

Index

- adjacency, 27, 39, 111, 177
- adjoint
 - operator, 167
 - PΔE, 167
- affine transformation, 111
- ansatz, 24, 50, 66, 67, 69, 86, 154–156
- antidifference, 5
- autonomous PΔE, 128
- auxiliary equation, 16

- backward OΔE, 35
- basic characteristic, 153, 185
- basis
 - canonical, 90–91, 92
 - for conservation laws, 167
 - for first integrals, 63
 - for Lie algebra, 88, 92
 - for tangent vectors, 79
- bidirectional Kovalevskaya form, 124–127
- blow-up, 35, 41
- border, 115
 - rightmost, 116, 123, 134
- boundary
 - condition, 96
 - of a stencil, 115
- boundary-value problem, 17, 30
- box, 112, 183

- canonical coordinates, 44–48, 78–82, 92
 - compatible, 45–48, 58, 80
 - prolongation, 45
- Casoratian, 31
- Cauchy–Euler equation, 21
- characteristic
 - basic, 153, 185
 - of a conservation law, 175
 - of a transformation group, 76, 150
 - of Lie symmetries, 44
- characteristic form of a conservation law, 175, 180
- characteristics, equivalent, 153
- closed-form solution, 5
- commutator, 87, 152
- computer algebra, 30, 56, *see also* LREtools,
 - rsolve, Rsolve
- conservation law, 165–166
 - equivalence, 167, 174–181
 - five-point, 173
 - three-point, 170, 173
 - trivial, 166–167
- constraints, difference, 188–190
- convex hull, 115
- cross-ratio equation, 163, 192

- Darboux integrability, 160, 162, 169
- degenerate PΔE, 117
- dependent variable, isolated, 121, 124–127, 163
- derived series, 90
- determining equation
 - first integrals, 62
 - for conservation laws, 170
 - for P_p , 69
- diffeomorphism, 40
- difference
 - cohomology, 166
 - constraints, 188–190
 - equation, *see* ordinary difference equation,
 - partial difference equation
 - operator, 4, 19, 139
 - relation, 186, 190

- differential elimination, 28, 53–58, 66, 69, 71, 80, 149, 154, 170–172
- dihedral group, 39
- discrete
- Heaviside function, 122
 - interval, 112
- discrete Lotka–Volterra equation, *see* dLV equation
- divergence, 165, 182
- dLV equation, 175, 178, 180, 193
- domain, 1, 112, 114
- regular, 117
 - solution, 116, 130
- dpKdV equation, 122, 123, 124, 127, 128, 133, 135, 147, 149, 156, 166, 172, 174, 182, 184, 197
- autonomous, 128, *see also* dpKdV equation
- eigenfunction of a difference operator, 20
- Einstein summation convention, 79
- elimination, *see* differential elimination
- equation
- Cauchy–Euler, 21
 - cross-ratio, 163, 192
 - dLV, 175, 178, 180, 193
 - dpKdV, 122, 123, 124, 127, 128, 133, 135, 147, 149, 156, 166, 172, 174, 182, 184, 197
 - Euler–Lagrange, 181
 - heat, 158, 161, 163
 - lattice KdV, 196
 - Lyness, 73–74
 - Riccati, 23, 24, 32, 33, 61, 67, 72, 75, 104
 - Toda-type, 137, 182, 184, 192
 - Todd’s, 74
 - wave, 130, 139, 169
- equivalent
- characteristics, 153
 - conservation laws, 167, 174–181
 - symmetries, 102, 131
- Euclidean algorithm, 135
- Euler operator, 99, 176–177
- Euler–Lagrange equation, 96, 181
- incompatible, 194
- even projector, 94
- evolutionary symmetry, 134
- exponential
- generating function, 32
 - of a generator, 79
- factorization, 20–24, 139–142
- Fibonacci number, 32
- fibre, 109, 128, 129, 131, 151
- representative, 131, 134
- fibre-preserving transformation, 113
- first integrals, 62–70, 101, 167
- functionally independent, 64, 68, 75
 - of a PΔE, 169
- forward
- difference operator, 4
 - OΔE, 35
 - shift operator, 18
- Forward Euler discretization, 36
- functional independence, 64, 68, 75
- Fundamental Theorem of Difference Calculus, 4
- gamma function, 10
- gauge symmetry, 188
- general linear group, 112
- generalized symmetry, 128–131, 151–156, 183
- generating function, 30, 32
- generator, group, 38–39
- generic initial conditions, 122
- geometric integration, ix, xii, 71
- Goursat problem, 127
- graph, 83, 116, 130
- group, 38
- harmonic numbers, 7
- heat equation, 158, 161, 163
- Heaviside function, 122
- homogeneous, linear PΔE, 139, 143–145, 158, 167
- ideal, 90
- identity
- element, 38
 - map, 38, 40, 41, 108
 - operator, 18
- incompatible Euler–Lagrange equations, 194
- indefinite sum, 5
- index notation, 138
- infinitesimal generator, 79, 86
- initial
- conditions, 4, 65, 84, 121–127
 - lines, 124–128
 - variables, 127, 152
- initial-value problem, 35, 36, 71, 77, 121–123
- recursively computable, 122–123
- inner product, 96

