

## Index

- AB5, 483, 504, **707**
- abelian category, 91, 93, 99, 112, 401, 423, 448,  
 460, 461, 484, 530, 555, 604, 606, 607,  
 632, 647, 671, 675, **706**, 716
- abelian group, 135
- abelian structure, 153, 658, 698
- abelian subcategory, 64, 431, 481, **707**,  
 712
- absolutely pure module/object, **57**, 65, 66, 76,  
 106, 115, 116, 145, 177, 206, 215, 231,  
 248, 292, 353, 357, 396, 397, 453, 457,  
 480, 487, 497–499, 501, 509, 611, 612,  
 642, 649, 651, 672, 701
- abstract elementary class, 701
- acc, *see* ascending chain condition, **187**
- accessible category, *see* finitely accessible  
 category, **604**
- acyclic complex, **652**
- additive category, 631, **706**
- additive functor, **704**
- adic module, 111, 136, 222, **224**, 266, 268, 269,  
 292, 293, 332, 334, 340–342, 346, 348,  
 372, 376, 389, 548, 566, 568, 569, 571,  
 572, 591
- adjoint, **705**
- admissible pair of ideals, **380**, 384
- affine, 555
- affine line, 367
- affine set, 604, 702
- affine variety, 550
- algebraically compact, *see* pure-injective  
 module/object, **128**, 143, 148, 236
- almost pure-injective module, **135**
- almost regular ring, **397**
- almost split map, 200, **248**, 248–250, 253,  
 255–257, 369, 386, 587
- almost split sequence, 204, 234, 329, 332, 370,  
 372, 376, 402, **587**
- amalgamation property, 701
- annihilator condition, *see* quantifier-free  
 condition, 4
- annihilator  
 in triangulated category, **648**  
 of element, xxii, xxvi, **4**, 17, 34, 35, 59, 63,  
 82, 105, 115, 145, 207, 223, 235, 364, 379,  
 551, 558, 576  
 of ideal of category, **408**, **488**, 596, 599  
 of morphism, 488
- approximation (of module), **118**
- artin algebra, xviii, xxi, 114, 186, 199, **201**,  
 201–205, 234, 250–252, 269, 309, 326,  
 401–410, 457, 500, 518, 539, 541, 548,  
 584–602, 689
- Artin–Rees Lemma, 188
- artinian module/object, 269
- artinian ring, 164, 190, 193, 323, 457, 458
- ascending chain condition, 23, 183, **187**, 198,  
 301, 354, 621
- associated prime, **551**, 571
- atomic boolean algebra, **394**
- atomic formula, **411**
- atomless boolean algebra, 306, 535
- Auslander’s Lemma, **690**
- Auslander–Bridger duality, **165**
- Auslander–Bridger transpose, 500
- Auslander–Reiten quiver, 195, 328, 332, **586**
- Auslander–Reiten sequence, *see* almost split  
 sequence, **587**
- Auslander–Reiten translate, 205, 328, 332, **587**,  
 589
- automorphism, language augmented by, 700
- axiom of choice, 102

- axiomatisable class, *see* elementary class, **413**  
 axiomatisation, 363, 701
- band, **345**, 407  
 band module, 344  
 basic algebra, 328, **585**  
 basic formula, 105  
 basis of  
   open neighbourhoods, 218, 224, 340, 348, 381, 564, 572  
   open sets, 213, 227, 228, 265, 269, 377, 386, 452, 486, 502, 513, 551, 623, 624, 627  
 Bezout ring, **354**  
 biendomorphism ring, 281, 282, 290–291, 296–297, 529, 567, 574  
 bimodule, 38, 54, 200, 276, 278, 586, 587  
 biregular ring, **392**  
 block, 634  
 boolean algebra, 66, 102, 319, 393, 394, 539  
 boolean ring, *see* boolean algebra, **394**  
 bound variable, **5**, **412**  
 boundary pair, **541**  
 breadth; *also see* width, **312**, 546, 664  
 bridge quiver, 350, 407  
 Brown representability, 654  
   for morphisms (BRM), **638**  
   for objects (BRO), **638**  
 butterfly quiver, **24**
- canonical algebra, 122, 308, **351**, 540  
 canonical basis, 698  
 Cantor sum of ordinals, 305, 381, **713**  
 Cantor–Bendixson analysis, 195, 244  
 Cantor–Bendixson derivative, xv, 244, **260**, 269, 349, 387, 566, 599, 655  
 Cantor–Bendixson rank, 209, 224, 226, 229, **260**, 266, 270, 272, 319, 340, 348, 349, 366, 372, 374, 376, 377, 380, 386, 391, 395, 541, 548, 689  
 cardinality of language, **416**  
 Casimir element, 364  
 categoricity, 700  
 category of  
   finite length modules/objects, 369  
   finitely presented modules/objects, 119–121, 166, 183, 205, 220, 250, 306, 309, 405, 409, 444–446, 449–450, 491, 500, 515, 517, 585, 659, 663, 674  
   pp sorts, *see* pp-pairs, category of, **91**  
   pp-pairs, *see* pp-pairs, category of, **91**  
 CB-rank, *see* Cantor–Bendixson rank, **260**
- central localisation, 158  
 central simple algebra, **222**, 336, 340  
 centre, **xxvi**  
 centre of ring, 157, 562  
 chain, 79, 83, 247, **298**, 298–302, 304, 312, 317, 318, 381, 382, 472, 536, 544, 545, 548  
 chain condition, 207  
 character, **183**  
 classical tilting module, 687  
 classifying topos, 658  
 clique, **335**, 340, 570  
 closed map, 693  
 closed point, 215, 245, 326, 381  
 closed unbounded subset, 393  
 closure, wrt torsion theory, **473**, 475  
 coalgebra, 604, 614, 701  
 coarseness, **386**  
 cocomplete category, 604, **704**  
 cocone, 641, **704**  
 cocritical, 271  
 cofinal, 62, 470, 619, **714**  
 cogenerated torsion theory, 271, **466**, 479, 510, 512, 526, 555, 561, 579  
 coherent formula, 105  
 coherent functor from triangulated category, **640**, 647, 648  
 coherent module/object, **63**, 431, 490, 536, **712**  
 coherent ring, 6, 59, **63**, 63–68, 82, 106, 115, 116, 122, 179, 207, 214, 229, 292, 353, 387, 396, 438, 457, 484, 555, 558, 561, 577–581, 619, 637, 640, 646, 649, 700, 701, 712  
 coherent theory, 658, 702  
 cohomological ideal, **648**  
 cohomology ring, 653, 655  
 cokernel, 109, 136, 207, 432, 604, 606  
 colimit, 647, **704**  
 commutative algebra, 700  
 commutative ring, xx, 54, 56, 74, 77, 80, 102, 117, 132, 134, 150, 199, 207, 268, 272, 306, 319, 383, 387, 394, 541, 550, 553, 561, 575–581, 585, 653, 699  
 comodule, 614, 701  
 compact element, 91  
 compact module, **133**  
 compact object, **636**, 644–647, 649, 655  
 compact space, 111, 196, 203, 219, 220, 225, 260, 265, 278, 326, 443, 558, 581, 620, 621, 623, 624, 626, 627  
 compact, meet-, 505  
 compactly generated, 258, **636**, 636–656

