1 Nomenclature

Is a rose a rose? To crudely paraphrase Shakespeare, "What's in a name?" Can the same flower be described and characterized differently when viewed from different perspectives? Different scientific and technical fields tend to develop variations of the nomenclature for the same physical quantities. Within each field there is generally a degree of consistency, but interdisciplinary topics that bring together scientists, engineers, and practitioners from disparate fields often face inconsistencies in the symbols, units, and terminology utilized for quantitative analysis. The field of biomedical optics has not been immune to this problem, and publications in the field have invoked a variety of terms and symbols. In some cases, the same terms and symbols, as used in different fields, carry subtle but crucial differences in their meanings. An important example is the different meaning of the term *intensity* in physics (power per unit area), in radiometry (power per unit solid angle), and in heat transfer (power per unit area, per unit solid angle). Another example is that of the molar extinction coefficient for optical absorption of hemoglobin, which is typically defined using the logarithm to base-10 in chemistry and biology, or to base-e in physics, and may refer to one functional heme group or to the full molecule (four heme groups), depending on the specific physiological or biochemical characterizations. This may lead to a mismatch by as much as a factor of 9.2 (i.e., $4 \times \ln(10)$) among the numerical values reported for the molar extinction coefficient of hemoglobin according to different conventions. For a long time, the three fields that most commonly make use of and describe (and teach) the methods of quantitative optical measurements have been physics, astronomy, and electrical engineering (or its subfield, optical engineering), although chemistry and biology also make use of optical characterization techniques. Because biomedical optics is a broad interdisciplinary field, which is based on contributions from researchers in a variety of specialty areas, it is important that a common language be used to describe and characterize its key quantitative parameters.

In this chapter, we define the nomenclature used in this book for the quantities that describe the optical radiation field and its interaction with biological tissue. In cases of ambiguity across disciplines, we have adopted the notation of physics rather than radiometry, such that, for example, we define the intensity (I) as the

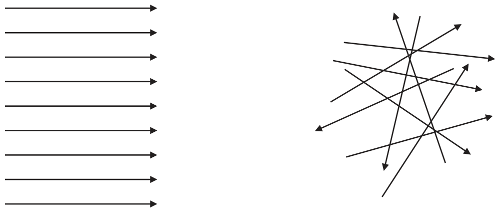

Figure 1.1
(a) A uniform, collimated optical field propagating in a vacuum or non-scattering medium. (b) Light strongly scattered in a turbid medium, featuring a net energy flow from left to right.

power per unit area incident on a surface, and we identify the power per unit solid angle emitted by a light source as radiant angular intensity ($\mathcal{J}$). In most cases, the physical dimensions of a parameter provide clear indications of its meaning, but this is not always the case: for example, intensity and fluence rate both have dimensions of power per unit area, but they represent different measures of an optical field. This is discussed below, and the definitions listed in this chapter provide a rigorous description of the main quantities and parameters used in biomedical optics.

1.1. Describing the optical radiation field and its interactions with tissue

Sometimes the units alone are not enough to clearly define a physical parameter because the units alone do not fully convey complex directional information, even when the parameters are expressed as vectors. Thus, while it is important to be cognizant of whether the quantity is a scalar or vector parameter, there may be more to the story. Thus, for vector parameters, a simple unit vector may provide information about the direction of a net flow of energy (or photons), for example, yet it may not provide all the necessary information to properly describe the flow; information about a *distribution* of directions may be needed. Consider, for example the two cases represented in Figure 1.1. Both optical fields illustrated in panels (a) and (b) of Figure 1.1 represent a net flux of optical energy from left to right, a quantity often referred to in physics in terms of the Poynting vector (named after the British physicist John Henry Poynting [1852–1914]).

Figure 1.1(a) represents the simple propagation, from left to right, of a collimated optical field in a vacuum or non-scattering medium. Figure 1.1(b) represents a field of diffusely scattered light in a turbid medium, with photons propagating in multiple directions; this field, nonetheless, has a net flow of optical energy from left to right as in Figure 1.1(a). Although these two figures represent optical fields with a net flow of energy in the same direction, the first case can be fully described by a single vector quantity, whereas the latter case would require at least two measures to properly describe the optical field: a vector quantity (similar to the first case) representing the net flux of energy, plus a scalar quantity to indicate what the total optical exposure would be, at a given point within the turbid medium, as a consequence of photons impinging from all directions.

