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Insurance data

Generalized linear modeling is a methodology for modeling relationships between variables. It generalizes the classical normal linear model, by relaxing some of its restrictive assumptions, and provides methods for the analysis of non-normal data. The tools date back to the original article by Nelder and Wedderburn (1972) and have since become part of mainstream statistics, used in many diverse areas of application.

This text presents the generalized linear model (GLM) methodology, with applications oriented to data that actuarial analysts are likely to encounter, and the analyses that they are likely required to perform.

With the GLM, the variability in one variable is explained by the changes in one or more other variables. The variable being explained is called the "dependent" or "response" variable, while the variables that are doing the explaining are the "explanatory" variables. In some contexts these are called "risk factors" or "drivers of risk." The model explains the connection between the response and the explanatory variables.

Statistical modeling in general and generalized linear modeling in particular is the art or science of designing, fitting and interpreting a model. A statistical model helps in answering the following types of questions:

- Which explanatory variables are predictive of the response, and what is the appropriate scale for their inclusion in the model?
- Is the variability in the response well explained by the variability in the explanatory variables?
- What is the prediction of the response for given values of the explanatory variables, and what is the precision associated with this prediction?

A statistical model is only as good as the data underlying it. Consequently a good understanding of the data is an essential starting point for modeling. A significant amount of time is spent on cleaning and exploring the data. This chapter discusses different types of insurance data. Methods for the display, exploration and transformation of the data are demonstrated and biases typically encountered are highlighted.

1.1 Introduction

Figure 1.1 displays summaries of insurance data relating to $n = 22\,036$ settled personal injury insurance claims, described on page 14. These claims were reported during the period from July 1989 through to the end of 1999. Claims settled with zero payment are excluded.

The top left panel of Figure 1.1 displays a histogram of the dollar values of the claims. The top right indicates the proportion of cases which are legally represented. The bottom left indicates the proportion of various injury codes as discussed in Section 1.2 below. The bottom right panel is a histogram of settlement delay.

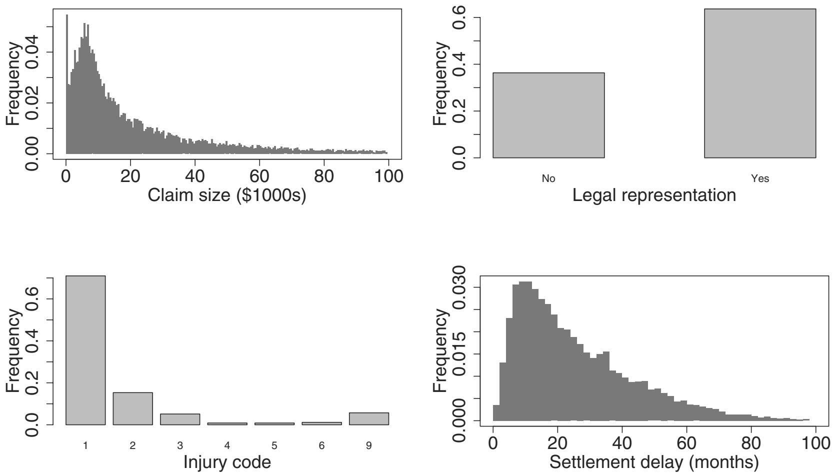

Fig. 1.1. Graphical representation of personal injury insurance data

This data set is typical of those amenable to generalized linear modeling. The aim of statistical modeling is usually to address questions of the following nature:

- What is the relationship between settlement delay and the finalized claim amount?
- Does legal representation have any effect on the dollar value of the claim?
- What is the impact on the dollar value of claims of the level of injury?
- Given a claim has already dragged on for some time and given the level of injury and the fact that it is legally represented, what is the likely outcome of the claim?

Answering such questions is subject to pitfalls and problems. This book aims to point these out and outline useful tools that have been developed to aid in providing answers.

Modeling is not an end in itself, rather the aim is to provide a framework for answering questions of interest. Different models can, and often are, applied to the same data depending on the question of interest. This stresses that modeling is a pragmatic activity and there is no such thing as the "true" model.

Models connect variables, and the art of connecting variables requires an understanding of the nature of the variables. Variables come in different forms: discrete or continuous, nominal, ordinal, categorical, and so on. It is important to distinguish between different types of variables, as the way that they can reasonably enter a model depends on their type. Variables can, and often are, transformed. Part of modeling requires one to consider the appropriate transformations of variables.