- integrability conditions, 69
- integrating factor, 11
- interior of a stencil, 115
- interval, discrete, 112
- invariant, 80
 - point, 77, 80
 - solution, 82, 148, 157–159
 - stretch, 25, 28
- isolated dependent variable, 121, 124–127, 163
- IVP, *see* initial-value problem
- Jacobi identity, 88, 151, 153
- Kovalevskaya form, 121, 123, 127, 152, 166, 169, 176, 179, 180, 185
 - bidirectional, 124–127
- ℓ^2 inner product, 96, 167
- Lagrange multipliers, 98, 189
- Lagrange's method, 143–144
- Lagrangian, 96
 - augmented, 189
 - null, 98, 99, 182
- lattice, 109
 - map, 110
 - rigid, 112, 133
 - point, 109
 - product, 112–113
 - symmetry, 131–133
 - transformation, 114
- lattice KdV equation, 196
- leading
 - term, 121
 - vertex, 121
- Leibniz rule (modified), 7, 19
- Lie algebra, 89–95, 153
 - canonical basis, 90
 - simple, 90
 - solvable, 90
- Lie group
 - canonical coordinates, 78–82
 - infinitesimal generator, 79
 - local, 41
 - multi-parameter, 87–89
 - one-parameter, 41
- Lie point transformation, *see* transformation, Lie point
- Lie subalgebra, 89
- line of ordered points, 111
- linear OΔE, 1
 - constant-coefficient, 15–18
 - factorization, 18–24
 - first-order, 9–12
 - homogeneous, 3
 - inhomogeneous, 3
 - orthogonal polynomial solutions, 25
 - reduction of order, 12–15, 59
 - special function solutions, 25
 - standard form, 2
 - transformation, 24–30
- linear PΔE, 139, 143–145, 158
- linear superposition, 3, 143, 145
- linearizable PΔE, 159–160
- linearized symmetry condition (LSC), 49, 68, 73, 83, 86, 149, 150, 183
 - for systems of OΔEs, 80
- LREtools (Maple), 30
- LSC, *see* linearized symmetry condition
- Lyness equation, 71, 73–74
- Möbius transformation, 105
- mask, *see* stencil
- mastersymmetry, 156, 158, 162
- maximal rank condition, 149, 180, 193
- McMillan map, 71, 74, 99, 102
- minimization of order, 118–121, 124–126
- Noether's Second Theorem, 186–188
- Noether's Theorem, 95, 183–185
 - null Lagrangian, 98, 99, 182
- odd projector, 94
- OΔE, *see* ordinary difference equation
- one-parameter local Lie group, 41
- orbit, 76–77
- order
 - minimization, 118–121, 124–126
 - of a PΔE, 115
 - of an OΔE, 1
- ordinary difference equation (OΔE), 1, 35, 80, *see also* linear OΔE
 - backward, 35
 - forward, 35
- orthogonal polynomials, 25
- partial difference equation (PΔE), 114, 138, 149
 - autonomous, 128
 - factorizable, 139–142
 - linear, 139, 143–145, 158
 - linearizable, 159–160

