Cambridge University Press
978-0-521-87429-8 - Advanced Gravitational Wave Detectors
Edited by David G. Blair, Eric J. Howell, Li Ju and Chunnong Zhao
Excerpt
More information

Part 1

An introduction to gravitational wave astronomy and detectors

Cambridge University Press
978-0-521-87429-8 - Advanced Gravitational Wave Detectors
Edited by David G. Blair, Eric J. Howell, Li Ju and Chunnong Zhao
Excerpt
More information

1

Gravitational waves

D. G. Blair, L. Ju, C. Zhao and E. J. Howell

This chapter describes the theory of gravitational waves. We first introduce gravitational waves and describe how they are generated and propagate through space. We then show how the luminosity, frequency and amplitude of a gravitational wave source can be defined. A brief mathematical summary of how gravitational waves are a natural consequence of Einstein's general theory of relativity is then provided. To finish, we summarise some important quantities that are used to describe gravitational wave signal strengths and the response of detectors to different types of signal.

1.1 Listening to the Universe

Our sense of the Universe is provided predominantly by electromagnetic waves. During the 20th century the opening of the electromagnetic spectrum successively brought dramatic revelations. For instance, optical astronomy gave us the Hubble law expansion of the Universe. Radio astronomy gave us the cosmic background radiation, the giant radio jets powered by black holes in galactic nuclei, and neutron stars in the form of radio pulsars. X-ray astronomy gave us interacting neutron stars and black holes. Infrared astronomy gave us evidence for a massive black hole in the nucleus of our own galaxy.

Gravitational waves offer us a new sense with which to understand our Universe. If electromagnetic astronomy gives us eyes with which we can see the Universe, then gravitational wave astronomy offers us ears with which to hear it. We are presently deaf to the myriad gravitational wave sounds of the Universe. Imagine you are in a forest: you see a steep hillside, massive trees and small shrubs, bright flowers and colourful birds flitting between the trees. But there is much more to a forest: the sound of the wind in the treetops, the occasional crash of a falling branch, the thump thump of a fleeing kangaroo, the pulse of cicadas, the whistles of parrots and honking of bell frogs. The sense of hearing dramatically enriches our experience.

The gravitational wave universe is likely to be rich with 'sounds' across a frequency range from less than one cycle per year (the nanohertz band) up to tens of kilohertz. Sources emitting in the audio frequency band are detectable using Earthbased detectors. Observations in the microhertz to millihertz range require space-based detectors, while radio pulsar timing can detect in the nanohertz band. At even lower frequencies a gravitational wave signature from the Big Bang is expected to be frozen in the cosmic microwave background radiation.

Gravitational waves are produced whenever there is a non-spherical acceleration of mass–energy distributions. The nanohertz to millihertz frequencies consist of highly redshifted signals from the very early Universe, the slow interactions of very massive black holes and a weak background from binary star systems. Signal frequencies often scale inversely with the mass of the relevant systems. Black holes below 100 solar masses and neutron stars will produce gravitational waves in the audio frequency range: nearly monochromatic whistles from millisecond pulsars, short bursts from their formation, and chirps as binary systems spiral together and finally coalesce. Past experience tells us that our imagination and ability to predict is often limited. The sources we predict today may be just a fraction of what we will hear when advanced detectors (under construction at the time of writing) and future third generation detectors are operating at sufficient sensitivity.

Gravitational waves are waves of tidal force. They are vibrations of spacetime which propagate through space at the speed of light. They are registered as tiny vibrations in the relative spacings of carefully isolated masses. Their detection is primarily an experimental science, consisting of the development of the necessary ultra-sensitive measurement techniques. While gravitational waves can be considered as classical waves, the measurement systems must be treated quantum mechanically since the expected signals generally approach the limits set by the uncertainty principle.

The binary pulsar system PSR 1913+16 has played a key role in the unfolding story of gravitational waves. This system has proved Einstein's theory of general relativity to high precision, including the quadrupole formula which states that the total emitted gravitational wave power from any system is proportional to the square of the third time derivative of the system's quadrupole moment. The system loses energy exactly as predicted by this formula (Weisberg and Taylor, 1984; Weisberg and Taylor, 2005). Figure 1.1 shows the impressive fit of the measured values with the relativistically predicted accumulated shift in periastron (point of closest approach) due to orbital decay. Hulse and Taylor, who discovered the system in 1974 (Hulse and Taylor, 1975), were rewarded by a Nobel prize almost 20 years later. By this time careful monitoring had shown a gravitational wave energy loss from the system in agreement with theory to better than 1%.