1.2 Types of variables

Insurance data is usually organized in a two-way array according to cases and variables. Cases can be policies, claims, individuals or accidents. Variables can be level of injury, sex, dollar cost, whether there is legal representation, and so on. Cases and variables are flexible constructs: a variable in one study forms the cases in another. Variables can be quantitative or qualitative. The data displayed in Figure 1.1 provide an illustration of types of variables often encountered in insurance:

- *Claim amount* is an example of what is commonly regarded as continuous variable even though, practically speaking, it is confined to an integer number of dollars. In this case the variable is skewed to the right. Not indicated on the graphs are a small number of very large claims in excess of \$100 000. The largest claim is around \$4.5 million dollars. Continuous variables are also called "interval" variables to indicate they can take on values anywhere in an interval of the real line.
- *Legal representation* is a categorical variable with two levels "no" or "yes." Variables taking on just two possible values are often coded "0" and "1" and are also called binary, indicator or Bernoulli variables. Binary variables indicate the presence or absence of an attribute, or occurrence or non-occurrence of an event of interest such as a claim or fatality.
- *Injury code* is a categorical variable, also called qualitative. The variable has seven values corresponding to different levels of physical injury: 1–6 and 9. Level 1 indicates the lowest level of injury, 2 the next level and so on up to level 5 which is a catastrophic level of injury, while level 6 indicates death. Level 9 corresponds to an "unknown" or unrecorded level of injury

and hence probably indicates no physical injury. The injury code variable is thus partially ordered, although there are no levels 7 and 8 and level 9 does not conform to the ordering. Categorical variables generally take on one of a discrete set of values which are nominal in nature and need not be ordered. Other types of categorical variables are the type of crash: (non-injury, injury, fatality); or claim type on household insurance: (burglary, storm, other types). When there is a natural ordering in the categories, such as (none, mild, moderate, severe), then the variable is called ordinal.

- The distribution of *settlement delay* is in the final panel. This is another example of a continuous variable, which in practical terms is confined to an integer number of months or days.

Data are often converted to counts or frequencies. Examples of count variables are: number of claims on a class of policy in a year, number of traffic accidents at an intersection in a week, number of children in a family, number of deaths in a population. Count variables are by their nature non-negative integers. They are sometimes expressed as relative frequencies or proportions.

1.3 Data transformations

The panels in Figure 1.2 indicate alternative transformations and displays of the personal injury data:

- **Histogram of log claim size.** The top left panel displays the histogram of log claim size. Compared to the histogram in Figure 1.1 of actual claim size, the logarithm is roughly symmetric and indeed almost normal. Historically normal variables have been easier to model. However generalized linear modeling has been at least partially developed to deal with data that are not normally distributed.
- **Claim size versus settlement delay.** The top right panel does not reveal a clear picture of the relationship between claim sizes and settlement delay. It is expected that larger claims are associated with longer delays since larger claims are often more contentious and difficult to quantify. Whatever the relationship, it is masked by noise.
- **Claim size versus operational time.** The bottom left panel displays claim size versus the percentile rank of the settlement delay. The percentile rank is the percentage of cases that settle faster than the given case. In insurance data analysis the settlement delay percentile rank is called operational time. Thus a claim with operational time 23% means that 23% of claims in the group are settled faster than the given case. Note that both the mean and variability of claim size appear to increase with operational time.
- **Log claim size versus operational time.** The bottom right panel of Figure 1.2 plots log claim size versus operational time. The relationship

between claim and settlement delay is now apparent: log claim size increases virtually linearly with operational time. The log transform has "stabilized the variance." Thus whereas in the bottom left panel the variance appears to increase with the mean and operational time, in the bottom right panel the variance is approximately constant. Variance-stabilizing transformations are further discussed in Section 4.9.