- Compactness Theorem, 126, 219, **415**  
 compatible (sections), **716**  
 complemented lattice, 360, 361, 363, 531, **713**  
 complemented over, 393  
 complete category, 604, **704**  
 complete resolution, 653  
 complete ring, 269  
 complete theory, **413**, 697  
 completion, 131, 134, 222  
 component, **587**  
   of natural transformation, **705**  
 concentrated (complex, in degree), **633**  
 condition, 412  
    $e$ -, **77**, 378  
 cone on morphism, **633**  
 congruence, on lattice, 88, 298, 301, 313, 524, 546, **713**  
 conjugate category, **611**  
 conjunction, 411  
 connected ring, **327**  
 constants, field of, **376**  
 constructible model, 699  
 continuous map, 278  
 continuous, *see* superdecomposable, **172**, **709**  
 Continuum Hypothesis, 699  
 continuum many, 242, 244, 293  
 contracting endomorphism, 347  
 contracting word, 349  
 contravariant functor, 422, 425, 453, 493, 638, 639, 706, 715  
 contravariantly finite subcategory, **119**, 206, 652  
 coordinate ring, 550  
 coproduct; *see* sum, direct, 636  
 coray, 114, 333, 340, **590**, 593  
 cosyzygy, **600**, 634  
 cotilting module, xxii, 151, **690**, 701  
 cotorsion, xxii, **205–208**, 500, 701  
 cotorsion envelope, 207, 208  
 cotorsion theory, 690, 701  
 countable, 175, 207, 215, 242, 244–245, 269, 307, 315–316, 318, 319, 321, 322, 324, 326, 395, 521, 680, 699  
 countable-dimensional, 175  
 covariantly finite subcategory, **118**, 599  
 covariantly finite with respect to, **120**  
 cover, **206**  
 Coxeter functor, 687, 689  
 Crawley–Jonsson–Warfield Theorem, 167, **717**  
 cut, 313, 352  
 CW spectrum, 634, 637  
 cyclically presented module, **68**, 77  
 dcc, *see* descending chain condition, **173**  
 decidability, 363, 687, 692, 698–700  
 Dedekind domain; *see also* PI Dedekind domain, 6, 55, 75, 111, 122, 158, 195, **221**, 323, 341, 388, 569  
 Dedekind prime ring, **75**, 224, 336, 569, 573  
 deductive closure, 412  
 definable category, xxii, 122, 560, 612, 656, **668**, 675–677, 682, 690  
 definable class, **413**  
 definable functor; *see also* interpretation functor, **678**, 682, 692  
 definable quotient category, **674**  
 definable scalar, 660  
   *see also* ring of definable scalars, 93, **281**, 347  
   category of, **530**  
 definable structure, 485, 669  
 definable subcategory, xviii, 39, 91, **105**, 105–118, 121, 122, 126, 148, 178, 179, 209–215, 219, 225, 231, 235, 258–259, 266, 274, 275, 286, 303, 315, 321, 359, 363, 373, 388, 395, 410, 413, 414, 445, 451, 457, 460, 473, 474, 476, 478–479, 482, 485, 499, 501, 505, 507–516, 528, 539, 543, 546, 555, 598, 607–608, 610–611, 621, 627, 648, 658, 662, 664, 665, 668, 672, 680, 688, 690, 692  
 characterisation, 107, 414, 445, 501, 509, 672  
   of finitely accessible category, **607**, **667**  
   of triangulated category, **647**  
 definable subclass, *see* definable subcategory, **105**  
 dense, 167, 251, 253, 261, 266, 326, 341, 355, 360, 362, 383, 458, 566, 598  
 dense functor, **704**  
 dense subcategory, **704**  
 dense, wrt torsion theory, 289, **462**, 467, 472–473, 490, 514, 620  
 densely ordered, 244, **301**, 302, 306, 310, 317, 320  
 derivation, **374**, 376  
 derived category, 581, 600, **633**, 636, 649, 653  
 derived equivalence, 205  
 derived functor, 56  
 descending chain condition, 116, 130, **173**, 183, 300, 301, 544, 621  
 deviation, 300  
 diagonal map, **101**, 103, 126  
 diagram, **704**  
 difference ring, 700

- differential operator, 75, 367, 370, 377, 637  
 differential polynomial ring, 374–377, 699  
 dihedral algebra, 16  
 dimension  
    $\Theta$ -, **534**  
    $\Theta_{\mathcal{L}}$ -, 536–538  
    $\mathcal{L}$ -, **299**  
   2dim, 304, 317  
   elementary Krull, 226, **300**, 302  
   finitistic, **456**  
   flat, 457  
   fp-injective, 457  
   Gabriel, **534**, 535, 542  
   global, 55, 370, 438, 650, **706**  
   injective, 55, 457, 640  
   Krull, 73, 81, **300**, 303, 312, 315, 320, 367, 375, 381, 386, 535, 540, 541  
   Krull–Gabriel, 195, 215, 226, 245, 273, 303, 306, 310, 314, 348–350, 360, 366, 374, 377, 382, 391, 395, 405, 406, 534, **539**, 541, 542, 544, 546, 548, 559, 599, 664  
    $\mathcal{L}$ -, 57  
   local Krull–Gabriel, **544**  
   m-, 20, 187, 194, 215, 226, 241–244, 262–263, 272, **302**, 306, 312, 314, 316, 317, 319, 349–350, 360, 366, 371, 373, 375, 377, 382, 385, 395, 405, 539, 540, 591, 664  
   of category/object wrt property, xxi, **534**  
   projective, 55, 121, 432, 436, 438, 457, 634, 637, 639, 687–689, **705**  
   pure global, **55**, 137, 639  
   pure global, for triangulated category, **639**  
   pure-injective, **55**, 640  
   pure-projective, **54**  
   uniserial, 312, 315, **546**, 664  
   weak, 457  
 diophantine geometry, 363  
 direct image sheaf, 568, 571  
 direct limit, 28, 44–45, 50, 56, 60, 83, 100, 107, 116, 117, 119, 225, 333, 372, 387, 396, 424, 446, 451, 464, 466, 467, 469, 479, 482, 578, 590–592, 599, 604, 605, 610, 622, 623, 663, 664, 667, 673, 675, 676, 690, 701, 704, **704**, 707, 709, 711, 715  
   to have, **714**  
 direct summand, 19, 52, 72, 75, 80, 108, 121, 134, 159, 170, 171, 176, 216, 251, 276, 294, 310, 313, 327, 386, 466, 651, 654, 692, 704, 717  
 directed poset, **714**  
 directed system, **714**  
 discrete space, 197, 211, 229, 235, 247  
 discrete valuation domain, **222**, 387–390  
 discrete valuation ring, 132  
 disjunction, 17, 411, 414  
 distinguished triangle, **631**  
 distributive lattice, 78, 247, 304, **713**  
 distributive module, **247**, 378  
 divisibility condition, 5, 12, 13, 17, 22, 26, 28, 30, 32, 44, 67, 73, 83, 225, 353, 368, 379, 564, 629, 649  
 divisible module, **58**, 76, 105, 113, 136, 182, 354, 369, 375, 575, 604  
 division ring, 199, 352  
 domain, 58, 61, 105, 113, 172, **221**, 352, 367  
 domestic (representation type), 16, **202**, 310, 345, 349, 350, 406, 540, 574  
 domination-equivalence, 697  
 dominion, **285**, 530  
 double dual, **37**, 113–114, 152, 646  
 dual category (elementary dual), 487, 517, 543, 546, 579, **612**, 671, 673  
 dual definable subcategory, **112**, 115, 612, 668, 669  
 dual functor, **453**  
 dual functor category, of definable (sub)category, 515, **515**, 543, 556, 659  
 dual module (hom-dual), 34, **35**, 63, 113, 133, 151, 182, 193, 194, 251, 255, 269, 272, 450, 585, 594  
 dual pp condition, **30**, 35–37, 40, 59, 69, 94–99, 112, 177, 225, 354, 396, 454  
 dual Serre subcategory, 491, **491**, 516  
 dual topology (Hochster dual), **278**  
 dual torsion theory, 512  
 dual-Ziegler spectrum, 283, 549  
 dual-Ziegler topology, xx, 200, **279**, 326, 553  
 dualising ring, 252  
 duality of  
   functor categories, 453–454, 501, 562, 595  
   module categories, 199, 500, 585, 612  
 dyadic fractions, 301, 306, 308  
 Dynkin diagram, 195, 311  
 eigenspace, 361, 363  
 eigenvector, 358  
 element, xviii, 442, **442**, 496, 609, 622, 626, 628, **657**, 660, 668, 683, 709  
 elementarily equivalent, 645  
 elementary class; *also see* definable class, 105, 209, 214, **413**, 474, 664, 682

