

# Index

- approximation of 2D function
  - mollifier, 110
  - piecewise linear type, 108
- Arzela–Ascoli theorem, 20
- Azencott-type estimate, 528
- Besov regularity, 39, 87
- Besov–Hölder embedding, 575
- Besov–Lévy modulus embedding, 576
- Besov-variation embedding, 575
- Bouleau–Hirsch criterion, 551, 614
- bounded variation, 21
- Brownian motion, 327
  - and its delay, 351
  - Cameron–Martin theorem for, 358
  - enhanced, 333
  - finite quadratic variation, 383
  - fractional, 405
  - infinite 2-variation, 382
  - natural lift of, 333
  - on a Lie group, 334
  - Schilder’s theorem for, 360
  - support theorem for, 368
- Brownian rough path, 333
  - large deviations for, 359
- Burkholder–Davis–Gundy inequality
  - for enhanced martingale, 389
  - for martingale, 388
  - in homogenous  $p$ -variation norm, 394
- Cameron–Martin embedding theorem, 408
- Campbell–Baker–Hausdorff formula, 137
- Cantor function, 27
- Carnot–Caratheodory metric, 148
- centre, 316
  - of Lie algebra, 317
- Chen’s theorem, 133
- Chow’s theorem, 140
- compactness, 94, 177
- concatenation of paths, 61
- control function, 22, 80
  - 2D, 105
- Coutin–Qian condition, 407
- Davie’s lemma, 216
- dilation, 133
- Dirac measure, 110
- directional derivative, 69, 72, 284, 598
- Dirichlet form, 617
- dissection, 21
- distance
  - $p$ -variation, 77
  - Hölder, 77, 166, 170
  - supremum or infinity, 19
- Doss–Sussman method, 316
- driving signal, 53
  - time reversed, 62
- Duhamel’s principle, 74
- ellipticity condition, 550
- enhanced Brownian motion, 333
  - Cameron–Martin theorem for, 358
  - definition, 333
  - Donsker’s theorem for, 354
  - exact variation, 338
  - geodesic approximation, 340
  - law of iterated logarithm, 339
  - Lévy modulus, 338
  - McShane approximation, 350
  - non-standard approximation, 347

- piecewise linear
    - approximation, 340, 343
  - rough path regularity, 336
  - scaling, 335
  - Schilder's theorem for, 362
  - Strassen's law for, 367
  - support theorem for, 370
  - support theorem in conditional form, 380
  - Sussmann approximation, 349
  - weak approximation, 354
- enhanced Gaussian process
  - definition, 429
  - existence, 429
  - Karhunen–Lóeve
    - approximation, 438
  - modulus and exact variation, 419
  - mollifier approximation, 437
  - piecewise linear
    - approximation, 436
  - weak approximation, 443
- enhanced Markov process, 465
  - geodesic approximation, 467
  - piecewise linear
    - approximation, 469
  - Schilder theorem for, 483
  - support theorem, Hölder topology, 486
  - support theorem, uniform topology, 484
  - weak convergence, 482
- enhanced martingale, 387
  - piecewise linear
    - approximation, 398
  - rough path regularity, 390
- equicontinuous set, 20
- Euler approximation
  - for ODEs, 55
  - for RDEs, 238
- Euler estimate
  - for ODEs, 213
  - for RDEs, 223
- Euler scheme, 212
  - for RDEs, convergence, 238
  - higher-order, 212
- explosion time, 55
- exponential map, 136
- finite  $\varphi$ -variation, 99
- finite  $p$ -variation, 77, 105
- finite Hölder regularity, 77
- flows of diffeomorphisms, 290
- Fréchet derivative, 73, 287, 598
- fractional Brownian motion, 405
  - Cameron–Martin space for, 410
- free Lie algebra, 139
- free nilpotent group, 143
- Garsia–Rodemich–Rumsey theorem, 573
- Gaussian process
  - Karhunen–Lóeve
    - approximation, 415
  - mollifier approximation, 413
  - natural lift of, 429
  - non-degeneracy, 549
  - of Volterra type, 555
  - piecewise linear
    - approximation, 411
- Gaussian rough path, 429
  - large deviations for, 445
- geodesic, 144
  - approximation, 88, 174, 340
  - space, 88
- geodesic scheme
  - for RDEs, 239
- geometric rough path, 195
- Gronwall's lemma, 54
- H-regularity, 545
  - definition, for abstract Wiener functionals, 613
  - of RDE solutions driven by Gaussian rough paths, 547
- Hörmander's condition, 553, 561

- heat kernel, 624
- Heisenberg group, 147, 196
- homogenous distance, 166
- homogenous norm, 146
- homogenous norms
  - equivalence of, 149
- Hopf–Rinow theorem, 88
- Hurst parameter, 1
  
- increments of a map, 30
- inhomogenous distance, 170
- integral
  - Riemann–Stieltjes, 45
  - rough, 253
  - Stratonovich, 507
  - Young, 116
  - Young–Wiener, 434
- interpolation, 78, 177
  
