

NONLINEAR CLIMATE DYNAMICS

This book introduces stochastic dynamical systems theory to synthesise our current knowledge of climate variability. Nonlinear processes, such as advection, radiation and turbulent mixing, play a central role in climate variability. These processes can give rise to transition phenomena, associated with tipping or bifurcation points, once external conditions are changed. The theory of dynamical systems provides a systematic way to study these transition phenomena. Its stochastic extension also forms the basis of modern (nonlinear) data analysis techniques, predictability studies and data assimilation methods. Early chapters present the stochastic dynamical systems framework and a hierarchy of climate models to study climate variability. Later chapters analyse phenomena such as the North Atlantic Oscillation, the El Niño/Southern Oscillation, Atlantic Multidecadal Variability, Dansgaard-Oeschger Events, Pleistocene Ice Ages and climate predictability. This book will prove invaluable for graduate students and researchers in climate dynamics, physical oceanography, meteorology and paleoclimatology.

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Cambridge University Press
978-0-521-87917-0 - Nonlinear Climate Dynamics
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CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City
Cambridge University Press
32 Avenue of the Americas, New York, NY 10013-2473, USA
www.cambridge.org
Information on this title: www.cambridge.org/9780521879170

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First published 2013

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data

Dijkstra, Henk A.
Nonlinear climate dynamics / Henk A. Dijkstra, Utrecht University.
pages cm
Includes bibliographical references and index.
ISBN 978-0-521-87917-0 (hardback)
1. Climatology – Statistical methods. 2. Dynamic climatology. I. Title.
QC874.5.D55 2013
551.601'175–dc23 2012044078

ISBN 978-0-521-87917-0 Hardback

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Preface

Dynamical systems theory is an extremely powerful framework for understanding the behavior of complex systems. Its concepts apply to many scientific fields, and hence its language provides a multidisciplinary and unifying communication tool. The theory provides a systematic approach for assessing the sensitivity of a mathematical model of a particular phenomenon to changes in parameters and initial conditions. As such, it finds application in stability problems, transition behavior and predictability studies. In addition, techniques and concepts from dynamical systems theory have led to the development of a diverse set of nonlinear methods of time series analysis.

For many phenomena, existing models cannot resolve all relevant spatial and temporal scales, and hence small-scale features are often represented as ‘noise’. As a result of the increase in computational power, solutions of the resulting stochastic partial differential equations are now within reach. Although stochastic dynamical systems are difficult to deal with, in recent years, the theory of stochastic dynamical systems has matured and is ready to be applied to many scientific areas.

This book developed from a course on climate dynamics that I taught at Colorado State University in 2005 and a course on stochastic climate models that I taught at Utrecht University in 2008. My main motivation in writing this book was to provide both an introduction into stochastic dynamical systems theory and to show the application of these methods to problems in climate dynamics.

The book is, therefore, logically divided into two parts. In the first part, Chapters 2 through 5, introductions to dynamical systems (Chapter 2), stochastic calculus (Chapter 3) and random dynamical systems (Chapter 4) are given. With the level of mathematical detail provided, this material should be accessible to graduate students in climate physics. Chapters 7 through 11 provide a description of how stochastic dynamical systems theory has been used to understand particular phenomena in climate physics. Chapter 6 illustrates the hierarchy of climate models used in the description of climate variability and climate change. Chapter 12 shows the application of dynamical systems theory to predictability studies. The choice of all the material is

quite a personal one and I apologise for certainly having forgotten to cite and discuss results of additional very relevant publications.

In the present scientific world in which everybody can be a ‘Google’ specialist on any topic in fifteen minutes, and where the focus appears to be more and more on concepts that are appealing rather than precise, it is good to know that there are frameworks available that can be used to fit pieces of a complex puzzle. The material in this book forms the basic material for such a framework related to problems in climate variability. I hope it finds its way to the younger generation of climate scientists.

Acknowledgements

The writing of this book has been a pleasure as a result of the interaction with many colleagues in the fields of climate dynamics and dynamical systems.

First, I thank Will de Ruijter (IMAU, Utrecht) for his unfaltering support of my research activities. He and the rest of the faculty, staff, postdocs and students at IMAU have been responsible for creating the environment in which this book could be written. In addition, I thank my colleagues at the Royal Dutch Meteorological Institute, Geert Jan van Oldenborgh and Sybren Drijfhout, for the nice discussions over the years. Special thanks go to Fred Wubs (RUG) for his collaboration in many joint projects and his contributions to the underlying numerical methodology to handle high-dimensional dynamical systems.

Joint work with Michael Ghil (LMD, Paris, France) and Eric Simonnet (INLN, Nice, France) has been very important for the development of the material in this book. Chapter 2 is based on a joint paper that we wrote for the *Handbook of Numerical Analysis* (Vol XIV). Chapter 4 is based on Eric's notes on random dynamical systems, which we discussed extensively during my many enjoyable visits to INLN (for which I thank the CNRS for support). I thank Eric Deleersnijder (Louvain-la-Neuve, Belgium) for inviting me to give several lectures on stochastic dynamical systems, which eventually led to Chapter 3. Most of the book was written during a four-month sabbatical at the Australian National University in 2011. I thank Andy Hogg and Ross Griffiths for a quiet and great time in Canberra and the ARC Centre of Excellence for Climate System Science for the generous support.

I thank members of my current group at IMAU (Elodie Burrillon, Andrea Cimattoribus, Leela Frankcombe, Lisa Hahn-Woernle, Anna von der Heydt, Dewi Le Bars, Maria Rugenstein, Matthijs den Toom, and Jan Viebahn) for providing feedback on early versions of this book. I want to mention in particular Matthijs den Toom and Jan Viebahn for essential contributions and Anna von der Heydt for creating several of the figures used in Chapter 11. Daan Crommelin is thanked for providing very useful

comments on a first version of Chapter 7. I thank many colleagues for providing me with original files for many of the figures in this book (see permissions).

The generous support from the Netherlands Organization for Scientific Research (NWO) over the years is much appreciated. This support has been essential in carrying out the research on the applications of dynamical systems theory in physical oceanography and climate dynamics. The support in supercomputing time on the machines at the Academic Computing Center in Amsterdam (SARA) through the projects from the National Computing Facilities Foundation (NCF) is also much acknowledged.

Matt Lloyd at Cambridge University Press has been very supportive and patient in waiting for the final version of the manuscript. I also thank Amanda O'Connor for going through the text and for providing useful comments on layout and style. Peggy Rote (Aptara, Inc.) did a fantastic job in the copyediting of the book; all remaining errors are my sole responsibility.

Finally, I thank Julia for always being around.