- elementary cogenerator, **258**, 290, 297, 511, 520, 529, 654  
 elementary dual, 595  
 elementary dual (module), 184, 185, 187, 199, 201, **271**, 270–273, 291, 304, 371, 539  
 elementary dual category, *see* dual category (elementary dual) and dual definable subcategory, **612**  
 elementary dual of functor (*also see* dual functor), **455**  
 elementary duality, 38, 90, 94, 190, 194, 228, 238, **268**, 272, 318, 355, 385, 449, 454–457, 487, 504, 515, 696, 701  
 elementary embedding, 118, 664  
 elementary equivalence, 108, 113, 215, 363, **413**, 414, 415, 698, 701  
 elementary extension, 127, **414**, 416  
 elementary radical series, 90  
 elementary socle series, **90**  
 elementary substructure, 149, **414**, 415  
 elementary torsion theory/localisation, 106, **474**, 474–485, 621  
 elimination of  
   imaginaries, 448, 450, 487, 661, 698, 700  
   quantifiers, 68, 487, 649, 700  
 endocategory, 698  
 endolength, 180, 182, 195, 203, 205, 216, 273, 592–593, 601  
 endomorphism ring, 664  
   module over, 10, 23, 35, 53, 79, 130, 132, 143, 150, 160, 174, 180, 187, 235, 247, 290, 294, 296, 315, 326, 378, 382, 455, 591, 662, 671  
   realise algebra as, 322  
 enough injectives, 162, 500, 674, **709**  
 enough invertible ideals, **75**, 355, 357  
 enough projectives, 674  
 envelope, **206**  
 epimorphism, 424  
   of rings, 22, 105, 113, 201, **274**, 283–287, 335, 359–360, 363, 365, 371, 373, 531–532, 566, 570, 685, 704  
 essential (in), 146, 156, 241, 248, 255, **708**  
 essentially coherent ring, **396**  
 Euclidean diagram, 311  
 evaluation, 93, 450, 495, 656, 660, 664, 670  
 exact functor, xx, 93, 99, 112, **276**, 448, 463, 464, 484, 530, 642, 661, 670, 671, 677–678, 682, 684, **705**  
   on triangulated category, *see* homological functor, **637**  
 exact sequence, 424, 465  
 exact subcategory, 432, **612**  
 exactly definable category, **671**  
 exactness of direct limits, **707**  
 exceptional uniserial ring, **81**, 386  
 exchange property, 501  
 expanding endomorphism, 347  
 expanding word, 349  
 $\text{Ext}^1$ , 357, 368, 446, 461, 497, 500, 587, 616, 646, 688–690  
 $\text{Ext}^n$ , 447, 633  
 $\text{Ext}$ -quiver, 323  
 extended Dynkin diagram, 56, 311, 328, 333, 340  
 extension, 342, 343, 463  
 extension, closed under, 132, 207, 408, **461**, 468, 490  
 factorisable system, **305**, 351, 404  
 factorisation of morphism, 119, 328, 349, 351, 401, 402, 410, 430, 437, 488, 596, 598, 644  
 factors initially, **85**  
 faithful module, 354, 383  
 FBN ring, 552, 560, 565  
 few types, 699  
 fg-ideals topology, **559**, 577, 578  
 field of fractions; *also see* quotient field, 122  
 filter, 21, 89, **100**, 117, 126, 135, 159–160, 352, 361, 462, 467, 470, 504–505, 507, 613, 615, **715**  
   cofinally relatively finitely generated, **614**  
   generated, **102**, 393  
   proper, **100**  
 filter quotient, 136  
 filtered category, **715**  
 filtered colimit, 60, 604, 647, **715**  
 filtered diagram, 644  
 filtration, 458  
 finite endolength, xviii, 105, 114, **180**, 184, 193, 197, 200, 215, 216, 222, 225, 235, 245, 262, 272, 286, 291, 297, 326–327, 362, 381, 498, 566, 592, 600, 601, 653  
   object in triangulated category, **645**  
 finite intersection property, **102**, 115, 128, 132  
 finite length, 174, 180, 199, 235, 245, 261, 302, 327  
 finite length module/object, 163, 190, 196, 251, 292, 326, 355, 370–371, 409, 534, 538, 586, 594, 598, 599  
 finite localisation, 565, 572