The field of physics tends to be rigorous about such distinctions – consider, for example the clear distinctions among *speed*, *velocity,* and *velocity distribution* – but newly developing interdisciplinary fields, for example those related to the biological sciences, are often less rigorous about units and nomenclature, especially when reporting physical quantities. As a consequence, it is not uncommon to hear the terms *power* and *intensity* utilized interchangeably in the field of radiology for describing the emission properties of an X-ray source.

In 1996 the American Association of Physicists in Medicine, under the auspices of the American Institute of Physics, published a reference document titled "Recommended Nomenclature for Physical Quantities in Medical Applications of Light" (Hetzel et al., 1996). This document recognized the disparate and inconsistent usage in the published literature, and proposed a codification of the definitions. The International Organization for Standardization (ISO) has also been publishing standard nomenclature and definitions for quantities associated with light and electromagnetic radiation (ISO, 2008). What follows below is a listing of the principal optical parameters (and associated units) of relevance in biomedical optics, attempting to be consistent with those documents, while nonetheless invoking some deviations that we have introduced for some of the symbols for the purpose of clarity and consistency with the majority of the biomedical optics literature. For a few of the parameters describing the optical radiation field itself, we also note whether the parameter is conserved in a non-dissipative system, and explain why. (Such conservation need not be limited to a closed system. This can be important for understanding the efficiency of optical system designs and methods for detection.) The modern metric system (or International System of Units, abbreviated SI from the French *Système International d'Unités*) is mainly used in this book, although centimeters (cm), grams (g), and seconds (s) (CGS system and the related Gaussian system of units, named after the German mathematician Carl Friedrich Gauss [1777–1855]) will sometimes take preference over meters (m), kilograms (kg), and seconds (s) (MKS units, the basis for the SI system), for convenience or because the associated dimensions are closer to those of observed quantities.

The most common terms used to describe the radiant field, the flow of optical energy, and the interactions of light with tissue and biological media are listed below, including indications of the preferences utilized in this book.

1.2 Quantities describing the optical radiation field

Electric field: $\mathbf{E}(\mathbf{r}, t)$ – SI units: V/m (V: volt, named after the Italian physicist Alessandro Volta [1745–1827]).

Magnetic field: $\mathbf{H}(\mathbf{r}, t)$ – SI units: A/m (A: ampere, named after the French physicist André-Marie Ampère [1775–1836]).

Magnetic induction: $\mathbf{B}(\mathbf{r}, t)$ – SI units: T (T: tesla: named after the Serbian-American engineer Nikola Tesla [1856–1943]).

The *electric field* (**E**) and the *magnetic field* (**H**), or *magnetic induction* (**B**), vectors are the basic vector quantities that describe electromagnetic radiation. **E** and **H** are not independent as the electric and magnetic fields are linked by Maxwell's equations, named after the Scottish physicist James Clerk Maxwell [1831–1879]. The electric and magnetic field vectors are orthogonal to each other and to the direction of propagation of the optical wave. The square of the electric field's magnitude and the square of the magnetic field's magnitude are both proportional to the energy density (and the intensity) associated with the optical radiation.

Radiant energy: $Q(\Delta t)$ – SI units: J (joule, named after the English physicist James Prescott Joule [1818–1889]).

Radiant energy describes the emission or delivery of optical energy. $Q(\Delta t)$ is the energy of an optical field that has been emitted by a source over a time span $\Delta t = t_2 - t_1$, or that is delivered to, reflected from, transmitted through, or absorbed by a medium over time Δt. $Q(\Delta t)$ is related to the radiant power $P(t)$, defined below, by the relationship:

$$Q(\Delta t) = \int_{t_1}^{t_2} P(t)dt. \tag{1.1}$$

The radiant energy can be expressed in terms of the number of photons (emitted, delivered, reflected, transmitted, or absorbed over time Δt), which are the energy quanta of the electromagnetic field, each carrying an energy hf, where h is Planck's constant ($h = 6.626 \times 10^{-34}$ J s) (named after the German physicist Max Planck [1858–1947], who received the Nobel Prize in physics in 1918) and f is the frequency of light. In this text, we will predominantly use the angular frequency, $\omega = 2\pi f$, for the optical field, which leads to the photon energy being represented as $\hbar\omega$, where $\hbar \equiv h/(2\pi)$ is the reduced Planck's constant. Therefore, the radiant energy associated with $N_{\Delta t}$ photons emitted, delivered, reflected, transmitted, or absorbed over time Δt at frequency f is given by:

$$Q(\Delta t) = N_{\Delta t} hf = N_{\Delta t}\hbar\omega. \tag{1.2}$$

In the absence of dissipative effects (conversion of optical energy to other forms, such as heat) total optical energy is conserved.

