# DYNAMICAL PROCESSES ON COMPLEX NETWORKS

The availability of large data sets has allowed researchers to uncover complex properties such as large-scale fluctuations and heterogeneities in many networks, leading to the breakdown of standard theoretical frameworks and models. Until recently these systems were considered as haphazard sets of points and connections. Recent advances have generated a vigorous research effort in understanding the effect of complex connectivity patterns on dynamical phenomena. This book presents a comprehensive account of these effects.

A vast number of systems, from the brain to ecosystems, power grids and the Internet, can be represented as large complex networks. This book will interest graduate students and researchers in many disciplines, from physics and statistical mechanics, to mathematical biology and information science. Its modular approach allows readers to readily access the sections of most interest to them, and complicated maths is avoided so the text can be easily followed by non-experts in the subject.

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To Luisa;
To Loulou, Simon, Théo;
To Martina

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# Preface

In the past few years, the study of large networked systems has received a boost from the ever-increasing availability of large data sets and computer power for their storage and manipulation. In particular, mapping projects of the World Wide Web and the physical Internet offered the first chance to study the topology of large complex networks. Gradually, other maps followed describing many networks of practical interest in social science, critical infrastructures, and biology. Indeed, large complex networks arise in a vast number of natural and artificial systems. The brain consists of many interconnected neurons; ecosystems consist of species whose interdependency can be mapped into intricate food webs. Social systems may be represented by graphs describing various interactions among individuals. Large networked infrastructures such as power grids and transportation networks are critical to our modern society. Finally, the living cell is not an exception, its organization and function being the outcome of a complex web of interactions among genes, proteins, and other molecules. In the search for the underlying laws governing the dynamics and evolution of these complex systems, researchers have started the systematic analysis and characterization of their network representations. A central result of these activities is that large-scale networks are generally characterized by complex topologies and very heterogeneous structures. These features usually find their signature in connectivity patterns statistically characterized by heavy tails and large fluctuations, scale-free properties, and non-trivial correlations such as high clustering and hierarchical ordering.

The large size and dynamic nature of complex networks has attracted the attention of the statistical physics community. In this context the statistical physics approach has been exploited as a very convenient strategy because of its deep connection with statistical graph theory and the possibility of characterizing emergent macroscopic phenomena in terms of the dynamical evolution of the basic elements of the system. In addition, the large scale analysis of networks appearing in very

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different fields has provided evidence for the sharing of several asymptotic properties and raised the issue of the supposed emergence in complex networks of general and common organizing principles that go beyond the particulars of the individual system, and are thus amenable to general modeling principles.

The advances in understanding large complex networks have generated increased attention towards the potential implications of their structure for the most important questions concerning the various physical and dynamical processes occurring on top of them. Questions of how pathogens spread in population networks, how blackouts can spread on a nationwide scale, or how efficiently we can search and retrieve data on large information structures are all related to spreading and diffusion phenomena. The resilience and robustness of large infrastructures can be studied by percolation models in which we progressively damage the network. Social behavior may be often modeled through simple dynamical processes and agent-based models. Since the early studies of percolation and spreading processes in complex networks, a theoretical picture has emerged in which many of the standard results were radically altered by the topological fluctuations and the complex features observed in most real-world networks. Indeed, complex properties often imply a virtual infinite heterogeneity of the system and large scale fluctuations extending over several orders of magnitude, generally corresponding to the breakdown of standard theoretical frameworks and models. Therefore, while the definitions of the basic models are expected to remain unchanged, the scientific community has become aware of the need to investigate systematically the impact of the various network characteristics on the basic features of equilibrium and non-equilibrium dynamical processes.

The study of models unfolding on complex networks has generated results of conceptual and practical relevance. The resilience of networks, their vulnerability to attacks, and their synchronization properties are all strongly affected by topological heterogeneities. Consensus formation, disease spreading, and our accessibility to information can benefit from or be impaired by the connectivity pattern of the population or infrastructure we are looking at. Noticeably, all these results have been obtained in different contexts and have relevance across several disciplines including physics, biology, and computer and information science. The purpose of this book is to provide a unified presentation of these results and their impact on our understanding of a wide range of networked systems.