1.2 Gravitational waves in stiff-elastic spacetime

In Newtonian physics spacetime is an infinitely rigid conceptual grid. Gravitational waves cannot exist in this theory. They would have infinite velocity and infinite energy density, because in Newtonian gravitation the metrical stiffness of space is infinite. Conversely, general relativity introduces a finite coupling coefficient between curvature of spacetime, described by the Einstein curvature tensor, and the stress energy tensor which describes the mass–energy which gives rise to the curvature. This coupling is expressed by the Einstein equation

$$\mathbf{T} = \frac{c^4}{8\pi G}\mathbf{G}, \tag{1.1}$$

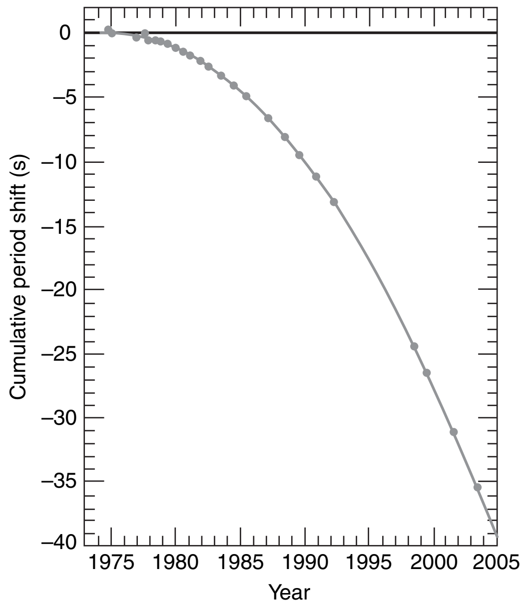
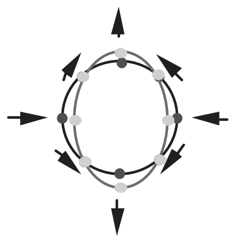


Figure 1.1 The curve indicates the general relativistically predicted accumulated shift in periastron for PSR 1913 + 16 due to the orbital decay by gravitational radiation. The data points indicate the measured values of the epoch of periastron (courtesy of Wikimedia Commons; data from Weisberg and Taylor, 2005).

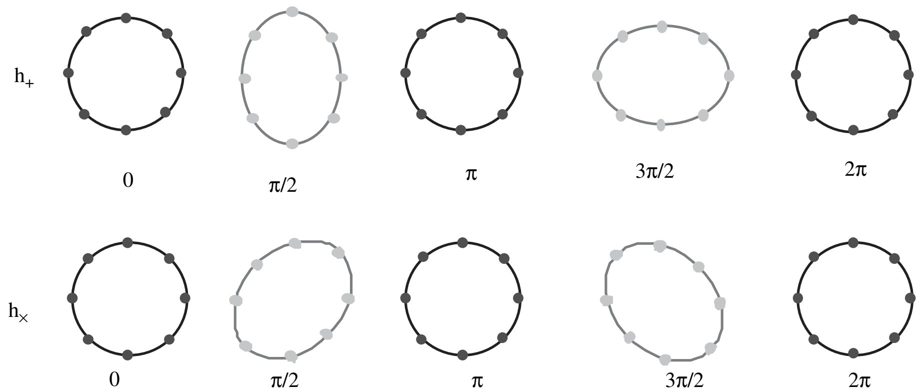
Here **T** is the stress energy tensor and **G** is the Einstein curvature tensor, c is the speed of light and G is Newton's gravitational constant. The coupling coefficient, $c^4/8\pi G$, is an enormous number of order 10^{43}. This expresses the extremely high stiffness of space, which is the reason that the Newtonian law of gravitation is an excellent approximation in normal circumstances, and why gravitational waves have a small amplitude, even when their energy density is very high.

The existence of gravitational waves is intuitively obvious as soon as one recognises that spacetime is an elastic medium. The basic properties of gravity waves can be easily deduced from our knowledge of Newtonian gravity, combined with knowledge of the fact that curvature is a consequence of mass distributions.

First, consider how gravitational waves might be generated. Electromagnetic waves are generated when charges accelerate. Because a negative charge accelerating to the left is equivalent to a positive charge accelerating to the right, it is impossible to create a time-varying electric monopole. The process of varying the charge on one electrode always creates a time-varying dipole moment. Hence it follows that electromagnetic waves are generated by time-varying dipole moments. In contrast to electromagnetism, gravity has only one charge: there is no such thing as negative mass! Hence it is not possible to create an oscillating mass dipole. Action equals reaction. That is, momentum is conserved and the acceleration of one mass to the left creates an equal and opposite reaction to the right. For two equal masses, their spacing can change but the centre of mass is never altered.