Radiant power: $P(t)$ – SI units: W (watt, named after the Scottish engineer James Watt [1736–1819]).

The *radiant power*, $P(t)$, is the radiant energy emitted, delivered, reflected, transmitted, or absorbed per unit time. The *average power* over time $\Delta t = t_2 - t_1$ is:

$$\langle P \rangle_{\Delta t} = \frac{1}{\Delta t} \int_{t_1}^{t_2} P(t) dt = \frac{Q(\Delta t)}{\Delta t}, \tag{1.3}$$

while the instantaneous power at time t is:

$$P(t) = \frac{dQ}{dt}, \tag{1.4}$$

where dQ is the infinitesimal radiant energy over time dt about t. The average power associated with $N_{\Delta t}$ photons emitted, delivered, reflected, transmitted, or absorbed over time Δt at angular frequency ω is:

$$\langle P \rangle_{\Delta t} = \frac{N_{\Delta t} \hbar \omega}{\Delta t}. \tag{1.5}$$

No directional information is conveyed by the radiant power, which is thus a scalar quantity. Radiant energy flux and radiant power are synonymous, and both terms are found in the literature. We make preferential use of the terms *power* or *photon rate* over *energy flux*, following the physics tradition. In the absence of dissipation, the optical radiant power is conserved for continuous sources and can only be altered for pulsed sources if the optical pulses are compressed or expanded temporally. Moreover, the average radiant power is conserved whenever the observation time, Δt, is longer than the emission time of the source.

Radiant angular intensity: $\mathcal{J}(\hat{\Omega})$ – SI units: W/sr (sr: steradian, SI unit for solid angle, from the Greek *stereos* [solid] and the Latin *radius* [ray]).

The *radiant angular intensity* describes the power per unit solid angle emitted by a light source along a given direction $\hat{\Omega}$. In the field of radiometry, the word "intensity" is used to describe the quantity defined here. By contrast, in physics, intensity is defined as the power incident on a surface per unit area. Because we have decided to adopt the latter definition of intensity, we have added the "angular" specifier here, to indicate the power emitted per unit solid angle as the "angular intensity." If $dP(\hat{\Omega})$ is the infinitesimal radiant power emitted within the infinitesimal solid angle $d\Omega$ about $\hat{\boldsymbol{\Omega}}$, then the radiant angular intensity is defined as:

$$\mathcal{J}(\hat{\boldsymbol{\Omega}}) = \frac{dP(\hat{\boldsymbol{\Omega}})}{d\Omega}. \tag{1.6}$$

The radiant angular intensity is not used in this book, but the reader is cautioned to take note of the actual units when the term "intensity" appears in the literature. Sometimes the term *brightness* is used for the angular intensity, although it can lead to confusion because without including the factor of area^{-1} (as in the definition of *intensity* below), the quantity is not conserved. It is important to note that even in a closed system this quantity is not conserved, because the value can be changed by simply using a lens to alter the divergence angle of the light.

Radiance: $L(\hat{\boldsymbol{\Omega}})$ – SI units: W/(m^2-sr).

The *radiance* emitted by a light source (or traveling through a medium) along direction $\hat{\boldsymbol{\Omega}}$ is defined as the radiant angular intensity per unit area perpendicular to the direction $\hat{\boldsymbol{\Omega}}$. If $d\mathcal{J}$ is the infinitesimal radiant angular intensity transmitted through a material (or emitted by a source) over the infinitesimal solid angle $d\Omega$ about $\hat{\boldsymbol{\Omega}}$ and over an infinitesimal surface area dA that is normal to the unit vector $\hat{\mathbf{n}}$, then the radiance is defined as:

$$L(\hat{\boldsymbol{\Omega}}) = \frac{d\mathcal{J}}{dA\hat{\boldsymbol{\Omega}} \cdot \hat{\mathbf{n}}} = \frac{d^2P}{d\Omega dA\hat{\boldsymbol{\Omega}} \cdot \hat{\mathbf{n}}}. \tag{1.7}$$

There is a long tradition in the fields of astronomy and physics of the term *brightness* for this parameter, and the term *specific intensity* can also be found in the literature in place of radiance. Directional information is indicated by the unit vector along the direction considered ($\hat{\boldsymbol{\Omega}}$), and the radiance may be expressed as the vector $\mathbf{L} = L\hat{\boldsymbol{\Omega}}$ to indicate optical flux along direction $\hat{\boldsymbol{\Omega}}$.

