

SEMICONDUCTOR QUANTUM OPTICS

The emerging field of semiconductor quantum optics combines semiconductor physics and quantum optics, with the aim of developing quantum devices with unprecedented performance. In this book researchers and graduate students alike will reach a new level of understanding to begin conducting state-of-the-art investigations.

The book combines theoretical methods from quantum optics and solid-state physics to give a consistent microscopic description of light–matter- and many-body-interaction effects in low-dimensional semiconductor nanostructures. It develops the systematic theory needed to treat semiconductor quantum-optical effects, such as strong light–matter coupling, light–matter entanglement, squeezing, as well as quantum-optical semiconductor spectroscopy. Detailed derivations of key equations help readers learn the techniques and nearly 300 exercises help test their understanding of the materials covered.

The book is accompanied by a website hosted by the authors, containing further discussions on topical issues, latest trends, and publications on the field. The link can be found at www.cambridge.org/9780521875097.

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Preface

A wide variety of quantum-optical effects can be understood by analyzing atomic model systems interacting with the quantized light field. Often, one can fully calculate and even measure the quantum-mechanical wave function and its dependence on both the atomic and the light degrees of freedom. By elaborating on and extending this approach, researchers perpetually generate intriguing results and new insights allowing for the exploration and utilization of effects encountered only in the realm of quantum phenomena.

By now, quantum-optical investigations have evolved from atoms all the way to complex systems, such as solids, in particular semiconductors. As a profound conceptual challenge, the optical transitions in semiconductors typically involve an extremely large number of electronic states. Due to their electric charge, the optically active electrons experience strong Coulomb interaction effects. Furthermore, they are coupled to the lattice vibrations of the solid crystal. For such an interacting many-body system, the overwhelmingly large number of degrees of freedom makes it inconceivable to measure the full wave function; we obviously need new strategies to approach semiconductor quantum optics. The combination of quantum-optical and many-body interactions not only leads to prominent modifications of the effects known from atomic systems but also causes new phenomena without atomic counterparts.

In this book, we develop a detailed microscopic theory for the analysis of semiconductor quantum optics. As central themes, we discuss how the quantum-optical approach can be systematically formulated for solids, which new aspects and prospects arise, and which conceptual modifications have to be implemented. The presented material is largely based on our own research and teaching endeavors on various topics in quantum mechanics, many-body theory, solid-state physics, optics, laser theory, quantum-optics, and semiconductor quantum optics. Our experience shows that one needs a systematic combination of optical and many-body theory to truly understand and predict quantum-optical effects in semiconductors. Therefore, we have implemented a multifaceted approach where we first discuss the basic quantum-theoretical techniques and concepts. We then present the central steps to quantize the light field and the many-body system. Altogether, we develop a systematic theory for semiconductor quantum optics and present its main consequences.

One of our major goals is to provide a bridge between “traditional” quantum optics and many-body theory. We naturally cannot present the final conclusion on this topic because

the combination of quantum optics and many-body quantum dynamics actually poses one of the most difficult problems in contemporary physics, which does not allow for general exact results. Therefore, the material in this book is designed such that it can be applied to generate new systematic approximations to the full many-body/quantum optics. We believe that this work will contribute to an expansion of the general knowledge base needed to diversify our understanding of quantum mechanics in complex many-body systems.

This book introduces all the central concepts and develops the main steps of the theory needed for a precise formulation and analysis of many relevant phenomena. We thoroughly discuss the emerging effects as we cross the boundary from the classical to the quantum-optical features of semiconductor systems. Even though we present a research outlook beyond the basic investigations in the last chapter of this book, the more detailed applications and many-body extensions of the presented theory are covered in our second book, *Semiconductor Quantum Optics: Advanced Many-Body Aspects* (Cambridge University Press, to be published). Whereas this first book develops a working knowledge up to a level where one can start doing research on semiconductor quantum optics, our second book deepens the analysis to an advanced level and examines intriguing new phenomena and details.

This first book has been designed in such a way that it can be used for self-study as well as classroom teaching for advanced undergraduate or regular postgraduate courses with a different emphasis on the topics. To follow many details of the theory development and to deepen the basic understanding, we recommend reflecting upon the presented material through the exercises given at the end of each chapter. Chapters 1–6 can serve as supplementary material in teaching quantum mechanics, especially, for keen beginners or to provide a complementary view besides the standard books. Chapters 7–10 are well suited for introducing light quantization and quantum field theory while chapters 11–15 present the elements of solid-state and many-body theory. Especially, the cluster-expansion method (Chapter 15) provides a common starting point to bridge the “traditional” and the semiconductor quantum optics.

After these foundations are carefully laid, one can design a pure quantum-optics course based on Chapters 16–24. Here, we have paid special attention to study atomic phenomena with the goal to provide a connection to semiconductor quantum optics. The remaining chapters, 25–30, present the material for lectures discussing the essence of semiconductor quantum optics. Naturally, we recommend combining the material from all the chapters for the full learning experience. Further information, figure downloads, and comments on the book can be found at <http://sqo.physik.uni-marburg.de>.

This series of two books has been written during an extended period from 2006–2011. Most of this work has been done at the Philipps-Universität Marburg – we truly appreciate the research-oriented infrastructure provided here. We also thank the members of our department, our collaborators, and students for the inspirational research and teaching interactions we have had during our efforts to reveal and explain new semiconductor quantum phenomena. We also have enjoyed the hospitality of several collaborating institutes. In particular, we want to thank Professor Steven Cundiff (M.K., JILA visiting fellow program,

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