- finite matrix topology, **135**  
 finite module, 131  
 finite representation type, xviii, xxv, 16, 55, 76, 173, 186, **195**, 211, 247, 252, 266, 304, 396, 403, 494, 539, 588, 640, 647, 649, 650, 689, 701  
 finite ring, 699  
 finite spectrum, 637, 638  
 finite-type torsion theory/localisation, 106, 107, 117, 161, 181, 212–214, 231, 271, 288–290, 460, **468**, 468–491, 509–512, 516–517, 529, 536, 549, 555, 557, 560, 579–581, 619, 621, 664, 669  
 finite-dimensional algebra, 118, 122, 202, 210, 266, 307, 321, 406, 430, 574, 585, 592, 639, 679, 687  
 finite-dimensional module, 131  
 finite-type presheaf, **555**, 561  
 finitely accessible category, xxi, 119, 207, 442, **604**, 603–612, 647, 656, 667, 682, 709, **711**  
 finitely generated functor, 428, 436, 505–507, 663  
 finitely generated module/object, 134, 269, 327, 471–472, 525, 559, 618, 626, **710**  
 finitely generated, relatively, **613**  
 finitely generated, wrt torsion theory, **472**, 474  
 finitely matrixable subgroup; *see* pp-definable subgroup, **5**  
 finitely presented algebra, 275  
 finitely presented functor, xvii, xix, 106, 217–220, 231–232, 235, 240, 322, 346, 409, 430–431, 443–448, 488, 497, 513–515, 518, 530, 564, 595, 663, 664, 671, 677, 696  
 finitely presented module/object, xx, 11, 12, 21–27, 29, 34, 37, 39, 41, 45, 50, 52, 56, 57, 60, 64, 68–77, 81, 83, 86, 106, 162–164, 166, 167, 192, 193, 200, 203, 237, 238, 248–250, 253, 272, 291, 305, 316, 356, 386, 396, 420, 440, 446, 468, 474–476, 481, 535, 537, 538, 552, 553, 558, 613–615, 618, 624, 640, 647, 686, **709**  
 finitely presented subfunctors, lattice of, 433, 435, 436, 455, 486, 504, 514, 521, 531, 536, 539, 663  
 finitely satisfied (pp-type), **125**  
 finiteness condition, 88  
 Finitistic Dimension Conjecture, 457  
 Fitting's Lemma, **163**  
 flat cover, 206, 207  
 flat module/object, xxii, xxvii, 59, **59**, 66, 76, 106, 115, 117, 122, 179, 205–207, 272, 292, 304, 372, 373, 457, 604, 605, **605**, 611, 619, 642, 646, 701  
 forgetful functor, 90, 91, 436, 504, 556, 663, **705**  
 formula (of a formal language), xxii, 19, 412, **412**, 413, 663, 683  
 $FP_2$ , 106, **446**, 483, 688  
 $FP_n$ , **447**, 637  
 fp-flat module, **397**  
 fp-idempotent ideal of category, **409**, 490, 518, 596, 597  
 fp-injective module; *see* absolutely pure module, **57**  
 fp-mono-injective module, **397**  
 fractional ideal, **75**  
 fractions  
   category of, 463, 651  
   ring of; *also see* localisation of ring, 105, 275, 460, 549  
 free abelian category, xxvii, 359, **448**, 453, 484, 527, 530, 661, 664, 684  
 free associative algebra, 172, 526  
 free module, 117  
 free realisation, **23**, 34, 38, 50, 69, 83, 89, 119, 165, 191, 220, 249, 356, 387, 433–435, 440, 443, 455–456, 486, 501  
 free variable, xxv, **5**, 19, 32, 92, 93, 125, 279, 412, **412**, 484, 521, 629, 683  
 Freyd's Generating Hypothesis, 647  
 Frobenius map, 700  
 fs topology, *see* full support topology, **525**  
 full embedding, 641  
 full functor, 157, 278, 316, 322, 423, 495, 684, **704**  
 full on pure-injectives, **679**, 684, 694  
 full subcategory, 118, 309, **704**  
 full support topology, **264**, 288, 295, **525**, 559, 562, 577  
 function defined by pp condition; *see* definable scalar, 660  
 functional (relation), **281**, 294  
 functor  
   commuting with .../preserving ..., **142**  
   defined by pp condition, **10**, 11, 19, 26, 28, 90, 94, 106, 165, 231, 419, 443, 445, 446, 448, 452, 515  
   reflecting ..., **142**

- functor category, xviii–xx, xxii, xxiii, xxv, xxvii,  
 29, 94, 144, 146, 186, 219, 228, 272, 279,  
 290, 333, 420, 423–456, 477, 493, 502,  
 510, **515**, 517, 539, 543, 553, 608, 640,  
 649, 656, 658, 669, 675, 676, 684, 686,  
 700, 701, 703, 705, 708  
 localised, 231, 242, 315, 503, 512–515,  
 519–520, 529, 555, 611, 664, 670  
 functor preserving (co)limits, 705  
 functor preserving direct limits, 451, 465, 476,  
 687  
 and products, 275, 495, 592, 641, 677, 682,  
 692  
 functor preserving direct products, 677  
 functor preserving induced structure, 680, **684**  
 functor preserving pure-exact sequences, 46, 678  
 functor preserving pure-injectivity, 142, 276,  
 278  
 functorial approach, summary, 107, 161, 231  
  
 G-set, 702  
 Gabriel filter/topology, 462, **468**, 619  
 Gabriel–Popescu Theorem, **477**, 708  
 Gabriel–Zariski spectrum, xviii, xx, 394, 527,  
**553**, 557, 574  
 Gabriel–Zariski topology, xx, **553**, 554, 558, 561  
 Gelfand–Ponomarev algebra, 7, **307**, 344, 387,  
 389, 584  
 generalised Weyl algebra, 135, 323, 358, 363,  
 364, 367–374, 575, 699  
 generated by objects, **401**  
 generated definable subcategory, xxvi, 36, 107,  
 109, **109**, 111, 126, 153, 160, 161, 176,  
 180, 182, 185, 186, 196, 211, 214, 215,  
 246, 252, 282, 359, 416, 451, 520, 578,  
 597, 647, 664, 695  
 generated filter, 353, 693  
 generated ideal, **399**, 693  
 generated pp-type, **21**, 212, 237, 441  
 generated subsheaf, **625**  
 generating (a category), 422, 427, 465, 467, 471,  
 478, 481, 485, 487, 618, 623, 625, 636,  
 657, 676, 684, **707**, 710, 712  
 generating, in triangulated category, **636**  
 generator, of subfunctor, **426**  
 generic module, xviii, 111, **200**, 327, 333, 334,  
 341, 342, 346, 352, 565, 566, 591–593, 689  
 generic point (of closed set), **269**, 270, 271, 278,  
 544, 551, 563  
 generic tuple, **293**, 295  
 generic type, 697  
  
 generically tame, **202**  
 generically wild, **202**  
 Giraud subcategory, **708**  
 global section, 562, 571, 574, **715**  
 Goldie ring, 182, 201  
 Goodearl’s localisation, 570  
 Goursat’s Theorem, 217  
 graded module, 581  
 Grassmannian, 700  
 Grothendieck category, 28, 45, 178, 422, 424,  
 461–487, 498, 533, 553, 606, 607, 612,  
 622, 642, 673, **708**, 709, 712, 716, 717  
 Grothendieck topology, 464, 468  
 group, 600  
 group ring, 195, 318, 398, 600–698  
  
 Harada–Sai Lemma, **192**, 309, 403  
 Hausdorff space, 134, 135, 392  
 hereditary algebra, 198, 202, 266, 311, 322,  
 327–343  
 hereditary artin algebra, 327–343, 540  
 hereditary noetherian prime ring, *see* HNP ring,  
 332  
 hereditary order, 222, 336, 570, 573  
 hereditary ring, 53, 56, **221**, 275, 335, 362, 365,  
 367, 371, 373, 639, 650  
 hereditary torsion class; *see* torsion (sub)class,  
**461**  
 hereditary torsion theory, *see* torsion theory, **462**  
 hereditary torsionfree class, *see* torsionfree  
 (sub)class, **461**  
 Herzog’s Criterion, **34**, 187  
 Heyting algebra, complete, **267**, 693  
 highest weight idempotent, **362**  
 highest weight space, 20  
 Hilbert Basis Theorem, 188  
 HNP ring, **75**, 183, 224, 332, 335, 341, 355–358,  
 378, 569, 572  
 Hochster dual, 550  
 holonomic dual, 371, **373**  
 home sort, 283  
 homeomorphic at level of topology, 267, **267**,  
 559  
 homeomorphic embedding, 274, 277, 655  
 homeomorphism, 267, 268, 270, 648, 689, 695  
 homogeneous tube, **590**  
 homological functor, **637**, 641  
 homology, 637, 649, 650, **706**  
 homology colimit, **641**, 644, 652  
 homology/cohomology, **633**  
 homomorphism of lattices, 713