(a)



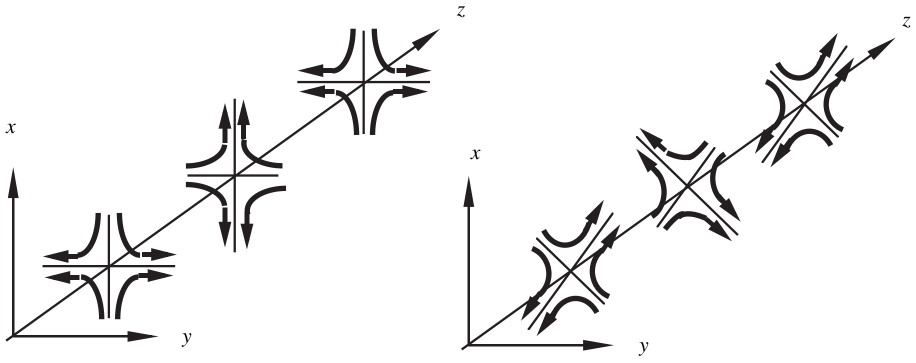
(b)

Figure 1.2 (a). The lowest order non-spherical deformation of a ring: the diagonal masses are not moved. (b) The deformation of a ring of test particles in one cycle of a gravitational wave field.

This means that there is a time-varying quadrupole moment, but there is no variation in monopole or dipole moment.

To be certain of the quadrupole nature of gravitational waves, think of a system which collapses under its own gravity. First think of a spherically symmetrical array of masses that collapse gravitationally towards a point. At a distance there is no difference between the gravitational field of a point mass and that of the same mass distributed in a uniform spherical distribution (this is a consequence of the inverse square law, and is also true for electric fields). Hence the process of gravitational collapse of a spherical distribution creates no variation in the external gravitational field, and hence no gravitational waves. Clearly gravitational waves must be created by non-spherical motions of masses. Consider a ring of eight test masses, such as that illustrated in Figure 1.2.

The simplest non-spherical motion is one in which the edge masses move inwards and the top and bottom masses move apart, as shown in Figure 1.2(a). Such a quadrupole motion does vary the external field and does create gravitational waves. For a small amount of vertical stretching, and an equal horizontal shrinking, it is obvious that the diagonally placed masses have no radial motion. There is clearly a second polarisation 45 degrees



(a) '+' polarisation

(b) '×' polarisation

Figure 1.3 Gravitational wave field force lines. (a) '+' polarisation; (b) '×' polarisation.

rotated from the first in which the diagonal masses move radially, and the top, bottom and edge masses have no radial motion. Unlike electromagnetic waves, gravitational wave polarisations are just 45 degrees apart.

Gravitational wave detection can be easily understood from the symmetry between sources and detectors – time reversal invariance. A gravitational wave will distort a ring of test masses in exactly the same way that the distortion of a ring of test masses creates gravitational waves. The non-spherical deformation pattern we just observed is exactly like the tidal deformation of the earth created by the gravity gradient due to the moon. A gravitational wave is indeed a wave of time varying gravity gradient. The amplitude of a gravitational wave is measured by the relative change in spacing between masses. That is, the wave amplitude, usually denoted h, is given by $\Delta L/L$, where L is the equilibrium spacing and ΔL is the change of spacing of two test masses. Whereas electromagnetic luminosity depends on the square of the second time derivative of the electric dipole moment, the gravitational wave luminosity is proportional to the square of the third time derivative of the mass quadrupole moment. The extra derivative arises because gravitational wave generation is associated with the differential acceleration of masses.

The above deformation patterns also apply to solid or fluid bodies. The rigidity of normal matter is so low compared with that of spacetime that the stiffness of the matter is utterly negligible. Considering the deformations of Figure 1.2(a) applied to a solid sphere, such as the Earth, it also follows that the 45° points must involve circumferential motions since the deformation shown acts to transfer matter from the 'equator' to the poles in the same way that the lunar tides act on the Earth.

The gravitational wave has an effective force field determined by the displacement vectors of the test masses. The force field is discussed further below, and is shown in Figure 1.3. The force field indicates that detectors can be designed to couple to gravity waves in several different ways. They may detect straight linear strains, orthogonal strains, or circumferential strains.

