

Statistical Physics of Particles

Statistical physics has its origins in attempts to describe the thermal properties of matter in terms of its constituent particles, and has played a fundamental role in the development of quantum mechanics. It describes how new behavior emerges from interactions of many degrees of freedom, and as such has found applications outside physics in engineering, social sciences, and, increasingly, in biological sciences. This textbook introduces the central concepts and tools of statistical physics. It includes a chapter on probability and related issues such as the central limit theorem and information theory, not usually covered in existing texts. The book also covers interacting particles, and includes an extensive description of the van der Waals equation and its derivation by mean-field approximation. A companion volume, *Statistical Physics of Fields*, discusses non-mean field aspects of scaling and critical phenomena, through the perspective of renormalization group.

Based on lectures for a course in statistical mechanics taught by Professor Kardar at Massachusetts Institute of Technology (MIT), this textbook contains an integrated set of problems, with solutions to selected problems at the end of the book. It will be invaluable for graduate and advanced undergraduate courses in statistical physics. Additional solutions are available to lecturers on a password protected website at www.cambridge.org/9780521873420.

MEHRAN KARDAR is Professor of Physics at MIT, where he has taught and researched in the field of statistical physics for the past 20 years. He received his B.A. at Cambridge, and gained his Ph.D. at MIT. Professor Kardar has held research and visiting positions as a junior Fellow at Harvard, a Guggenheim Fellow at Oxford, UCSB, and at Berkeley as a Miller Fellow.

In this much-needed modern text, Kardar presents a remarkably clear view of statistical mechanics as a whole, revealing the relationships between different parts of this diverse subject. In two volumes, the classical beginnings of thermodynamics are connected smoothly to a thoroughly modern view of fluctuation effects, stochastic dynamics, and renormalization and scaling theory. Students will appreciate the precision and clarity in which difficult concepts are presented in generality and by example. I particularly like the wealth of interesting and instructive problems inspired by diverse phenomena throughout physics (and beyond!), which illustrate the power and broad applicability of statistical mechanics.

Statistical Physics of Particles includes a concise introduction to the mathematics of probability for physicists, an essential prerequisite to a true understanding of statistical mechanics, but which is unfortunately missing from most statistical mechanics texts. The old subject of kinetic theory of gases is given an updated treatment which emphasizes the connections to hydrodynamics.

As a graduate student at Harvard, I was one of many students making the trip to MIT from across the Boston area to attend Kardar's advanced statistical mechanics class. Finally, in *Statistical Physics of Fields* Kardar makes his fantastic course available to the physics community as a whole! The book provides an intuitive yet rigorous introduction to field-theoretic and related methods in statistical physics. The treatment of renormalization group is the best and most physical I've seen, and is extended to cover the often-neglected (or not properly explained!) but beautiful problems involving topological defects in two dimensions. The diversity of lattice models and techniques are also well-illustrated and complement these continuum approaches. The final two chapters provide revealing demonstrations of the applicability of renormalization and fluctuation concepts beyond equilibrium, one of the frontier areas of statistical mechanics.

Leon Balents, Department of Physics, University of California, Santa Barbara

Statistical Physics of Particles is the welcome result of an innovative and popular graduate course Kardar has been teaching at MIT for almost twenty years. It is a masterful account of the essentials of a subject which played a vital role in the development of twentieth century physics, not only surviving, but enriching the development of quantum mechanics. Its importance to science in the future can only increase with the rise of subjects such as quantitative biology.

Statistical Physics of Fields builds on the foundation laid by the *Statistical Physics of Particles*, with an account of the revolutionary developments of the past 35 years, many of which were facilitated by renormalization group ideas. Much of the subject matter is inspired by problems in condensed matter physics, with a number of pioneering contributions originally due to Kardar himself. This lucid exposition should be of particular interest to theorists with backgrounds in field theory and statistical mechanics.

David R Nelson, Arthur K Solomon Professor of Biophysics, Harvard University

If Landau and Lifshitz were to prepare a new edition of their classic *Statistical Physics* text they might produce a book not unlike this gem by Mehran Kardar. Indeed, Kardar is an extremely rare scientist, being both brilliant in formalism and an astoundingly careful and thorough teacher. He demonstrates both aspects of his range of talents in this pair of books, which belong on the bookshelf of every serious student of theoretical statistical physics.

Kardar does a particularly thorough job of explaining the subtleties of theoretical topics too new to have been included even in Landau and Lifshitz's most recent Third Edition (1980), such as directed paths in random media and the dynamics of growing surfaces, which are not in any text to my knowledge. He also provides careful discussion of topics that do appear in most modern texts on theoretical statistical physics, such as scaling and renormalization group.

H Eugene Stanley, Director, Center for Polymer Studies, Boston University

This is one of the most valuable textbooks I have seen in a long time. Written by a leader in the field, it provides a crystal clear, elegant and comprehensive coverage of the field of statistical physics. I'm sure this book will become "the" reference for the next generation of researchers, students and practitioners in statistical physics. I wish I had this book when I was a student but I will have the privilege to rely on it for my teaching.

Alessandro Vespignani, Center for Biocomplexity, Indiana University

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Preface

Historically, the discipline of *statistical physics* originated in attempts to describe thermal properties of matter in terms of its constituent particles, but also played a fundamental role in the development of quantum mechanics. More generally, the formalism describes how new behavior emerges from interactions of many degrees of freedom, and as such has found applications in engineering, social sciences, and increasingly in biological sciences. This book introduces the central concepts and tools of this subject, and guides the reader to their applications through an integrated set of problems and solutions.

The material covered is directly based on my lectures for the first semester of an MIT graduate course on statistical mechanics, which I have been teaching on and off since 1988. (The material pertaining to the second semester is presented in a companion volume.) While the primary audience is physics graduate students in their first semester, the course has typically also attracted enterprising undergraduates, as well as students from a range of science and engineering departments. While the material is reasonably standard for books on statistical physics, students taking the course have found my exposition more useful, and have strongly encouraged me to publish this material. Aspects that make this book somewhat distinct are the chapters on probability and interacting particles. Probability is an integral part of statistical physics, which is not sufficiently emphasized in most textbooks. Devoting an entire chapter to this topic (and related issues such as the central limit theorem and information theory) provides valuable tools to the reader. In the context of interacting particles, I provide an extensive description of the van der Waals equation, including its derivation by mean-field approximation.

An essential part of learning the material is doing problems; an interesting selection of problems (and solutions) has been designed and integrated into the text. Following each chapter there are two sets of problems: solutions to the first set are included at the end of the book, and are intended to introduce additional topics and to reinforce technical tools. Pursuing these problems should also prove useful for students studying for qualifying exams. There

are no solutions provided for a second set of problems, which can be used in assignments.

I am most grateful to my many former students for their help in formulating the material, problems, and solutions, typesetting the text and figures, and pointing out various typos and errors. The support of the National Science Foundation through research grants is also acknowledged.