While several techniques and methods have been used in the field, the massive size and heterogeneity of the systems have favored the use of techniques akin to the analysis of non-linear, equilibrium, and non-equilibrium physical systems. As previously mentioned, the statistical physics approach has indeed revealed its convenience and has proven very useful in this context, particularly regarding its ability to shed light on the link between the microscopic dynamical evolution of the basic elements of the system and the emergence of macroscopic phenomena. For this reason we will follow this approach and structure the book according to the general framework provided by statistical physics. In doing this, however, we will make a special effort in defining for each phenomenon or system under study the appropriate language used in the field and offer to the reader a mapping between languages and techniques used in the different disciplines.

For the sake of clarity and readability we have used a modular approach to the material that allows the reader interested in just one phenomenon to refer directly to the corresponding specific chapter. The basic definitions and methods used throughout the book are introduced in the first four chapters that set the stage for the remaining chapters, each one clearly self-contained and addressing a specific class of processes. The material is ordered in a path that goes from equilibrium to non-equilibrium phenomena, in accordance with the general framework of statistical physics and in terms of the homogeneity of the techniques used in each context.

More precisely, we start in Chapter 1 by defining the basics of graph theory and some tools for the statistical characterization and classification of large networks. These tools are used in Chapter 2, where we discuss the complex features which characterize a large number of real-world networks. Chapter 3 is devoted to a brief presentation of the various network modeling techniques and the general approaches used for their analysis. Approximations and computational techniques are discussed as well. Chapter 4 provides an introduction to the basic theoretical concepts and tools needed for the analysis of dynamical processes taking place on networks. All these introductory chapters will allow researchers not familiar with mathematical and computational modeling to get acquainted with the approaches and techniques used in the book.

In Chapter 5, we report the analysis of equilibrium processes and we review the behaviors of basic equilibrium physical models such as the Ising model in complex networks. Chapter 6 addresses the analysis of damage and attack processes in complex networks by mapping those processes into percolation phase transitions. Chapter 7 is devoted to synchronization in coupled systems with complex connectivity patterns. This is a transition chapter in that some of the phenomena analyzed and their respective models can be considered in the equilibrium processes domains while others fall into the non-equilibrium class. The following four chapters of the book are devoted to non-equilibrium processes. One chapter considers search, navigation, and exploration processes of complex networks; the three other chapters concern epidemic spreading, the emergence of social behavior, and traffic avalanche and congestion, respectively. These chapters, therefore, are considering far from equilibrium systems with absorbing states and driven-dissipative

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dynamics. Finally, Chapter 12 is devoted to a discussion of the recent application of network science to biological systems. A final set of convenient appendices are used to detail very technical calculations or discussions in order to make the reading of the main chapters more accessible to non-expert readers. The postface occupies a special place at the end of the book as it expresses our view on the value of complex network science.

We hope that our work will result in a first coherent and comprehensive exposition of the vast research activity concerning dynamical processes in complex networks. The large number of references and research focus areas that find room in this book make us believe that the present volume will be a convenient entry reference to all researchers and students who consider working in this exciting area of interdisciplinary research. Although this book reviews or mentions more than 600 scientific works, it is impossible to discuss in detail all relevant contributions to the field. We have therefore used our author privilege and made choices based on our perception of what is more relevant to the focus and the structure of the present book. This does not imply that work not reported or cited here is less valuable, and we apologize in advance to all the colleagues who feel that their contributions have been overlooked.

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# Abbreviations

| | |
|---|---|
| AS | Autonomous System |
| BA | Barabási–Albert |
| BGP | Border Gateway Protocol |
| CAIDA | Cooperative Association for Internet Data Analysis |
| CPU | central processing unit |
| ER | Erdős–Rényi |
| IATA | International Air Transport Association |
| DIMES | Distributed Internet Measurements and Simulations |
| GIN | giant in-component |
| GOUT | giant out-component |
| GSCC | giant strongly connected component |
| GWCC | giant weakly connected component |
| HOT | Heuristically Optimized Trade-off |
| ICT | Information and communication technology |
| LAN | Local Area Network |
| ME | master equation |
| MF | mean-field |
| P2P | peer-to-peer |
| SCN | scientific collaboration network |
| SI | Susceptible-Infected model |
| SIR | Susceptible-Infected-Removed model |
| SIS | Susceptible-Infected-Susceptible model |
| TTL | time-to-live |
| WAN | worldwide airport network |
| WS | Watts–Strogatz |
| WWW | World Wide Web |