Radiance is often the most important parameter to use in describing the emission properties of a light source, especially for tissue spectroscopy. It is important to understand how the radiance of a source governs the amount of light that gets through an optical system, a spectrometer, etc. For example: this parameter is the only true measure that allows one to determine how much light can be coupled from a light source into an optical fiber. This is because both the divergence angle *and* the effective area of the emitting surface are invoked. Thus, lasers generally feature a much higher radiance than incoherent sources, even if incoherent sources may generate a higher optical power. As such, in the absence of dissipation (or temporal manipulation of pulsed light), radiance is conserved, because it is a fundamental property of the emitted field that cannot be altered (increased or diminished) by use of non-dissipative optical manipulations (e.g., lenses, mirrors, etc.). It is interesting to note that if one attempts to increase the power per unit area (or intensity, as defined below) of a beam of light by focusing it (say, with a lens) down to a smaller area, this is inevitably accompanied by a commensurate increase in the solid angle of the converging beam, such that the radiance remains unchanged. Texts on spectroscopy often refer to this phenomenon as "conservation of *étendue*," after the French word meaning *extent* or *scope*.

Radiant exposure: $H(\Delta t)$ – SI units: J/m^2.

The *radiant exposure* is a useful parameter to describe the optical energy delivered per unit area on a surface, and is sometimes referred to as incident energy, exposure, or absorbed energy. If A is the surface area to which a radiant energy $Q(\Delta t)$ is delivered, then the radiant exposure is defined as:

$$H(\Delta t) = \frac{Q(\Delta t)}{A}. \tag{1.8}$$

$H(\Delta t)$ is related to the intensity $I(t)$, defined below, by the relationship:

$$H(\Delta t) = \int_{t_1}^{t_2} I(t)dt. \tag{1.9}$$

Intensity: $I(t)$ – SI units: W/m^2.

The *intensity* describes the amount of optical energy delivered per unit time, per unit area on a surface. In radiometry, the term *irradiance* is used for this parameter, but here we follow the physics nomenclature that is also commonly adopted in biomedical optics. If A is the surface area to which a radiant power $P(t)$ is delivered, then the intensity is defined as:

$$I(t) = \frac{P(t)}{A}. \tag{1.10}$$

The average intensity over time $\Delta t = t_2 - t_1$ is:

$$\langle I \rangle_{\Delta t} = \frac{1}{\Delta t} \int_{t_1}^{t_2} I(t)dt = \frac{H(\Delta t)}{\Delta t}, \tag{1.11}$$

and the instantaneous intensity at time t is:

$$I(t) = \frac{dH}{dt}, \tag{1.12}$$

where dH is the infinitesimal radiant exposure over time dt about t. For a surface (actual or conceptual) normal to a unit vector $\hat{\mathbf{n}}$, the intensity resulting from all directions having positive ($I_{+\hat{\mathbf{n}}}$) or negative ($I_{-\hat{\mathbf{n}}}$) components along $\hat{\mathbf{n}}$ is given by the following angular integrals involving the radiance $L(\hat{\Omega})$:

$$I_{+\hat{\mathbf{n}}} = \int\limits_{\hat{\boldsymbol{\Omega}}\cdot\hat{\mathbf{n}}>0} L(\hat{\boldsymbol{\Omega}})\hat{\boldsymbol{\Omega}} \cdot \hat{\mathbf{n}} d\Omega, \tag{1.13}$$

$$I_{-\hat{\mathbf{n}}} = \int\limits_{\hat{\boldsymbol{\Omega}}\cdot\hat{\mathbf{n}}<0} L(\hat{\boldsymbol{\Omega}})\hat{\boldsymbol{\Omega}} \cdot (-\hat{\mathbf{n}}) d\Omega. \tag{1.14}$$

Like radiant angular intensity, in a non-dissipative system the intensity can easily be altered with a lens, and thus is not conserved.

Fluence rate: $\phi(\mathbf{r}, t)$ – SI units: W/m^2.