- homotopy, **632**
- homotopy category, **633**, 634, 637, 645, 647, 652
- homotopy theory, 643
- Hopf algebra, 653
- hull, 170
  - of element/tuple, **154**, 314, 375, 658, 698
  - of pp-type, **154**, 212, 219, 236, 380, 506, 663
  - of pp-type modulo a pp condition, **169**
- ideal of
  - category, **399**, 399–410, 488, 500, 596, 634, 646, 674
  - lattice, **102**, 160, 352, 361
  - ring, 117, 286, 320, 394, 517, 651
- ideals topology, **559**, 578
- idempotent, 66, 67, 102, 117, 165, 228, 287, 306, 319–320, 443, 496, 583, 716
- idempotent ideal, **221**, **408**
- idempotent ideal of category, 205, **408**
- idempotent morphism, 410, **704**
- idempotent object, 653
- idempotents lift, **165**, 208, **498**
- idempotents split, **428**, 605, 606, **704**
- image, 424
  - of functor, **704**
- imaginaries, xix, 91, 555, 604, 656, 658, 664, 692, 696, 698
- imaginaries structure, 450, 661, 664, 692
- implies (of pp conditions); *see* pp conditions, ordering on, **11**
- indecomposable idempotent; *see* primitive idempotent, **77**
- indecomposable module/object, 191, 192, 195, 204, 496
- indexed system, **714**
- indiscrete ring, **395**, 395–398, 700, 701
- induction, 601, 701
- inessential subobject, **116**, 440
- infinitary formula, 38
- infinitary language, 628, 668, 701
- infinite word, 344, 346–350
- injective
  - indecomposable, 29, 181, 186, 213, 214, 218, 231, 235, 328, 387, 392, 478, 502, 510, 525, 552–554, 557, 559, 575, 577, 654, 709, 716, 717
  - over arbitrary morphism, 517
  - wrt torsion theory, 289, **462**, 465, 474, 620
- injective cogenerator, 36, 116, 152, 177, 193, 241, 259, 290, 520, 585, **709**
- injective hull, 29, 80, 108, 115, 146, 153, 206, 207, 212, 231, 236, 245, 352, 356, 362, 374, 403, 406, 461, 463, 501, 506, 519, 526, 551, 554, 560, 600, 663, **708**
- injective hull, wrt torsion theory, **463**
- injective module/object, xx, xxii, xxvii, 35, 58, 63, 66, 76, 80, 91, 106, 116, 130, 139, 144–146, 173, 177, 206, 241, 265, 272, 354–356, 360, 369, 373, 383, 389, 391, 428, 429, 456, 461, 465, 466, 480, 497, 500, 510, 513, 526, 550, 558, 579, 587, 611, 616, 644, 651, 653, 668, 673, **708**
- injective resolution, 653
- injectives, category of, 500
- integral group ring, 118, 323, 388, 698
- internality, 697
- interpretation, xx, 8, 601, 657, **681**, 699
- interpretation functor, 311, 334, 570, 678, **682**, 682–690, 692
- interval in lattice of
  - pp conditions, 182, 199, 231, 242, 245, 261, 313, 435
  - pp-pairs, 522
- invariant basis number, 202
- invariant,  $\Gamma$ -, **393**
- invariants statement, 110, 413, 697
- inverse image sheaf, 568
- inverse limit, 56, 59, 101, 114, 133, 333, 372, 590, 591, 593, 690, **704**
- inverse limits, to have, **714**
- invertible ideal, **75**
- irreducible,  $\wedge$ -, 551, 578
- irreducible closed set, **269**, 278, 551, 563, 580
- irreducible element, 375
- irreducible morphism, 402, 435, **587**
- irreducible pp condition, **29**, 501
- irreducible pp-type, **159**, 170, 218, 219, 254, 265, 313, 379, 382, 505, 663, 693
- irreducible (right) ideal, **29**
- isolated by minimal pair, 167, 194, **234**, 235–239, 246, 248, 250, 271, 349, 357, 374, 381, 383, 392, 518, 545, 664
- isolated by simple functor, **519**, 664
- isolated in closed set by simple functor, **519**
- isolated point, 111, 194, 196, 199, 224, 231–250, 252–254, **260**, 261, 266, 269, 326, 341, 355, 358, 360, 362, 369–370, 372–374, 381, 383, 386, 392, 526, 554, 565, 598, 648
- isolation condition, 195, **239**, 262–263, 271, 272, 317, 319, 349, 371, 381, 391, 521, 541, 544