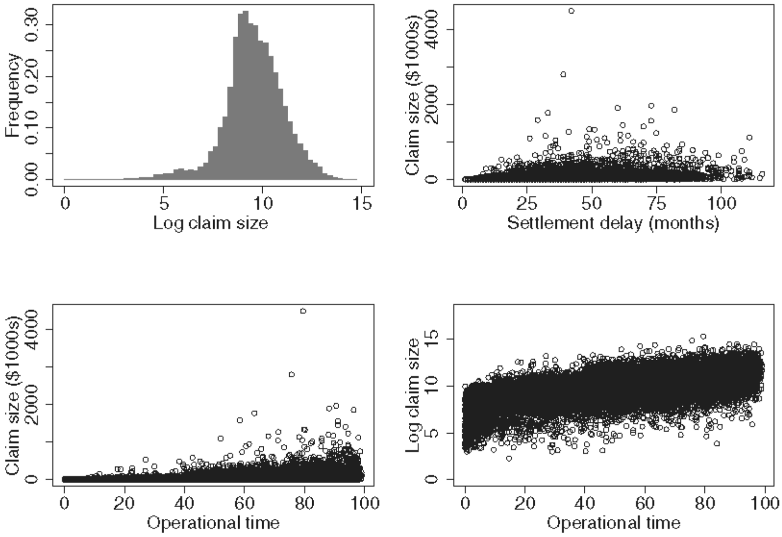

Fig. 1.2. Relationships between variables in personal injury insurance data set

The above examples illustrate ways of transforming a variable. The aim of transformations is to make variables more easily amenable to statistical analysis, and to tease out trends and effects. Commonly used transformations include:

- **Logarithms.** The log transform applies to positive variables. Logs are usually "natural" logs (to the base $\mathrm{e} \approx 2.718$ and denoted $\ln y$). If $x = \log_b(y)$ then $x = \ln(y)/\ln(b)$ and hence logs to different bases are multiples of each other.
- **Powers.** The power transform of a variable y is y^p. For mathematical convenience this is rewritten as $y^{1-p/2}$ for $p \neq 2$ and interpreted as $\ln y$ if $p = 2$. This is known as the "Box–Cox" transform. The case $p = 0$ corresponds to the identity transform, $p = 1$ the square root and $p = 4$ the reciprocal. The transform is often used to stabilize the variance – see Section 4.9.
- **Percentile ranks and quantiles.** The percentile rank of a case is the percentage of cases having a value less than the given case. Thus the percentile

rank depends on the value of the given case as well as all other case values. Percentile ranks are uniformly distributed from 0 to 100. The quantile of a case is the value associated with a given percentile rank. For example the 75% quantile is the value of the case which has percentile rank 75. Quantiles are often called percentiles.

- **z-score.** Given a variable y, the z-score of a case is the number of standard deviations the value of y for the given case is away from the mean. Both the mean and standard deviation are computed from all cases and hence, similar to percentile ranks, z-scores depend on all cases.
- **Logits.** If y is between 0 and 1 then the logit of y is $\ln\{y/(1-y)\}$. Logits lie between minus and plus infinity, and are used to transform a variable in the (0,1) interval to one over the whole real line.

1.4 Data exploration

Data exploration using appropriate graphical displays and tabulations is a first step in model building. It makes for an overall understanding of relationships between variables, and it permits basic checks of the validity and appropriateness of individual data values, the likely direction of relationships and the likely size of model parameters. Data exploration is also used to examine:

(i) relationships between the response and potential explanatory variables; and
(ii) relationships between potential explanatory variables.

The findings of (i) suggest variables or risk factors for the model, and their likely effects on the response. The second point highlights which explanatory variables are associated. This understanding is essential for sensible model building. Strongly related explanatory variables are included in a model with care.

Data displays differ fundamentally, depending on whether the variables are continuous or categorical.

Continuous by continuous. The relationship between two continuous variables is explored with a scatterplot. A scatterplot is sometimes enhanced with the inclusion of a third, categorical, variable using color and/or different symbols. This is illustrated in Figure 1.3, an enhanced version of the bottom right panel of Figure 1.2. Here legal representation is indicated by the color of the plotted points. It is clear that the lower claim sizes tend to be the faster-settled claims without legal representation.

Scatterplot smoothers are useful for uncovering relationships between variables. These are similar in spirit to weighted moving average curves, albeit more sophisticated. Splines are commonly used scatterplot smoothers. They

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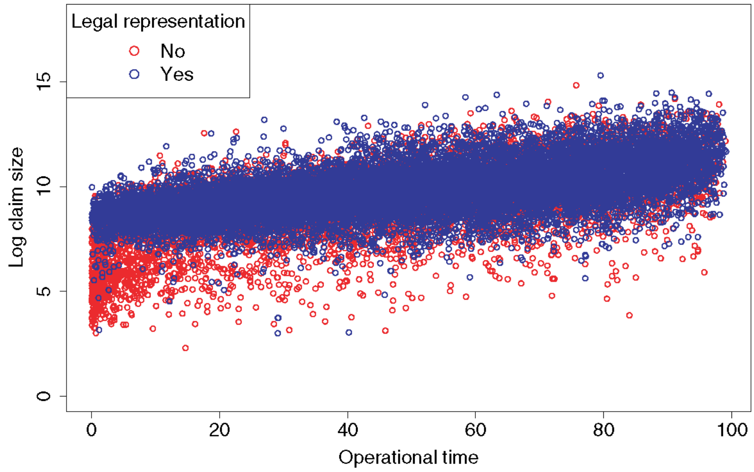


