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978-0-521-88219-4 - The Mathematics of Logic: A Guide to Completeness Theorems  
and their Applications

Richard Kaye

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