Elastic Waves at High Frequencies

Techniques for Radiation and Diffraction
of Elastic and Surface Waves

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Do not go gentle into that good night,
Old age should burn and rave at close of day;
Rage, rage against the dying of the light.

Though wise men at their end know dark is right,
Because their words had forked no lightning they
Do not go gentle into that good night.

Do not go gentle into that good night
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Foreword

This volume is dedicated to the memory of John G. Harris, whose life ended prematurely on the 6th of May, 2006. John's friendship and research impacted many people – he was a dedicated and loving husband, an accomplished scientist and applied mathematician, a passionate teacher, and an important mentor to many young scientists. This book was originally intended to be John's second book on elastic wave theory and diffraction. It grew from four lectures that were given at the Department of Mathematics and Mechanics, within IIMAS, at the National Autonomous University of Mexico, in January 2004. After John's passing, several of his colleagues, inspired by his wife Beatriz, began to convert these unfinished notes into a form suitable for publication. We have worked to combine the existing chapters with additional, contributed chapters from experts in the field of elastic wave theory.

Born and raised in Toronto, John entered McGill University as a mature student and graduated with a Bachelors in Electrical Engineering (Honours). After receiving a Masters of Science in Applied Physics from Stanford University, John traveled to Northwestern University to work toward a doctorate in Applied Mathematics with Jan Achenbach, which he completed in 1979. J. D. Achenbach had a lasting impact on John's work in elastic wave scattering, which formed the basis of much of John's research as a professor at the Department of Theoretical and Applied Mechanics at the University of Illinois at Urbana–Champaign between 1979 and 2005.

John's initial research focused on the scattering of ultrasonic elastic waves from cracks, in particular the use of surface waves to interrogate curved shells and surface-breaking cracks in cases of interest to acoustic microscopy. He developed a great interest in WKBJ theory and

asymptotic approximations of wavefields, which allowed him to share his clear and elegant understandings of diffraction in an extensive list of publications. John also wrote a graduate-level textbook, *Linear Elastic Waves*, and a monograph on elastic wave theory during his visit to Mexico. He began this volume in Quaid-i-Azam University in Islamabad, Pakistan, where he spent a semester lecturing in 1994.

Two unifying themes are used throughout this book. The first is that wave propagation and scattering are among the most fundamental processes that we use to comprehend the world around us. The second is that waves are best understood in an asymptotic approximation, where they are free of the complications of their excitation and are governed primarily by their propagation environment. This book is not intended as a textbook, in the sense that it is not written to accompany a specific course. However, the chapters do follow a course of increasing complexity, beginning with plane-wave propagation and spectral analyses, which allows for the development of advanced techniques for studying diffraction. A short synopsis of the chapters is as follows.

John's writing forms the basis of Chapters 1, 2, and 3. Chapter 1 introduces the topics of elastic wave theory, energy flux, and Fourier and Laplace representations of time-harmonic wavefields. Chapters 2 and 3 solve canonical scattering problems using asymptotic approximations to Fourier integrals.

Chapter 4, written by A. Norris, explores the use of reciprocity identities and mechanical impedance to describe radiation and scattering problems. Radiation of elastic waves is examined using Gaussian beam solutions as a model for transducers, while reciprocity is used to derive the force on a particle caused by an incident elastic wave.

Chapter 5, written by P. A. Martin, concerns integral formulations of scattering from cracks. In particular, a special case of scattering from a screen is solved in the low-frequency limit by approximating a hypersingular integral equation, and a general strategy for solving more complex problems is proposed to analyze scattering from curved cracks and non-planar geometries.

Chapter 6, written by J. G. Harris, develops techniques for scanned acoustic imaging that utilize a converging spherical wave generated by a transducer above a fluid–solid interface. The incident and scattered fields are written in terms of an angular spectrum of plane waves, and a reciprocity relation is used to express unknown material variability (in a thin film attached to the interface, for example) in terms of the measured transducer voltage.

Chapter 7, written by A. Davis, explores the effects of viscosity on acoustic diffraction. Compressibility and viscosity are seen to be intertwined, as dilatational waves become coupled to vorticity disturbances near scattering interfaces. Scattering solutions are derived for plane waves diffracted by a half-plane, and for spherical waves scattered by a plane interface and by an elastic sphere.

Chapter 8, written by R. Craster, elucidates the phenomenon of channeling of wave energy along guided structures. The chapter first summarizes guided waves in acoustics, and then proceeds to describe elastic waves in straight waveguides (including the Rayleigh–Lamb modes). Asymptotic expressions for wave propagation in waveguides that are inhomogeneous, possibly bent or of varying thickness, are the ultimate focus of this chapter.

Appendix A, written by J. G. Harris, discusses asymptotic expansions and asymptotic approximations of integrals, methods that are used routinely throughout the book. Appendix B, also by J. G. Harris, lists, without derivation, properties of the special functions that arise in the book.

These nine chapters cover both the necessary introductory material and a broad survey of applications in diffraction and scattering theory. John's dedication, creativity, and clear understanding of these subjects have inspired us to take on the task of completing his final work. We thank him dearly for his friendship and collaboration, and hope that future readers will find the topics as compelling and captivating as he did.

Sincerely,

Gareth I. Block
Richard V. Craster
Anthony M. J. Davis
Paul A. Martin
Andrew N. Norris

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