

## Fundamentals of Digital Communication

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This textbook presents the fundamental concepts underlying the design of modern digital communication systems, which include the wireline, wireless, and storage systems that pervade our everyday lives. Using a highly accessible, lecture style exposition, this rigorous textbook first establishes a firm grounding in classical concepts of modulation and demodulation, and then builds on these to introduce advanced concepts in synchronization, non-coherent communication, channel equalization, information theory, channel coding, and wireless communication. This up-to-date textbook covers turbo and LDPC codes in sufficient detail and clarity to enable hands-on implementation and performance evaluation, as well as “just enough” information theory to enable computation of performance benchmarks to compare them against. Other unique features include the use of complex baseband representation as a unifying framework for transceiver design and implementation; wireless link design for a number of modulation formats, including space-time communication; geometric insights into noncoherent communication; and equalization. The presentation is self-contained, and the topics are selected so as to bring the reader to the cutting edge of digital communications research and development.

Numerous examples are used to illustrate the key principles, with a view to allowing the reader to perform detailed computations and simulations based on the ideas presented in the text.

With homework problems and numerous examples for each chapter, this textbook is suitable for advanced undergraduate and graduate students of electrical and computer engineering, and can be used as the basis for a one or two semester course in digital communication. It will also be a valuable resource for practitioners in the communications industry.

Additional resources for this title, including instructor-only solutions, are available online at [www.cambridge.org/9780521874144](http://www.cambridge.org/9780521874144).

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Cambridge University Press  
978-0-521-87414-4 - Fundamentals of Digital Communication  
Upamanyu Madhow  
Frontmatter  
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[More information](#)

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**CAMBRIDGE**  
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Published in the United States of America by Cambridge University Press, New York

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9780521874144](http://www.cambridge.org/9780521874144)

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First published 2008, 2011

Second Edition 2012

Reprinted 2013

*A catalogue record for this publication is available from the British Library*

ISBN 978-0-521-87414-4 Hardback

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**To my family**

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Cambridge University Press  
978-0-521-87414-4 - Fundamentals of Digital Communication  
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# Preface

The field of digital communication has evolved rapidly in the past few decades, with commercial applications proliferating in wireline communication networks (e.g., digital subscriber loop, cable, fiber optics), wireless communication (e.g., cell phones and wireless local area networks), and storage media (e.g., compact discs, hard drives). The typical undergraduate and graduate student is drawn to the field because of these applications, but is often intimidated by the mathematical background necessary to understand communication theory. A good lecturer in digital communication alleviates this fear by means of examples, and covers only the concepts that directly impact the applications being studied. The purpose of this text is to provide such a lecture style exposition to provide an accessible, yet rigorous, introduction to the subject of digital communication. This book is also suitable for self-study by practitioners who wish to brush up on fundamental concepts.

The book can be used as a basis for one course, or a two course sequence, in digital communication. The following topics are covered: complex baseband representation of signals and noise (and its relation to modern transceiver implementation); modulation (emphasizing linear modulation); demodulation (starting from detection theory basics); communication over dispersive channels, including equalization and multicarrier modulation; computation of performance benchmarks using information theory; basics of modern coding strategies (including convolutional codes and turbo-like codes); and introduction to wireless communication. The choice of material reflects my personal bias, but the concepts covered represent a large subset of the tricks of the trade. A student who masters the material here, therefore, should be well equipped for research or cutting edge development in communication systems, and should have the fundamental grounding and sophistication needed to explore topics in further detail using the resources that any researcher or designer uses, such as research papers and standards documents.

## Organization

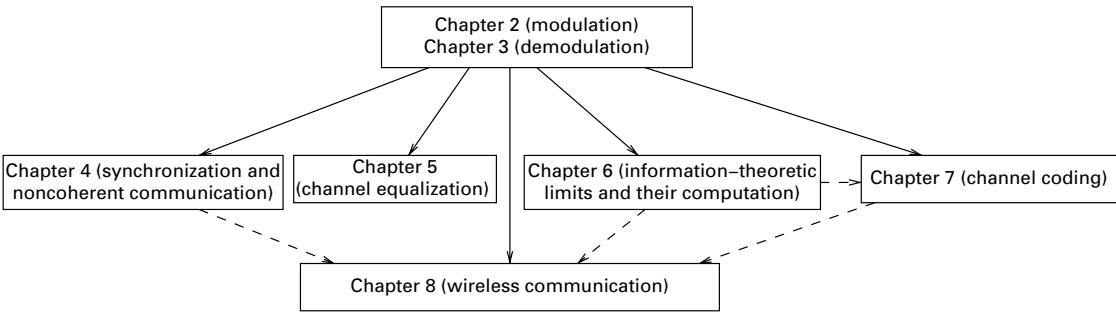
Chapter 1 provides a quick perspective on digital communication. Chapters 2 and 3 introduce modulation and demodulation, respectively, and contain

material that I view as basic to an understanding of modern digital communication systems. In addition, a review of “just enough” background in signals and systems is woven into Chapter 2, with a special focus on the complex baseband representation of passband signals and systems. The emphasis is placed on complex baseband because it is key to algorithm design and implementation in modern digital transceivers. In a graduate course, many students will have had a first exposure to digital communication, hence the instructor may choose to discuss only a few key concepts in class, and ask students to read the chapter as a review. Chapter 3 focuses on the application of detection and estimation theory to the derivation of optimal receivers for the additive white Gaussian noise (AWGN) channel, and the evaluation of performance as a function of  $E_b/N_0$  for various modulation strategies. It also includes a glimpse of soft decisions and link budget analysis.