- Kolmogorov criterion, 582
- Kolmogorov–Lamperti tightness
  - criterion, 583
  
- Lévy’s area, 329
  - as time-changed Brownian
    - motion, 332
- large deviations, 603
  - contraction principle, 604
  - for anticipating SDE, 540
  - for Brownian rough path, 359
  - for Gaussian rough paths, 445
  - for Markovian rough path, 483
  - for SPDE, 543
  - for stochastic flow, 541
  - for symmetric diffusion, 629
- Lemma A, 215
- Lemma B, 216
- Lie algebra, 136
  - free, 139
- Lie group, 135
- lift
  - of  $(p, q)$ -type, 204
  - of geometric rough path
    - (Lyons), 185
  - of smooth path, 129
  
- limit theorem
  - for stochastic flows, 522
  - strong, 517
  - weak, 520
- Lipschitz map, 213
- Lyons-lift, 185
  - of  $(p, q)$ -type, 204
  
- Malliavin covariance matrix, 551
- Malliavin derivative, 566
- Markov process
  - natural lift of, 465
- Markovian rough path, 465
  - large deviations for, 483
- martingale
  - continuous local, 387
  - techniques, 341, 441, 610
- moderate function, 388
- modulus of continuity, 80, 83
  
- natural lift
  - of a Markov process, 465
  - of Brownian motion, 333
  - of Gaussian process, 429
- neo-classical inequality, 211
- non-degeneracy condition on
  - Gaussian driving signal,
    - 549
- non-explosion condition on vector
  - fields, 69
  
- ordinary differential equation,
  - 55
  - continuity of solution map, 62,
    - 65
  - Euler approximation for, 55
  - Euler estimate for, 213
  - existence, 55
  - uniqueness, 59
- ordinary differential equations
  - different starting points, same
    - driving signal (Lemma B),
      - 216
  - same starting point, different
    - driving signals (Lemma
      - A), 215

- path
- $\alpha$ -Hölder, 77
  - approximation to
    - piecewise geodesic, 88
    - piecewise linear, 32
  - concatenation, 61
  - continuous, 19
    - 1-Hölder, 28
    - absolutely, 26, 34
    - absolutely of order  $p$ , 86
    - Lipschitz, 28
  - continuously differentiable, 30
  - lift of, 129
  - of Besov regularity, 39, 87
  - of bounded variation, 21
  - of finite  $\varphi$ -variation, 99
  - of finite  $p$ -variation, 77
  - of finite Hölder regularity, 77
  - of Sobolev regularity, 39, 42, 87
  - rectifiable, 44
  - time reversal of, 61
- Poincaré inequality
- on free nilpotent groups, 495
- quadratic form, 615
- Riemann–Stieltjes integral, 45
- rough differential equation
- definition, 224
  - directional derivative, 284
  - Euler estimate for, 223
  - Euler scheme for, 238
  - existence, 222
  - Fréchet derivative, 287
  - full, 241
  - geodesic scheme for, 239
  - linear, 265
  - perturbed, 317
  - uniqueness, 233
  - with drift, 303
- rough integral, 253
- rough partial differential equation, 295
- rough path
- geometric, 195
- Schilder’s theorem, 629
- semi-martingale, 386
- signature, 129
- Sobolev regularity, 39, 42, 87
- stochastic differential equation
- anticipating, 523, 540
  - driven by Gaussian signal, 515, 537
  - driven by Markovian signal, 516, 537
  - in Stratonovich sense, 510
  - with delay, 524
- stochastic flow
- large deviations, 541
  - support theorem, 535
- stochastic partial differential equation, 525
- large deviations, 542
  - support theorem, 542
- stochastic Taylor expansion
- strong remainder estimate, 528
  - weak remainder estimate, 531
- Stratonovich integral, 507
- sub-Riemannian manifold, 149
- superadditive map, 22
- support theorem
- for enhanced Markov process, Hölder topology, 486
  - for Brownian motion, 368
  - for enhanced Brownian motion, 370
  - for enhanced Brownian motion, conditional, 380
  - for enhanced Markov process, uniform topology, 484
  - for SDE driven by Gaussian signal, 537

656

*Index*

- support theorem (*cont.*)
  - for SDE driven by Markovian signal, 537
  - for SPDE, 543
  - for stochastic flow, 535
  - Stroock–Varadhan, 533
- symmetric diffusion, 625
- time reversal of paths, 61
- translation operator, 209
- upper gradient lemma, 495
- vector fields
  - $C^k$  bounded, 68
  - 1-Lipschitz continuous, 53
- Lipschitz regular in sense of Stein, 213
  - non-explosion condition on, 69
- Wiener’s characterization, 96
- Wong–Zakai theorem, 511
- Young integral, 116
- Young pairing, 204
- Young regularity, complementary, 449, 546
- Young–Lôève estimate, 114
- Young–Lôève–Towghi estimate, 122
- Young–Wiener integral, 434