1.3 The luminosity of gravitational waves

The weak coupling of gravitational waves to matter is a consequence of the enormous elastic stiffness of spacetime. If the elastic stiffness of spacetime were infinite (Newtonian physics) the coupling would be zero. In general relativity, the generation of gravitational waves is given quantitatively by combining the third time derivative of the quadrupole moment, D, described previously, with the appropriate coupling constant. The latter can only depend on the constants G and c (for classical waves), and by dimensional analysis this constant must have the form G/c^5. The gravitational wave luminosity of a source is given by

$$L_{\mathrm{G}} \sim \frac{G}{c^5}\left(\frac{\mathrm{d}^3 D}{\mathrm{d}l^3}\right)^2. \tag{1.2}$$

Except for a numerical factor, this is the Einstein quadrupole formula (Einstein, 1916). There are two useful formulae one can derive from equation (1.2). The first is the formula for a hypothetical terrestrial source or binary star system. The second is for an interacting black hole system. The terrestrial source might be a pair of oscillating masses joined with a spring. Ideally the spacing of the masses should change from zero to L. This is achieved in the edge-on view of a rotating dumbbell or binary star system in a circular orbit, as shown in Figure 1.4. Viewed edge-on, the masses appear to move in and out periodically twice per rotation cycle. The quadrupole moment for two masses a distance x apart is Mx^2. If the motion is sinusoidal at an angular frequency of ω, the square of the third time derivative is $\sim M^2L^4\omega^6$. Thus the gravitational wave luminosity of such a system is

$$L_{\mathrm{G}} \sim \frac{G}{c^5} M^2 L^4 \omega^6. \tag{1.3}$$

This equation applied to any natural or artificial source in our Solar System gives a depressingly small luminosity. This is a consequence of the extraordinarily small value of G/c^5. However, the situation is different in an astrophysical context.

Suppose that the system is a similar binary system, except that it consists of a pair of gravitationally bound masses, of size such that their escape velocity approaches c and each has a radius near to the Schwarzschild radius: that is, a pair of black holes. In this case, using the Schwarzschild radius, $r_S = 2GM/c^2$, the luminosity can be expressed in relativistic units:

$$L_{\mathrm{G}} \sim \frac{c^5}{G}\left(\frac{v}{c}\right)^6\left(\frac{r_S}{r}\right)^2. \tag{1.4}$$

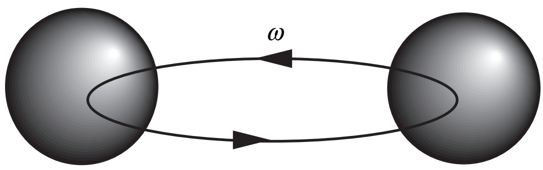


Figure 1.4 A rotating dumbbell or a binary star system, viewed edge-on, has a maximal variation of quadrupole moment.

Cambridge University Press
978-0-521-87429-8 - Advanced Gravitational Wave Detectors
Edited by David G. Blair, Eric J. Howell, Li Ju and Chunnong Zhao
Excerpt
More information

The remarkable difference between equation (1.3) and equation (1.4) expresses the difference between the physics of normal matter and that of black holes. Equation (1.3) is scaled by the tiny factor G/c^5, while equation 1.4 is scaled by its enormous reciprocal. Normal matter in our Solar System creates negligible curvature of spacetime. A black hole creates an extreme distortion of spacetime. Hence normal matter sources are intrinsically extremely weak, while very large amplitude waves are created in events such as the coalescence of a pair of black holes (for which we would expect $v \sim c$ when $r_S \sim r$). The factor c^5/G is roughly the total electromagnetic luminosity of the Universe. This is the upper limit to the gravitational wave luminosity of black hole systems. In reality equation (1.4) does not take into account the gravitational redshift effects and other spacetime curvature effects which act to reduce the maximum luminosity. However, to an order of magnitude, equation (1.4) indicates the extreme luminosity of gravitational waves that can be expected in short bursts when gravitationally collapsed systems with strong gravity, such as black holes (escape velocity $= c$) and neutron stars (escape velocity $\sim 0.1c$), are involved.