The concept of *fluence rate* is particularly relevant in biomedical optics. It provides a measure of the optical energy per unit time, per unit area, incident from *any* direction within optically turbid media (such as most biological tissues) as opposed to the case of surface illumination (in which case the intensity is the proper parameter). Inside optically turbid media, generally, optical fields (or photons) propagate along directions covering the entire solid angle. Therefore, to obtain a measure of the power per unit area about a given point $\mathbf{r}$, it is appropriate to consider the optical energy per unit time that reaches an infinitesimal area about the point from all possible directions. Sometimes, the term *total exposure rate* is used for the fluence rate. Because the radiance $L(\hat{\boldsymbol{\Omega}})$ describes the optical power per unit area propagating along direction $\hat{\boldsymbol{\Omega}}$, it is natural to define the fluence rate by integrating $L(\hat{\boldsymbol{\Omega}})$ over the entire solid angle (4π) as follows:

$$\phi(\mathbf{r}, t) = \int\limits_{4\pi} L(\mathbf{r}, t, \hat{\boldsymbol{\Omega}}) d\Omega. \tag{1.15}$$

It is important to understand the difference between the integral in Eq. (1.15) and those in Eqs. (1.13) and (1.14). First, the integral in Eq. (1.15) is carried out over the entire solid angle 4π, whereas those in Eqs. (1.13) and (1.14) are carried out over a half-space solid angle of 2π. Second, the integrands in Eqs. (1.13) and (1.14) contain the additional factor $\hat{\boldsymbol{\Omega}} \cdot \hat{\mathbf{n}}$ or $\hat{\boldsymbol{\Omega}} \cdot (-\hat{\mathbf{n}})$. This means that while $I_{+\hat{\mathbf{n}}}$ and $I_{-\hat{\mathbf{n}}}$ provide a power per unit area associated with a flat surface element normal to $\hat{\mathbf{n}}$, ϕ provides a power per unit area associated with the surface of an infinitesimal sphere centered at position $\mathbf{r}$. This is illustrated in Figure 1.2.

Fluence: $\psi(\mathbf{r}, \Delta t)$ – SI units: J/m^2.

The *fluence* extends the concept of fluence rate to describe the optical energy delivered per unit area over a time interval $\Delta t = t_2 - t_1$ from *all* directions about a given point $\mathbf{r}$ within an optically turbid medium. Sometimes, the symbol H_0 is

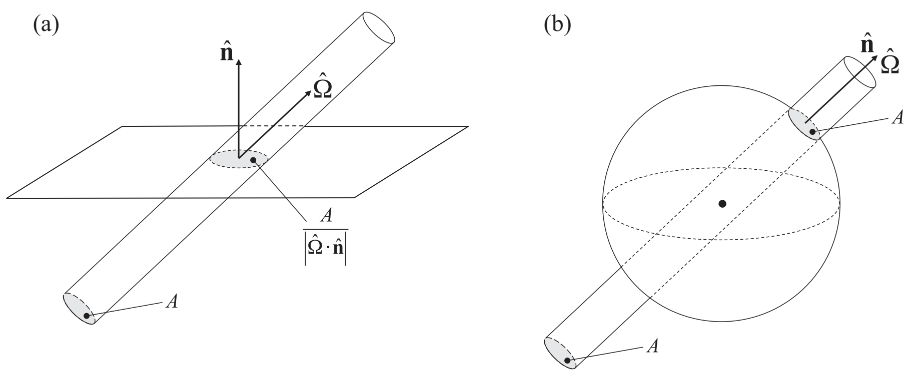

Figure 1.2

To translate the radiance L along direction $\hat{\mathbf{\Omega}}$ (power per unit area normal to $\hat{\mathbf{\Omega}}$, per unit solid angle) into the intensity (power per unit area) on a surface, one needs to take into account the relationship between the area of a general surface element and the area A of the corresponding surface element normal to $\hat{\mathbf{\Omega}}$. (a) In the case of a planar surface characterized by a normal unit vector $\hat{\mathbf{n}}$, the area of the surface element is larger than A by a factor $1/|\hat{\mathbf{\Omega}}\cdot\hat{\mathbf{n}}|$ so that the intensity on the surface over solid angle $d\Omega$ about $\hat{\mathbf{\Omega}}$ is $L(\hat{\mathbf{\Omega}})\hat{\mathbf{\Omega}}\cdot\hat{\mathbf{n}}d\Omega$. (b) In the case of a sphere for which $\hat{\mathbf{\Omega}}$ is a radial direction, the area of the surface element is equal to A so that the intensity on the surface over solid angle $d\Omega$ about $\hat{\mathbf{\Omega}}$ is simply $L(\hat{\mathbf{\Omega}})d\Omega$.