Once students are firmly grounded in the material of Chapters 2 and 3, the remaining chapters more or less stand on their own. Chapter 4 contains a framework for estimation of parameters such as delay and phase, starting from the derivation of the likelihood ratio of a signal in AWGN. Optimal non-coherent receivers are derived based on this framework. Chapter 5 describes the key ideas used in channel equalization, including maximum likelihood sequence estimation (MLSE) using the Viterbi algorithm, linear equalization, and decision feedback equalization. Chapter 6 contains a brief treatment of information theory, focused on the *computation* of performance benchmarks. This is increasingly important for the communication system designer, now that turbo-like codes provide a framework for approaching information-theoretic limits for virtually any channel model. Chapter 7 introduces channel coding, focusing on the shortest route to conveying a working understanding of basic turbo-like constructions and iterative decoding. It includes convolutional codes, serial and parallel concatenated turbo codes, and low density parity check (LDPC) codes. Finally, Chapter 8 contains an introduction to wireless communication, and includes discussion of channel models, fading, diversity, common modulation formats used in wireless systems, such as orthogonal frequency division multiplexing, spread spectrum, and continuous phase modulation, as well as multiple antenna, or space-time, communication. Wireless communication is a richly diverse field to which entire books are devoted, hence my goal in this chapter is limited to conveying a subset of the concepts underlying link design for existing and emerging wireless systems. I hope that this exposition stimulates the reader to explore further.

## How to use this book

My view of the dependencies among the material covered in the different chapters is illustrated in Figure 1, as a rough guideline for course design or self-study based on this text. Of course, an instructor using this text



**Figure 1** Dependencies among various chapters. Dashed lines denote weak dependencies.

may be able to short-circuit some of these dependencies, especially the weak ones indicated by dashed lines. For example, much of the material in Chapter 7 (coding) and Chapter 8 (wireless communication) is accessible without detailed coverage of Chapter 6 (information theory).

In terms of my personal experience with teaching the material at the University of California, Santa Barbara (UCSB), in the introductory graduate course on digital communication, I cover the material in Chapters 2, 3, 4, and 5 in one quarter, typically spending little time on the material in Chapter 2 in class, since most students have seen some version of this material. Sometimes, depending on the pace of the class, I am also able to provide a glimpse of Chapters 6 and 7. In a follow-up graduate course, I cover the material in Chapters 6, 7, and 8. The pace is usually quite rapid in a quarter system, and the same material could easily take up two semesters when taught in more depth, and at a more measured pace.

An alternative course structure that is quite appealing, especially in terms of systematic coverage of fundamentals, is to cover Chapters 2, 3, 6, and part of 7 in an introductory graduate course, and to cover the remaining topics in a follow-up course.

# Acknowledgements

This book is an outgrowth of graduate and senior level digital communication courses that I have taught at the University of California, Santa Barbara (UCSB) and the University of Illinois at Urbana-Champaign (UIUC). I would, therefore, like to thank students over the past decade who have been guinea pigs for my various attempts at course design at both of these institutions. This book is influenced heavily by my research in communication systems, and I would like to thank the funding agencies who have supported this work. These include the National Science Foundation, the Office of Naval Research, the Army Research Office, Motorola, Inc., and the University of California Industry-University Cooperative Research Program.

A number of graduate students have contributed to this book by generating numerical results and plots, providing constructive feedback on draft chapters, and helping write solutions to problems. Specifically, I would like to thank the following members and alumni of my research group: Bharath Ananthasubramaniam, Noah Jacobsen, Raghu Mudumbai, Sandeep Ponnuru, Jaspreet Singh, Sumit Singh, Eric Torkildson, and Sriram Venkateswaran. I would also like to thank Ibrahim El-Khalil, Jim Kleban, Michael Sander, and Sheng-Luen Wei for pointing out typos. I would also like to acknowledge (in order of graduation) some former students, whose doctoral research influenced portions of this textbook: Dilip Warriar, Eugene Visotsky, Rong-Rong Chen, Gwen Barriac, and Noah Jacobsen.

I would also like to take this opportunity to acknowledge the supportive and stimulating environment at the University of Illinois at Urbana-Champaign (UIUC), which I experienced both as a graduate student and as a tenure-track faculty. Faculty at UIUC who greatly enhanced my graduate student experience include my thesis advisor, Professor Mike Pursley (now at Clemson University), Professor Bruce Hajek, Professor Vince Poor (now at Princeton University), and Professor Dilip Sarwate. Moreover, as a faculty at UIUC, I benefited from technical interactions with a number of other faculty in the communications area, including Professor Dick Blahut, Professor Ralf Koetter, Professor Muriel Medard, and Professor Andy Singer. Among my



UCSB colleagues, I would like to thank Professor Ken Rose for his helpful feedback on Chapter 6, and I would like to acknowledge my collaboration with Professor Mark Rodwell in the electronics area, which has educated me on a number of implementation considerations in communication systems. Past research collaborators who have influenced this book indirectly include Professor Mike Honig and Professor Sergio Verdu.

I would like to thank Dr. Phil Meyler at Cambridge University Press for pushing me to commit to writing this textbook. I also thank Professor Venu Veeravalli at UIUC and Professor Prakash Narayan at the University of Maryland, College Park, for their support and helpful feedback regarding the book proposal that I originally sent to Cambridge University Press.

Finally, I would like to thank my family for always making life unpredictable and enjoyable at home, regardless of the number of professional commitments I pile on myself.