Any source can be characterised by an amplitude h and flux F detected at the Earth or by a luminosity L_G, which characterises the total rate of energy loss from the system. The energy flux (in W m^{-2}) in terms of the amplitude h is given by:

$$F = \frac{\pi}{4}\frac{c^3}{G} f^2 h^2 . \tag{1.5}$$

In general h is the amplitude for two polarisations $h^2 = h_+^2 + h_\times^2$. Numerically, we can write

$$F = 30\, W\, m^{-2} \left(\frac{f}{1\, kHz}\right)^2 \frac{h^2}{10^{-20}} . \tag{1.6}$$

This value represents a considerable energy flux, 3% of the solar intensity on Earth, although such high flux densities can only be sustained in short bursts. Hence, the energy of a gravitational wave is extremely high for a very small amplitude.

1.4 The amplitude and frequency of gravitational wave sources

As we saw in the previous section, a gravitational wave is a wave of gravity gradient which causes relative displacements, or strains, between test masses. The detection of gravitational waves requires the detection of small strain amplitudes. We should now consider the typical size of such strain amplitudes. One can very crudely estimate this by scaling the amplitude of the gravitational wave relative to the extreme amplitude at the point of coalescence of two masses to form a black hole. At the point of black hole formation spacetime curvature is very large. For example, the deflection of light for a light beam passing near to the event horizon can approach a complete orbit of a black hole. At the point of generation the dynamic curvature of space that will become the outgoing gravitational wave is unlikely to be able to exceed the static curvature represented by the maximal deflections of light past a black hole. The strain $\Delta L/L$ represented by such deflections can be estimated from the difference in light travel time for the deflected path around the black hole (say half an orbit) and the direct path between the same points in the absence of the black hole. For a half orbit (in Euclidean geometry) the circular path is $\pi/2$ longer than the direct path, so roughly $\Delta L \sim L$, and the maximum possible strain amplitude is $\sim$ unity. But by the inverse square law, the amplitude of the wave reduces as $1/r$. (The energy density, which is proportional to the square of the amplitude, reduces as $1/r^2$.) So for such a black hole source we can give the strain amplitude at distance r as simply $h \sim r_S/r$. For more realistic sources only a fraction of the total energy will participate in quadrupole motion. Thus it is more reasonable to include an efficiency factor ϵ, which characterises the fraction of the total system rest mass which can convert to gravitational waves. In this case we can write

$$h \sim \epsilon^{1/2} \frac{r_S}{r} \, . \tag{1.7}$$

Since the Schwarzschild radius of a solar mass is a few kilometres, the maximum strain amplitude that can be expected from any stellar mass source is numerically equal to the reciprocal of its distance in kilometres. Because r_S is linearly proportional to the mass, gravitational wave amplitudes from very high mass sources, such as colliding blackholes of 10^9 solar mass in galactic nuclei, will be of correspondingly larger amplitude. Putting in numbers, equation (1.7) gives $h \sim 10^{-16}$ for 10 solar masses and 100% efficiency at the galactic centre, and $h \sim 10^{-13}$ for 3 billion solar masses at 3 Gpc (towards the edge of the visible Universe).

Clearly these maximal numbers are very small. It might seem that the supermassive black hole sources might be much more detectable than the stellar mass source. The strain amplitude in this case corresponds to the detection of a motion equal to the size of an atomic nucleus on a one metre baseline, or one metre between here and Neptune. In fact the detection of such small strains on Earth is probably impossible. This is because the frequency of the waves from supermassive black hole sources must always be very low. The peak frequency, or its reciprocal, the burst duration, can be estimated from the time the binary black hole system takes to complete its final orbit before coalescence. Its value is about 10 kHz for one solar mass, reducing inversely as the mass. Thus the peak frequency will be about 1 kHz for the above galactic centre source, and 3×10^{-6} Hz for the distant massive black holes. The latter frequency will be reduced towards 10^{-6} Hz by cosmological redshifts. At such low frequencies, environmental effects, in particular gravity gradients associated with tides and weather variations in the surrounding environment, create perturbations which greatly exceed the desired signal.

There are two known ways to get around this obstacle. One is by using drag-free satellite technology in which a spacecraft is servo controlled by thrusters to follow a central protected freely floating test mass. In this way it is possible to create very stable free floating masses in space. Laser interferometry between the spacecraft and the test mass can then measure the gravitational wave strains. In this case detection does look relatively straightforward, though expensive, since it requires several widely separated spacecraft.

For frequencies even lower than 10^{-6} Hz, radio pulsars can replace man-made spacecraft in detection systems. The pulsar ideally provides a perfect monochromatic timing signal. The radio beams from the pulsar are traversed by incoming gravitational waves. If several