also used to indicate that the fluence is a radiant exposure (H) associated with light propagating along all possible directions, or radii of an infinitesimal sphere (suggested by the subscript 0) around the point considered. The fluence is given by a time integral of the fluence rate as follows:

$$\psi(\mathbf{r}, \Delta t) = \int_{t_1}^{t_2} \phi(\mathbf{r}, t)dt. \tag{1.16}$$

An alternative term for the fluence is *total exposure*.

Angular energy density: $u(\mathbf{r}, \hat{\mathbf{\Omega}}, t)$ – SI units: J/(m^3-sr).

The *angular energy density* $u(\mathbf{r}, \hat{\mathbf{\Omega}}, t)$ is defined as the energy per unit volume, per unit solid angle that is propagating about direction $\hat{\mathbf{\Omega}}$, position $\mathbf{r}$, and time t. It is related to the radiance by the relationship:

$$u(\mathbf{r}, \hat{\mathbf{\Omega}}, t) = \frac{L(\mathbf{r}, \hat{\mathbf{\Omega}}, t)}{c_n}, \tag{1.17}$$

where c_n is the speed of light in the medium, given by c/n, with c speed of light in vacuum and n index of refraction of the medium. The angular photon density, defined as the number of photons per unit volume, per unit solid angle traveling along direction $\hat{\mathbf{\Omega}}$ can be indicated as $u_N(\mathbf{r}, \hat{\mathbf{\Omega}}, t)$.

Radiant energy density: $U(\mathbf{r}, t)$ – SI units: J/m^3.

This is a volume *energy density* and it is important in describing the spatial distribution of the optical energy within optically turbid media such as most tissues. It is defined as the radiant energy per unit volume, considering all directions of light propagation. Therefore, it can be defined either by the fluence rate $\phi(\mathbf{r}, t)$ divided by the speed of propagation of light in the medium, or by the integral of the angular energy density over the entire solid angle:

$$U(\mathbf{r}, t) = \frac{\phi(\mathbf{r}, t)}{c_n} = \int_{4\pi} u(\mathbf{r}, \hat{\boldsymbol{\Omega}}, t) d\Omega. \tag{1.18}$$

An important alternative, often found in the literature, is to express the energy density in terms of the photon number density, which has units of m^{-3} and lacks the information about the photon energy ($\hbar\omega$). The photon number density may be represented by the symbol $U_N(\mathbf{r}, t)$.

Net flux: $\mathbf{F}(\mathbf{r})$ – SI units: W/m^2.

We have seen above (in the definition of radiance) how an elemental flux vector can be defined as $\mathbf{L} = L\hat{\boldsymbol{\Omega}}$ to indicate optical flux along a given direction $\hat{\boldsymbol{\Omega}}$. If light propagates along different directions through a point $\mathbf{r}$, such as the case in optically turbid media, it is useful to define a *net flux* vector $\mathbf{F}(\mathbf{r})$ by integrating the elemental flux vector over the entire solid angle:

$$\mathbf{F}(\mathbf{r}) = \int_{4\pi} L(\mathbf{r}, \hat{\boldsymbol{\Omega}}) \hat{\boldsymbol{\Omega}} d\Omega. \tag{1.19}$$

The magnitude of the net flux vector provides the net energy per unit time, per unit area that propagates through an elemental area at point $\mathbf{r}$, while the direction of the net flux vector provides its direction of propagation. Thus, in the limit of perfectly isotropic, diffusely scattered light, the net flux can be zero, even though the fluence rate and energy density may be finite. In this text the net flux, $\mathbf{F}(\mathbf{r})$, takes the place of the Poynting vector that is frequently used in physics texts.

1.3 Quantities describing the optical properties of tissue and the interactions between the radiation field and tissue

Index of refraction: n – dimensionless.

The *index of refraction* of a medium is the ratio of the speed of light in vacuum to the speed of light in the medium. It can vary with the frequency of light (to be discussed in Section 4.8), but the spatial dependence of n on a microscopic scale is of particular importance in biomedical optics. In fact, the discontinuities and