Fig. 1.3. Scatterplot for personal injury data

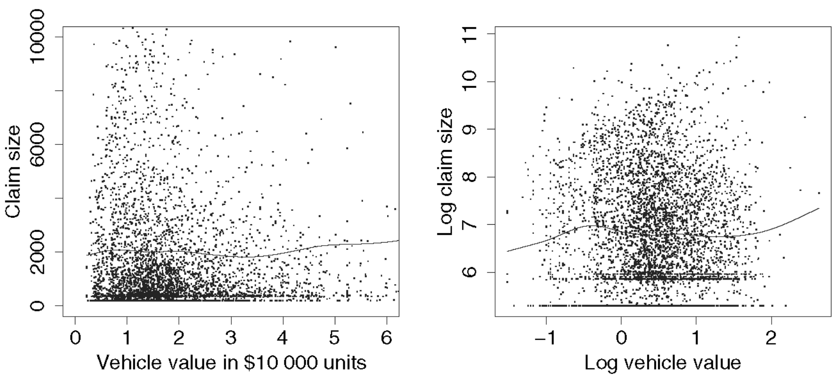
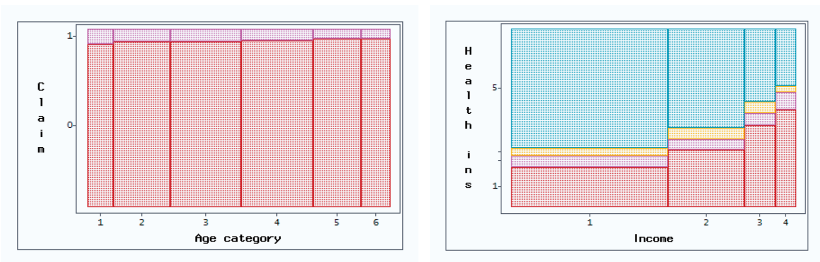


Fig. 1.4. Scatterplots with splines for vehicle insurance data

have a tuning parameter controlling the smoothness of the curve. The point of a scatterplot smoother is to reveal the shape of a possibly nonlinear relationship. The left panel of Figure 1.4 displays claim size plotted against vehicle value, in the vehicle insurance data (described on page 15), with a spline curve superimposed. The right panel shows the scatterplot and spline with both variables log-transformed. Both plots suggest that the relationship between claim size and value is nonlinear. These displays do not indicate the strength or statistical significance of the relationships.

Table 1.1. *Claim by driver's age in vehicle insurance*

	Driver's age category						
Claim	1	2	3	4	5	6	Total
Yes	496 8.6%	932 7.2%	1 113 7.1%	1 104 6.8%	614 5.7%	365 5.6%	4 624 6.8%
No	5 246 91.4%	11 943 92.8%	14 654 92.9%	15 085 93.2%	10 122 94.3%	6 182 94.4%	63 232 93.2%
Total	5 742	12 875	15 767	16 189	10 736	6 547	67 856



Vehicle insurance

Private health insurance

Fig. 1.5. Mosaic plots

Categorical by categorical. A frequency table is the usual means of display when examining the relationship between two categorical variables. Mosaic plots are also useful. A simple example is given in Table 1.1, displaying the occurrence of a claim in the vehicle insurance data tabulated by driver's age category. Column percentages are also shown. The overall percentage of no claims is 93.2%. This percentage increases monotonically from 91.4% for the youngest drivers to 94.4% for the oldest drivers. The effect is shown graphically in the mosaic plot in the left panel of Figure 1.5. The areas of the rectangles are proportional to the frequencies in the corresponding cells in the table, and the column widths are proportional to the square roots of the column frequencies. The relationship of claim occurrence with age is clearly visible.

A more substantial example is the relationship of type of private health insurance with personal income, in the National Health Survey data, described on page 17. The tabulation and mosaic plot are shown in Table 1.2 and the right panel of Figure 1.5, respectively. "Hospital and ancillary" insurance is coded as 1, and is indicated as the red cells on the mosaic plot. The trend for increasing uptake of hospital and ancillary insurance with increasing income level is apparent in the plot.

