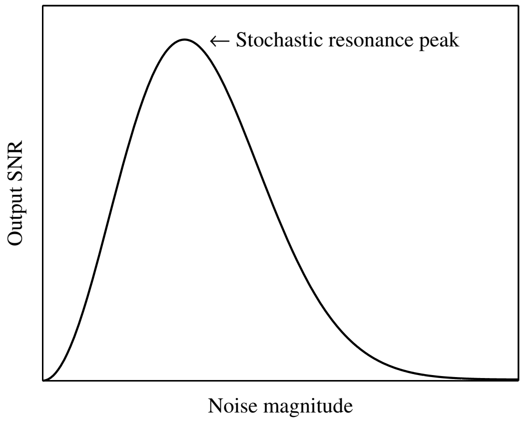

Fig. 2.2. Typical curve of output SNR vs. input noise magnitude, for systems capable of stochastic resonance. For small and large noise values, the output SNR is very small, while some intermediate nonzero noise value provides the maximum output SNR.

Shatokhin *et al.* 2004, Moss *et al.* 2004), editorial works (Bulsara *et al.* 1993, Astumian and Moss 1998, Petracchi *et al.* 2000, Abbott 2001, Gingl 2002), book chapters (Wiesenfeld 1993a), and magazine articles (Wiesenfeld 1993b, Moss and Wiesenfeld 1995, Bulsara and Gammaitoni 1996).

Some of the most influential workshops and conferences on SR include those held in San Diego, USA, in 1992 (published as a special issue of the journal *Il Nuovo Cimento*) (Moss *et al.* 1992) and 1997 (Kadtke and Bulsara 1997), Elba Island, Italy, in 1994 (published as a special issue of *Journal of Statistical Physics*) (Bulsara *et al.* 1994), and Ambleside, UK, in 1999 (Broomhead *et al.* 2000). Various other special issues of journals have been devoted to SR, such as the September 1998 issue of *Chaos*, the September 2000 issue of *Chaos Solitons and Fractals*, and the September 2002 issue of *Fluctuation and Noise Letters*.

There have been articles and letters on SR published in the prestigious journal, *Nature* (Douglass *et al.* 1993, Moss *et al.* 1993, Wiesenfeld and Moss 1995, Moss and Pei 1995, Bezrukov and Voydanoy 1995, Collins *et al.* 1995b, Noest 1995, Collins *et al.* 1995c, Levin and Miller 1996, Bezrukov and Voydanoy 1997b, Astumian *et al.* 1997, Dykman and McClintock 1998, Collins 1999, Russell *et al.* 1999, Moss and Milton 2003, Bulsara 2005, Badzey and Mohanty 2005). A book has been written exclusively on SR (Andò and Graziani 2000), as well as a large section of another book (Anischenko *et al.* 2002), and sections on SR written in popular science books (von Baeyer 2003, Kosko 2006). It has also started to appear in more general textbooks (Gerstner and Kistler 2002).

SR has been widely observed throughout nature – it has been quantified in such diverse systems as climate models (Benzi *et al.* 1982), electronic circuits (Fauve

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and Heslot 1983, Anishchenko *et al.* 1994), differential equations (Benzi *et al.* 1985, Hu *et al.* 1990), ring lasers (McNamara *et al.* 1988), semiconductor lasers (Iannelli *et al.* 1994), neural models (Bulsara *et al.* 1991, Longtin *et al.* 1991), physiological neural populations (Douglass *et al.* 1993), chemical reactions (Leonard and Reichl 1994), ion channels (Bezrukov and Voydanoy 1995), SQUIDs (Superconducting Quantum Interference Devices) (Hibbs *et al.* 1995), the behaviour of feeding paddlefish (Russell *et al.* 1999, Freund *et al.* 2002), ecological models (Blarer and Doebeli 1999), cell biology (Paulsson and Ehrenberg 2000, Paulsson *et al.* 2000), financial models (Mao *et al.* 2002), psychophysics (Ward *et al.* 2002), carbon-nanotube transistors (Lee *et al.* 2003, Lee *et al.* 2006), nanomechanical oscillators (Badzey and Mohanty 2005, Bulsara 2005), and even social systems (Wallace *et al.* 1997).