- partitioned ODE, 93
- path, 177
- PΔE, *see* partial difference equation
- point
 - reachable, 125
 - symmetry, 29
 - transformation, 41
 - Lie, 42, 76–80, 150
- preservation of an object, 108
- principal variable, 176
- product
 - lattice, 112–113
 - operator, 11
 - rule, 19
- projector, even/odd, 94, 121
- prolongation, 62, 83, 114, 128, 151
 - canonical coordinates, 45
 - formula, 44
- prolonged space, 83–85

- QRT map, 71, 147

- reachable point, 125
- recurrence relation, 1, 25
- recursively computable IVP, 122–123
- reduction of order, 12–15, 58–61
 - by first integrals, 64
 - multiple, 91–92
- reflection, 28, 38
- regular
 - domain, 2, 35, 117
 - point, 1, 35
- relation
 - difference, 186, 190
 - group, 38–39
- Riccati equation, 23, 24, 32, 33, 61, 67, 72, 75, 104
- rightmost
 - border, 116, 123, 134
 - vertex, 117, 120–121
- rigid lattice map, 39, 112, 133
- root of a conservation law, 178–181
- rotation, 38, 112
- `rsolve` (Maple), 30
- `Rsolve` (Mathematica), 30

- scalar
 - OΔE, 1
 - PΔE, 114
 - product, 112
- scaling, 29, 40

- separation of variables, 144–145
- shadow
 - diagram, 125
 - of a vertex, 125
- shear transformation, 112
- shift operator, 138
 - forward, 18
 - restricted, 62, 176
- simple Lie algebra, 90
- singular point, 1, 35, 117
- singularity, 122
 - confinement, 122
- solution domain, 116, 130
- solvable Lie algebra, 90
- special functions, 25
- special linear group, 112
- staircase, 124
- stencil, 114, 166
 - at a point, 116
 - rightmost point, 115
 - translation, 114
- stretch invariant, 25, 28
- structure constants, 88
- subsidiary variable, 176
- sum, indefinite, 5
- summation, 5–9
 - by parts, 7, 101
 - conditions, 177
 - convention, 79
 - operator, 8
- symmetries, equivalent, 102
- symmetry, 29, 37
 - condition, 45, 52, 80, 128
 - linearized, *see* linearized symmetry condition
- difference equation, 40
- differential equation, 40
- dynamical, 85–86
- evolutionary, 134
- fibre representative, 134
- generalized, 128–131, 151–156, 183
- generator, 38–39
- group, 38
- lattice, 131–133
- Lie, 42
- Lie point, 43, 80, 82, 150
- trivial, 42–43, 129–131, 153
- variational, 99–101, 183

- tangent vector field, 77–79
- Toda-type equation, 137, 182, 184, 192

Cambridge University Press

978-0-521-87852-4 - Difference Equations by Differential Equation Methods

Peter E. Hydon

Index

[More information](#)

206

Todd's equation, 74
 total space, 109
 trailing vertex, 124
 transformation, 10, 42, 108–114
 active view, 108
 affine, 111
 fibre-preserving, 27, 113
 identity, 38
 lattice, 114
 Lie point, 42, 76–80, 150
 linearizing, 72
 of a linear OΔE, 25–30
 passive view, 108
 point, 41, 83
 reflection, 28, 38
 rigid, 39
 rotation, 38, 112
 scaling, 29, 40
 shear, 112
 stretch, 28
 translation, 27, 42–43
 translation, of stencil, 114
 trivial
 conservation law, 166–167

Index

 symmetry, 42–43, 129–131, 153
 two-point boundary-value problem, 17

 unimodular matrix, 112
 unit step function, 122

 valid lattice map, 110
 variation of parameters, 34
 variational
 complex, 191
 integrator, 191
 problem, 95
 symmetry, 99–101, 183
 vertex, 115
 leading, 121
 shadow, 125
 trailing, 124
 visibility, 110

 wave equation, 130, 139, 169
 wavelike solutions of PΔEs, 145–148
 weighted average discretization, 158, 161

 z-transform, 30