Table 1.2. *Private health insurance type by income*

Private health insurance type		Income <\$20 000 1	\$20 000 –\$35 000 2	\$35 000 –\$50 000 3	>\$50 000 4	Total
Hospital and ancillary	1	2 178 22.6%	1 534 32.3%	875 45.9%	693 54.8%	5 280 30.1%
Hospital only	2	611 6.3%	269 5.7%	132 6.9%	120 9.5%	1 132 6.5%
Ancillary only	3	397 4.1%	306 6.5%	119 6.2%	46 3.6%	868 4.9%
None	5	6 458 67.0%	2 638 55.6%	780 40.9%	405 32.0%	10 281 58.5%
Total		9 644	4 747	1 906	1 264	17 561

Mosaic plots are less effective when the number of categories is large. In this case, judicious collapsing of categories is helpful. A reference for mosaic plots and other visual displays is Friendly (2000).

Continuous by categorical. Boxplots are appropriate for examining a continuous variable against a categorical variable. The boxplots in Figure 1.6 display claim size against injury code and legal representation for the personal injury data. The left plots are of raw claim sizes: the extreme skewness blurs the relationships. The right plots are of log claim size: the log transform clarifies the effect of injury code. The effect of legal representation is not as obvious, but there is a suggestion that larger claim sizes are associated with legal representation.

Scatterplot smoothers are useful when a binary variable is plotted against a continuous variable. Consider the occurrence of a claim versus vehicle value, in the vehicle insurance data. In Figure 1.7, boxplots of vehicle value (top) and log vehicle value (bottom), by claim occurrence, are on the left. On the right, occurrence of a claim ($1 = \text{yes}$, $0 = \text{no}$) is plotted on the vertical axis, against vehicle value on the horizontal axis, with a scatterplot smoother. Raw vehicle values are used in the top plot and log-transformed values in the bottom plot. In the boxplots, the only discernible difference between vehicle values of those policies which had a claim and those which did not, is that policies with a claim have a smaller variation in vehicle value. The plots on the right are more informative. They show that the probability of a claim is nonlinear, possibly quadratic, with the maximum probability occurring for vehicles valued

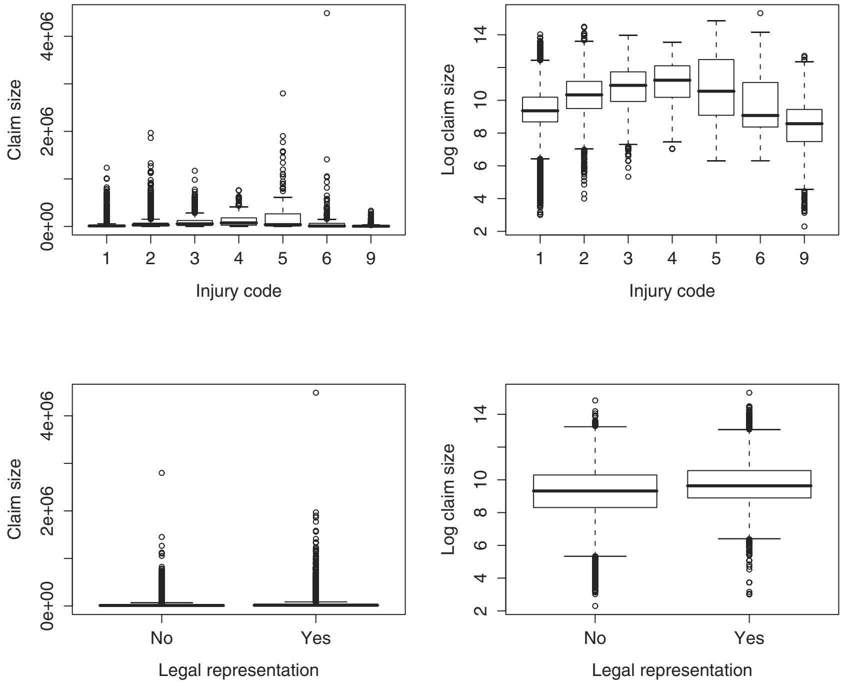


Fig. 1.6. Personal injury claim sizes by injury code and legal representation

around \$40 000. This information is important for formulating a model for the probability of a claim. This is discussed in Section 7.3.

1.5 Grouping and runoff triangles

Cases are often grouped according to one or more categorical variables. For example, the personal injury insurance data may be grouped according to injury code and whether or not there is legal representation. Table 1.3 displays the average log claim sizes for such different groups.

An important form of grouping occurs when claims data is classified according to year of accident and settlement delay. Years are often replaced by months or quarters and the variable of interest is the total number of claims or total amount for each combination. If i denotes the accident year and j the settlement delay, then the matrix with (i, j) entry equal to the total number or amount is called a runoff triangle. Table 1.4 displays the runoff triangle corresponding to the personal injury data. Runoff triangles have a triangular structure since $i + j > n$ is not yet observed, where n is current time.