

## DOUBLE-DIFFUSIVE CONVECTION

Double-diffusive convection is a mixing process driven by the interaction of two fluid components that diffuse at different rates. This phenomenon has important ramifications in oceanography and in numerous other fields, from crystal growth to magma chambers and stellar interiors. Nevertheless, several aspects of double-diffusive convection still remain unclear and controversial.

Leading expert Timour Radko presents the first systematic overview of the classical theory of double-diffusive convection, in a coherent narrative which brings together the disparate literature in this developing field. The book begins by exploring idealized dynamical models and illustrating key principles through examples of oceanic phenomena. Building on the theory, it then explains the dynamics of structures resulting from double-diffusive instabilities, such as the little-understood phenomenon of thermohaline staircases. The book also surveys non-oceanographic applications, such as industrial, astrophysical and geological manifestations, and discusses the climatic and biological consequences of double-diffusive convection.

Providing a balanced blend of fundamental theory and real-world examples, this is an indispensable resource for academic researchers, professionals and graduate students in physical oceanography, fluid dynamics, applied mathematics, astrophysics, geophysics and climatology.

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In memory of Melvin Stern. Half a century ago he had this idea . . .

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## Preface

Ever since I attended lectures on double-diffusion delivered by the founder of the field, a humble genius by the name of Melvin Stern, there has been no doubt in my mind that double-diffusive convection is the most intriguing subject of fluid dynamics. Even the manner in which double-diffusion was discovered is unusual. It represents a rare instance where a major physical phenomenon was predicted without the motivation of preceding field observations. In 1959, Melvin Stern, using little more than his imagination and physical intuition, formulated a theory for salt fingers – thin, alternating fluid filaments that appear when warm and salty water overlies fresh and cold. The subsequent publication of *The Salt Fountain and Thermohaline Convection* (Stern, 1960) officially marked the birth of a new field of fluid mechanics – double-diffusive convection.

Double-diffusion operates in a counterintuitive way; it is a mixing process that makes dense fluid denser and light fluid lighter. It is driven by the difference in the molecular diffusivities of heat and salt – something that is completely ignored in the vast majority of oceanographic theories and numerical models. The dynamics of double-diffusion are still surrounded by controversy. For instance, the most dramatic signature of oceanic double-diffusion comes from its ability to form stepped structures in vertical temperature and salinity profiles. These so-called thermohaline staircases have been observed in the ocean for forty years and are routinely generated in the laboratory. Yet, in recent literature on the subject, one still finds at least half a dozen different hypotheses for their origin. Extreme opposing views have been expressed on the global consequences of double-diffusion. Overall, it is hard to name any other field in fluid mechanics that can excite such passion amongst the experts and, at the same time, would be found so regrettably confusing by non-specialists.

Nevertheless, the growth of interest in double-diffusion is apparent. In ocean science, the last few years have seen marked improvements in the reliability of microstructure measurements. New observations, most notably the Salt Finger

Tracer Release Experiment, have made it possible to better quantify the role of double-diffusive mixing in water-mass transformation – and hence its impact on the climate system. It has been shown that vertical salt-finger diffusivities in the central thermocline often exceed, by as much as an order of magnitude, the diffusivity of overturning gravity waves, the other primary candidate for internal mixing. The significance of lateral mixing induced by double-diffusion has also become increasingly clear. Thermohaline intrusions, so spectacularly manifested in recent seismic observations, can be essential in explaining the elusive link between stirring of the ocean by mesoscale eddies and the ultimate dissipation of temperature and salinity variance by molecular diffusion. Concurrent developments in non-oceanographic realms underscore the breadth of the subject. A surge of interest in astrophysical applications has produced credible theories for double-diffusive control of the composition of main-sequence stars and giant planets. A seemingly countless variety of geological double-diffusive phenomena continue to stimulate explorations of the relevant dynamics. Chemical double-diffusion is another area in which research activities have markedly intensified in the past decade, triggered by a series of imaginative experimental studies of multicomponent reacting systems.

At the same time, it is generally realized that our insight into the physics and consequences of double-diffusion remains inadequate and that more effort should be invested in all aspects of the problem. Given all the motivations and intellectual challenges of double-diffusive research, one cannot help but wonder why we have not witnessed an explosion of knowledge and new ideas similar to what has happened in many other, less critical branches of geophysical fluid dynamics. My answer to this could be biased and personal. I believe that a significant obstacle to the development of our field is that not one single text has been devoted entirely to double-diffusion. A newcomer, trying to build up intuition and a broad understanding of the subject, is left searching for information in dozens of specialized articles. Of course, brief discussions of double-diffusive convection can be found in some outstanding general fluid dynamical books, such as *Buoyancy Effects in Fluids* (Turner, 1979) and *Ocean Circulation Physics* (Stern, 1975). However, they provide only basic information and make no attempt to systematically explore the rich dynamics of double-diffusive structures. Almost fifteen years have passed since I completed my graduate studies. However, I still remember a feeling of frustration that there wasn't one self-contained treatise that would efficiently guide a student through all the knowledge acquired from numerous laboratory experiments, fields programs and theoretical studies. This oversight becomes particularly striking when we recall that the sister subject of double-diffusion – thermal convection – has been discussed in at least thirty dedicated texts.