- isolation property, category has, **521**  
 isotropic module, **339**
- Jacobson radical, *see* radical, of ring, 716  
 Jacobson–Zariski topology, **362**  
 Jensen–Lenzing Criterion, **140**, 644, 645, 674, 679  
 join, 267, 298, 394, **712**  
 jointly initial, **120**
- kernel, 424, 432, 609  
 of functor, 506, 514, **705**  
 of lattice morphism, 261, **299**, 524, **713**  
 Kronecker algebra, 573  
 Kronecker quiver, **8**, 56, 58, 276, 328–334, 342, 345–346, 348, 583, 681  
 Krull Intersection Theorem, 188  
 Krull–Gabriel dimension, 602  
 Krull–Remak–Schmidt–Azumaya Theorem, 80, 164, 171, **717**  
 Krull–Schmidt category, **164**, 400  
 Krull–Schmidt ring, 29, 53, **164**, 191, 193, 229, 250, 253, 257, 380, 441, 501, 541, 601, 648
- language (formal), xxii, 168, 411, 414, 485, 627, 649, 656, 657, 664, 682, 700, 703  
 large in, **170**  
 lattice, 298  
 of morphisms, **86**  
 of pp conditions, *see* pp conditions, lattice of, **16**  
 of pp-definable subgroups, *see* pp-definable subgroups, lattice of, **87**  
 lattice (module), 122, 690, **691**, 698  
 lattice, freely generated, 79, 304, 318  
 Laurent polynomial ring, 700  
 Laurent series, 376  
 Lawvere–Tierney sheafification, 464  
 left exact functor, 462, 607, 672, 705, **705**  
 length of tuple, **xxiv**  
 Lie algebra, 358  
 linearly compact module, **131**, **132**  
 linked, **167**, 243, 382  
 linking, 664  
 local endomorphism ring, module with; *see* strongly indecomposable module/object, 29  
 local functor, 191, 249, **441**  
 local module/object, **164**, 165, 441  
 local object in triangulated category, **652**  
 local ring, 56, 135, 150, 160, 176, 179, 269, 547, 562, 565, 593, **716**
- locale, **267**  
 localisation, 93, 99, 231, 232, 241, 359, 633, 656, 657  
 of ring, 54, 75, 118, 135, 157, 207, 222, 288, **288**, 369, 372, 373, 567, 573  
 localisation functor, 118, **463**, 463–491, 536  
 localisation/localised category, xxv, 99, 106, 107, 255, 259, 460, **463**, 491, 508, 524, 527, 533, 554, 708  
 localising subcategory  
 in triangulated category, 637, **651**  
 of Grothendieck category, *see* torsion subcategory, **461**  
 locally artin algebra, **599**  
 locally atomic model, 680, 699  
 locally closed, 625, **626**  
 locally coherent category, 431, 481–486, 497, 499, 512, 521, 534, 535, 542, 553, 606, 608, 611, 619, 668, 672, **712**  
 locally coherent module/object, 42, **63**, 82–83, 448, 614  
 locally coirreducible category, **521**, 526  
 locally compact space, **626**  
 locally finite group, 398  
 locally finitely generated category, 462, 466, 467, 470–473, 525, 625–627, 709, **712**  
 locally finitely presented category, xviii, xx, xxi, 45, 105, 186, 207, 470, 485, 529, 553, **604**, 603–629, 656, 678, 701, 709, **711**  
 locally matricial, **393**  
 locally noetherian category, 606, **712**  
 locally noetherian space, **625**  
 locally simple functor, 346, **544**  
 Łoś’ Theorem, 102, **414**, 507  
 Löwenheim–Skolem Theorem, 51, 396, 415, 416, 676
- mapping cone, 634  
 maximal ideal, 118, 134, 158, 224, 332, 387, 392, 394, 566, 653  
 maximal order, 336, 339–340, 570  
 meet, 91, 267, 298, 394, **712**  
 minimal free realisation, 24, 166, **440**  
 minimal module over a tuple, **440**  
 minimal pair, **88**, 90, 93, 234, 239, 246, 248–250, 302, 314, 317, 355, 435, 518  
 relative, 151, 160, **233**, 234–236, 261, 272, 519  
 type has, **233**

- Mitchell Embedding Theorem, 423  
 Mittag–Leffler (module), **40**, 387, 448, 698  
 model, **412**, 415  
   of set of sentences, **413**  
 model category, 634  
 model theory, xviii–xxi, 18, 19, 39, 44, 83, 91,  
   102, 104, 105, 107, 136, 153, 168, 218, 231,  
   244, 247, 254, 255, 411, 415, 444, 450,  
   474, 550, 601, 657, 660, 664, 671, 680, 683  
   of modules, xviii, xix, xxi, xxiii, 106, 158,  
   209, 214, 300, 377, 411, 524, 696, 700,  
   701  
 model-companion, 700  
 model-complete, 701  
 model-completion, 700  
 modular lattice, 86, 91, 246, 298, 299, 312, 535,  
   **712**  
 modular model, non-, 697  
 module  
   *k*, *l*-presented, **68**  
   dual gen,rel, **69**  
   gen,rel, **69**  
 module  
   over a sheaf of rings; *see* sheaf of modules,  
   **716**  
   over preadditive category, **421**  
 monic, **423**  
 monomorphism, 424  
 monopresheaf, **620**  
 Morita equivalent, 76, 88, 196, **216**, 224, 275,  
   277, 283, 340, 551, 570, 585  
 Morita-invariance, **46**, 564  
 morphism of  
   degree 0, **632**  
   lattices, **300**, **713**  
   presheaves, **715**  
   triangles, **631**  
 natural transformation, 420, **423**, 425, 495, 638,  
   678, 686, 703, **705**  
 nearly simple ring, **81**  
 nearly simple uniserial domain, 81, 384  
 neg-isolated, 194, 232, **254**, 254–259, 266, 272,  
   519, 664  
   by, **254**  
   by simple functor, **520**  
   in closed set, **520**  
 negation, 17, 102, 411, 414  
 nine(=  $3 \times 3$ ) lemma, 484  
 noetherian algebra, **200**, 202  
 noetherian object, 238, **712**  
 noetherian ring, 6, 58, 63, 73, 80, 116, 132, 135,  
   173, 177, 187, 201, 207, 304, 324, 335,  
   352, 356, 359, 367, 378, 457, 541, 550,  
   551, 553, 561, 575, 577, 614, 655  
 noetherian space, **621**  
 non-additive context, 604, 658, 701  
 non-singular ring, 135  
 normal space, 392  
 one-point compactification, 554  
 open (pp-pair), **26**  
 opposite category, 612, **706**  
 orbit, 367, 370, 372  
 order, 122  
 Ore condition, 183  
 Ore domain, **352**, 354, 356, 368  
 Ore ring, **356**  
 orthogonal category, 465, 652, 690, 701  
 orthogonal idempotents, **77**, 217, 420, 540  
 orthogonal type, 697  
 p-base, 700  
 pair, of groups, 699  
 path, **583**  
 path algebra, 24, 56, 196, 494, **583**, 650, 684,  
   685  
 path category, **421**, 422, 443, 494, 554  
 perfect complex, 581, 637  
 perfect field, 700  
 perfect ring, 60, **116**, 179, 190, 207, 208, 457  
 Pext, 55, 646  
 phantom cover, 646  
 phantom map of modules, **646**  
 phantom morphism, **636**, 641, 643, 645–647  
 PI Dedekind domain, 136, 223, 234, 272, 292,  
   376, 540, 547, 559, 564, 566, 592  
 PI ring, 199, 201, **222**, 332, 335, 336, 341, 569,  
   572, 592  
 Pierce sheaf, **394**  
 Pierce spectrum, **394**  
 pointed module, **85**, 305, 316, 321, 322  
   A-, **85**, 88, 316, 324  
 pointed modules, morphism of, **85**  
 polynomial identity ring; *see* PI ring, **222**  
 poset, 298, **712**, 715  
 positive existential formula, 702  
 positive primitive (pp) formula; *see* pp  
   condition, **412**  
 power series ring, 139, 377  
 power set, **715**  
 power, direct, 140, 179, 180, 187, 293, 294, 529