The first highly successful application inspired by SR involves the use of electrically generated subthreshold stimuli in biomedical prosthetics to improve human balance control and somatosensation (Priplata *et al.* 2004, Priplata *et al.* 2003, Collins *et al.* 2003, Moss and Milton 2003, Harry *et al.* 2005). This work led to James J. Collins winning a prestigious MacArthur Fellowship in October 2003 (Harry *et al.* 2005).

Prior to this, Collins was also an author on a correspondence to *Nature* (Suki *et al*. 1998) on a noise-enhanced application, described as analogous to SR. The *Nature* paper used a model to verify the experimental results of Lefevre *et al.* (1996). Random noise was introduced into the operation of mechanical life-support ventilators, in order to more closely replicate natural breathing. It was found that added noise enhanced artificial ventilation in several ways. See Brewster *et al.* (2005) for a review.

2.2 Questions concerning stochastic resonance

There are a number of misconceptions and controversies about stochastic resonance. The following list of questions attempts to encapsulate the main points of contention:

(i) What is the definition of stochastic resonance?
(ii) Is stochastic resonance exploited by the nervous system and brain as part of the neural code?
(iii) Does stochastic resonance occur only if a signal's power is weak compared with the power of the noise in a system?
(iv) Can stochastic resonance lead to a signal-to-noise ratio gain?
(v) Was stochastic resonance known prior to the first use in 1981 of the term 'stochastic resonance'?
(vi) How and when is stochastic resonance different from a signal processing technique called *dithering*?

Although question (ii) is quite clearly the interesting scientific question, and seemingly the motivation behind much SR research, it would appear the other questions in the above list have sometimes provided a diversion. The problem is that reaching a consensus on the answers to questions (ii)–(vi) really depends on an agreed answer to question (i).

The broadest possible definition of stochastic resonance is that it occurs when randomness may have some positive role in a signal processing context. Given this definition, *we believe* that the answers to these questions are: (ii) yes, although it is difficult to prove, the brain – or at least, some parts of the nervous system – would almost certainly not function as it does if it were completely deterministic; (iii) no, randomness can have a positive role even if it is only a small amount of randomness; (iv) yes, in the information-theoretic sense, random noise in a system can lead to a less noisy output signal, provided that the system is nonlinear; (v) yes, randomness has been known to have a positive role in many circumstances for decades, if not centuries; and (vi) stochastic resonance occurs when dithering is used – dithering can be described as the exploitation of SR.

On the other hand, if the definition of stochastic resonance is restricted to its original narrow context, then the answers to questions (ii)–(vi) change to: (ii) maybe – this is yet to be conclusively answered, (iii) yes, (iv) no, (v) no, and (vi) dithering is quite different from SR, as is comprehensively explained in Gammaitoni (1995a).

This discussion is intended to illustrate that the debate on the topics listed above can depend crucially on what we mean by *stochastic resonance*. In the remainder of this chapter we discuss some of these issues in more detail in order to bring some clarity to the debate and to illustrate the pitfalls and controversies for readers new to the stochastic resonance field, or who have held only a peripheral interest in the area.

2.3 Defining stochastic resonance

SR is often described as a counter-intuitive phenomenon. This is largely due to its historical background; in the first decade and a half since the coining of the term in 1980, virtually all research into SR considered only systems driven by a combination of a periodic single-frequency input signal and broadband noise. In such systems, a natural measure of system performance is the output signal-to-noise ratio (SNR), or, more precisely, often the ratio of the output power spectral density (PSD) at the input frequency to the output noise floor PSD measured with the signal present. The noise floor is measured with the signal present, rather than absent, as the output noise may change if the signal is not present. This is because