When a commissioning editor for Cambridge University Press suggested that I write a book on double-diffusion, I was delighted and flattered. This proposition

has strengthened my conviction that progress in our field can be greatly accelerated by the double-diffusive book, an organized and self-contained entity accessible to a wide range of scientists with different levels of expertise and backgrounds. The timing appeared to be most appropriate for such an undertaking. All branches of fluid dynamics continuously evolve, but recent shifts in our understanding of double-diffusion have been particularly profound. Theoretical and modeling advances, motivated by recent oceanographic field programs, have led to new insights into the origin and dynamics of thermohaline staircases. The problem of nonlinear equilibration of salt fingers looks increasingly more tractable, with fresh ideas defining the way towards physically based parameterizations of double-diffusive mixing. At the same time, many traditional views, such as the significance of large-scale shear and finger/wave interaction, have to be critically reevaluated. Clearly, the extant descriptions of double-diffusion need considerable updating, as does our perception of its role in ocean mixing.

This book is designed as a comprehensive review of the field that combines the basic theory with major up-to-date findings, touches upon ongoing research, and offers some suggestions for future developments. It consists of three distinct parts. The first part (Chapters 1–5) presents the fundamental theory of double-diffusion at a level suitable for a diligent non-expert with some background in the physical sciences. Chapter 1 opens with a discussion of the basic dynamic principles of double-diffusion, traces the history of events leading to its discovery, and introduces key governing parameters. Chapter 2 summarizes the linear instability theory, and establishes the relevant spatial and temporal scales. While the primary double-diffusive instability operates on scales of several centimeters, it affects the evolution of much larger scales through the vertical fluxes of temperature and salinity. Accordingly, the next three chapters are concerned with the formulation of the flux laws. The properties of vertical transport and the associated nonlinear dynamics depend on the geometry of a double-diffusive system. Specific examples are presented for the unbounded gradient configuration (Chapter 3), the two-layer system (Chapter 4) and the vertically bounded layer model (Chapter 5).

The discussion of flux laws establishes the framework for the second part of this book (Chapters 6–9), where the focus is on structures resulting *from* the primary double-diffusive instability – collective instability waves, intrusions and thermohaline staircases. This component is more specialized and will be of particular interest to oceanographers and applied mathematicians who are already familiar with some convection models. Other readers may still follow the narrative by focusing on relevant observations, numerical results and physical arguments. The presentation weaves together the classical theory with recent developments and can be used as a starting point for graduate students and as a professional reference for active

researchers. While the dynamics of collective instabilities (Chapter 6) and thermohaline intrusions (Chapter 7) are reasonably well understood, the origin of thermohaline staircases (Chapter 8) is a more controversial topic. Therefore, Chapter 8 presents several competing views on staircase dynamics; the emphasis being on theoretical and modeling advances made in the past decade. Chapter 9 attempts to unify the analysis of secondary double-diffusive instabilities, discussed separately in Chapters 6–8.

The last, but not least, part of this book (Chapters 10–13) explains the role of double-diffusion within the broader context of environmental and physical sciences. Chapter 10 considers the interaction of double-diffusion with the active oceanic environment, perpetually forced by externally driven shears and turbulence. Here we also examine the most common techniques for identifying double-diffusive signatures in field measurements. Chapter 11 highlights the relatively new and widening interest in the climatic and biological consequences of double-diffusive mixing. Non-oceanographic applications, which include advances in astrophysics, geology, chemistry and engineering, are summarized in Chapter 12. We conclude (Chapter 13) by speculating on the prospects and challenges that face our field in years to come. Chapters 10–13 are more descriptive than the rest of the monograph and, in principle, can be read and understood independently. However, readers who may decide to proceed in this manner should be warned that the assessment of the significance of any phenomenon without a firm grasp of its basic physics is fraught with potential pitfalls.

Any systematic and objective review of double-diffusion inevitably conveys a sense of the singular influence of one person whose ideas permeate all theoretical developments in our field. Not only did Melvin Stern discover double-diffusion but – perhaps even more impressively – he remained the subject’s undisputed leader throughout his long and distinguished career. If not fate, then perhaps it was a random stroke of luck that I had Melvin as a graduate adviser and afterward was able to continue our collaboration until his recent passing away. Melvin was a huge inspiration for everyone who worked with him. One realization that came to me upon getting to know Melvin is that science is extremely emotional. It is possible to relate to very abstract concepts on a very personal level. Melvin just loved to do what he did and he was brilliant at it – maybe it is a definition of happiness in a way. I cannot even begin to express how profoundly his ideas influenced my own professional development. I hope that this monograph can reflect, at least to some extent, Stern’s vision of our field and his scientific approach.

I am also deeply grateful to everyone who contributed to this project either directly, by offering valuable comments on the earlier version of the text, or through inspiring discussions: Jason Flanagan, Pascale Garaud, William

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