- pp condition, xx, xxii, **4**, 4–39, 43, 46, 69–74, 89, 91–99, 103, 115, 119, 125, 130, 159, 161, 165, 227, 243, 254, 264, 275, 281, 284, 290, 313, 316, 326, 338, 346, 360, 380, 383, 396, 412, 433, 436, 437, 440, 442, 446, 454, 475, 484, 485, 505, 509, 520, 526, 577, 608, 627–629, 649, 656, 658, 660, 663, 683, 690
- conjunction of, 15, **16**, 27, 32, 70–72
- defines a scalar, **281**
- equivalent, **12**, 36, 92, 514
- implication between, *see* pp conditions, ordering on, **11**
- large, *see* large in, **170**
- lattice of, xxvi, **16**, 21, 31, 67, 78, 79, 83, 87–91, 113, 189, 190, 196–198, 231, 235, 240, 244, 247, 261, 267, 303, 310, 319, 320, 322, 323, 353, 371, 384, 387, 410, 434, 436, 445, 486, 514, 521, 539, 546, 663, 693, 701
- linking; *also see* linked, **167**
- ordering on, **11**, 12–15, 26, 31, 35, 79, 83, 90, 92, 163, 169, 305, 353, 384, 445, 485
- sum of, **16**, 32, 69, 70, 72, 74, 78, 165
- with parameters, **124**, 379
- pp-definable complement, 287, **532**
- pp-definable coset, **125**, 174–175
- pp-definable function, **93**, 343, 444, 445, 487, 669, 684
- pp-definable subgroup, **4**, 4–11, 15, 23, 25, 26, 34, 35, 53, 59, 61, 74, 77, 90, 118, 125, 128, 135, 137, 173, 177, 180, 183, 187, 201, 217, 287, 352, 354, 364, 368, 413, 487, 532, 598, 629, 697
- examples, 5–10
- lattice of, xxvi, 15–18, 36, 57, 59, 64, **87**, 99, 112, 149, 160, 180, 272, 300, 314–315, 360, 378, 382, 514, 602
- pp-elimination of quantifiers, 108, 110, **413**, 657, 658, 697
- pp-essential embedding, **156**, 168
- pp- $n$ -type, **19**
- pp-pair, **12**, **26**, 39, 65, 88, 91, 105, 107, 110, 112, 149–151, 185, 213, 217–221, 231, 262, 268, 283, 314, 354, 359, 409, 414, 443–445, 451, 455, 473, 486, 499, 513, 514, 516, 529, 663, 677, 687
- (closed), **26**, 50, 92, 105, 110, 121, 186, 211, 215, 231, 282, 499, 508, 514, 627, 658, 664, 668, 679
- (open), 92, 149, 211, 213, 214, 313, 375, 691
- category of, xxvi, xxvii, **91–99**, 112, 186, 221, 272, 283, 285, 444, 487, 515, 529–530, 656, 660, 663, 683
- pp-type, xxii, **18–38**, 50, 83, 86, 125–131, 153–157, 159, 174, 181, 212, 236, 249, 293, 356, 375, 379, 380, 382, 434, 435, 437, 440, 504–508, 520, 526, 663, 692, 700, 703
- finitely generated, **21**, 34, 40–42, 87, 91, 163, 189, 237, 295, 387
- homogeneous, **130**
- modulo a pp condition, **168**, 506, 507
- with parameters, **125**, 143, 148, 175, 236
- pp-type-definable scalar, 281, **294**
- pp-type-increasing (morphism), 19, **22**, 158, 256, 305, 309, 402, 409, 440, 441
- pp-type over, **125**
- pp-types
- lattice of, 190, 504
- ordering on, xxv, 22, 89, 91, 142, 293
- Prüfer module, 111, 222, **224**, 266, 268, 281, 292, 332, 334, 340–343, 346, 348, 372, 376, 389, 465, 548, 566, 568, 569, 571, 572, 590
- Prüfer ring, **74**, 247
- preenvelope, **206**
- premodule, **716**
- preadditive category, 219, 399, **420**, 423, 444, 496, 516, 553, 605, 684, **706**
- precover, **206**
- predecessor-closed, 121
- preinjective component, 331
- preinjective module, 58, **198**, 266, 328, **328**, 334, 338, 572
- preprint, **xxv**
- preprojective component, 330, 587
- preprojective module, 122, **198**, 266, **328**, 334, 338, 572
- prescheme, 625
- presentation, have a, 676, **676**
- presheaf, 549, 604, **715**
- of categories of definable scalars, 554, **556**
- of definable scalars, 570, 572, 695
- of rings, 282, 618
- of rings of definable scalars, 556
- presheaves, category of, 464, 471, 618–622
- prime ideal, xx, 118, 134, 157, 343, 377, 388, 547, 551, 552, 560, 561, 565, 566, 569, 572, 576, 654, 699

- prime ring, **75**  
 primitive factor ring, 365, 367  
 primitive factor rings artinian, 392  
 primitive ideal, 362  
 primitive idempotent, **77**, 379, 540, 691  
 principal filter, **102**  
 principal ideal, 383  
 principal ideal domain, 369, 374, 376  
 product of rings, 217  
 product, direct, 19, 40, 44, 48, 100, 111, 118,  
 120, 134, 136, 196, 207, 251, 258, 259,  
 282, 290, 327, 342, 347, 350, 360, 413,  
 461, 466, 574, 578, 597, 606–608, 610,  
 638, 654, 664, 667, 675–676, 715  
 product-complete, 260  
 projective cover, **116**, 165, 206, 403, 406, 440,  
 501, 600  
 projective generator(s), 75, 276, 442  
 projective line, 370, 574  
 projective module/object, xxvii, 28, 60, 62, 66,  
 82, 106, 116, 122, 165, 206, 328, 422,  
 427–429, 443, 450, 494, 531, 554, 556,  
 583, 585, 587, 605, 637, 646, **705**  
 projective presentation, 335, 337, 410, 432,  
 438, 444, 446, 451, 489, 494, 497, 518,  
 686  
 projective resolution, 63, 165, 186, 600, 634,  
 653, **705**  
 pseudo-finite-dimensional (representation), **363**,  
 699  
 pseudocokernel, 27, 427, 607–608, 635, 649,  
**704**  
 pseudokernel, **704**  
 pseudoweight, 362  
 pullback, 49, 51, 218, 243, 388, 466, 472, 479,  
 482, **707**  
 pullback ring, 323, 387–390, 698  
 pure embedding, 37, **44**, 44–51, 53, 57, 66, 68,  
 73, 76, 90, 103, 114, 117, 118, 138, 145,  
 153, 156, 164, 207, 208, 238, 251, 285,  
 438, 480, 597, 664, 691  
   in definable category, **676**  
   in finitely accessible category, **609**  
   in triangulated category, **643**  
 pure epimorphism, **49**, 51, 62, 109, 119, 151,  
 251  
   in finitely accessible category, **609**  
   in triangulated category, **643**  
 pure submodule/subobject, 19, 42, **44**, 51, 56,  
 88, 101, 107, 112, 120, 126, 131, 176, 178,  
 285, 413, 414, 667, 679  
 pure triangle, 641  
 pure-essential, **145**  
 pure-exact sequence, **45**, 54, **54**, 61, 62, 66, 71,  
 100, 136, 152, 205, 391, 495, 497, 600,  
 610, 616, 649, 673, 678  
   in finitely accessible category, **609**  
   in triangulated category, **643**  
 pure-exact triangle, 647  
 pure-injective  
   indecomposable, 155, 157–159, 161, 170,  
 171, 178, 181, 210, 211, 213, 259, 263,  
 274, 278, 307, 313, 350, 352, 382, 384,  
 389, 458, 518, 553, 557, 559, 562, 566,  
 574, 579, 645, 680, 692, 717  
   infinite-dimensional indecomposable, 197,  
 252, 326, 333, 337, 339, 352, 403, 568,  
 592, 598, 653  
 pure-injective hull, xvii, 80, 108, 109, 111, 131,  
 134, 135, **145**, 145–151, 153, 162–164,  
 172, 207, 212, 226, 237, 238, 248, 262,  
 278, 282, 304, 315, 348–350, 355, 365,  
 370, 372–374, 376, 383, 389, 452, 497,  
 498, 510, 617, 644, 658, 679  
 pure-injective hull, in triangulated category,  
**645**  
 pure-injective module/object, xvii, xxii, xxvi,  
 22, 43, 52, 53, 55, 56, 66, 74, 79, 80, 111,  
 132, 134–137, **138**, 143, 145, 148, 151,  
 153, 167, 194, 201, 205, 236, 247, 251,  
 258, 269, 293, 294, 313, 318, 323, 327,  
 332, 341–343, 354, 370, 377, 406, 458,  
 497, 507, 513, 520, 521, 529, 593, 601,  
 611, 616, 629, 645, 646, 649, 650, 653,  
 662, 674, 677, 678, 690, 697–699, 701  
 pure-injective object, in triangulated category,  
 641, **644**, 644–645, 648  
 pure-injective resolution, **54**  
 pure-injective ring, 135, 139, 176, 179, 207,  
 208, 374  
 pure-injectives, category of, 500  
 pure-projective module/object, 41–42, **51**, 55,  
 79–84, 121, 167, 187, 327, 387, 601, 698,  
 699  
 pure-projective object, in triangulated category,  
 641  
 pure-projective resolution, **54**  
 pure-semisimple ring, xviii, 55, 56, **189**, 196,  
 197, 199, 202, 253, 647, 649, 701  
 Pure-Semisimplicity Conjecture, 55, **198**  
 purely equivalent, **215**  
 purely opposed, 271

- purity, theory of, xx, xxii, 42, 44, 109, 207, 480, 493, 610, 616, 629, 643, 648, 658, 667, 668, 674, 676, 690
- pushout, 27, 48, 86, 89, 461, 704, **707**
- quantifier-free condition, xxii, **5**, 11, 28, 30, 32, 50, 66, 67, 83, 129, 185, 221, 224, 353, 368, 379, 391, 410, 436, 439, 445, 450, 473, 487, 505, 564, 629, 649
- quantifier-free formula, **412**
- quantum plane, 363
- quantum Weyl algebra, 324, 699
- quasi-coherent sheaf, 625, 637
- quasi-Frobenius ring, 207, 401, **634**, 637, 645, 649
- quasi-isomorphism, **633**, 652
- quasi-simple module, 111, 328–343, 572, 573, **590**, 592
- quasi-small module, **81**, 83
- quiver, 103, 321, 421, 429, 554, **582**  
 with relations, 343, **584**
- quotient category  
 by Serre subcategory, 484, 528  
*see* localisation/localised category, **463**
- quotient ring, 158, 182, 201, 222, 292, 352, 370, 373, 374, 376, 573
- radical  
 $\omega$ -, **402**, 598  
 of category, 310, **400**, 401–410, 592  
 of module/object, 194, 439, **441**  
 of ring, 150, 165, 200, 263, 383, 400, 540, 585, 716  
 of endomorphism ring, 19, 158, 204, 498  
 transfinite, 307, 349, 381, **402**, 404, 406, 407, 540
- rank function, Sylvester matrix, **184**
- rank of morphism, 401, 403
- ray, 333, 340, **590**, 592
- RD domain, 353, 356
- RD ring, 44, **73**, 227, 375, 384
- realisation (of pp-type), 104, **126**, 155
- reduced lattice, **691**
- reduced power, **101**, **715**
- reduced product, 56, **100**, 102–104, 107, 115, 126, 148, 647, 664, **715**
- reflect (isomorphism), **276**
- reflexive, 114, 194, **271**, 271–273, 291, 304, 371
- regular module, **328**, 330, 334, 337, 339
- regular type, 697, 699
- relation, **584**
- relation symbol, 661
- relativisation, 231, 508
- rep-Zariski spectrum, 195, 200, 326–327, 350, **553**, 558, 563–575, 577, 655
- representable functor, 11, 221, 256, **425**, 427–434, 436, 445, 451, 486, 531, 538, 556, 605, 637, 642, 676, **706**, 709
- representation embedding, 111, **276**, 309, 321, 334, 336, 343, 648
- representation of quiver, 186, 422, **582**, 683, 692
- restriction, 601, 664, 701  
 in presheaf, **715**  
 of scalars, 89, 112, 141, 161, 274, 277, 278, 284, 335, 530, 679
- right exact functor, 495, 498, 705, **705**
- ring  
 of definable scalars, xx, xxv, 279, **281**, 353, 359, 365, 371, 373, 527, 549, 560, 562, 566–573, 664  
 of definable scalars, in sort, **283**  
 of quotients; *see* localisation of ring; *also see* quotient ring, **288**  
 of type-definable scalars, **295**  
 with many objects, 55, 399, 421, **421**, 449, 488, 556, 608
- ringed space, 394, 471, 550, 561, 581, 625, 627, **716**
- ringoid, *see* ring with many objects, **421**
- rotation, 634
- $\Sigma$ -algebraically compact, *see*  $\Sigma$ -pure-injective, **173**
- $\Sigma$ -injective, **177**, 354, 391, 498
- $\Sigma$ -pure-injective, 90, 132, **173**, 173–187, 189, 207, 216, 222, 238, 263, 273, 290, 295, 343, 354, 374, 387, 498, 591, 592, 700
- $\sigma[M]$ , 612
- $\sigma[M]$ , 604, 617
- satisfies (of structures and sentences), **412**
- satisfies (of tuples and formulas/conditions), xxvii, 9, 34, 44, **412**
- saturated, **136**, 698, 699
- Schanuel's Lemma, 56
- scope of quantifier, **412**
- section, of presheaf, 618, 622, **715**
- self-duality, **199**
- self-injective ring, 208, 498
- semantic, 92, 415
- semiduo ring, **541**

- semiartinian  
 category, **534**  
 object, **534**  
 ring, 319, 391, 395, **534**, 535, 543
- semihereditary ring, 66, **75**, 353
- semilocal ring, **165**
- seminoetherian  
 category, **534**, 542  
 ring, 319, 543
- semiperfect ring, **165**
- semiprimary ring, 176, 207
- semiprime ring, 182
- semisimple artinian ring, 58, 66, 75, 195, 691
- semisimple category, 617
- sentence, **412**, 413, 415
- separably closed field, 700
- separated module, **388**, 389
- separated presheaf, **620**, **621**
- separated representation, **389**
- separating tubular family, 351
- serial module/object, **77**, 80, 350, 386
- serial ring, 53, 74, 76, **77**, 164, 167, 215, 228, 315, 319, 377–387, 539–541
- Serre subcategory, 99, 219, 359, 401, 409, 460, **463**, 482–484, 487–491, 514, 516, 527, 534, 535, 543, 555, 581, 597, 648, 651, 663, 668, 674
- sheaf, 571, **716**  
 of locally definable scalars, xx, 195, 394, **556**, 567–568, 574, 669  
 of modules, 207, 394, 619, 629, 697, **716**
- sheafification, 464, 471, 555, 619, 621, 623
- sheaves, category of, 464, 471, 619–629
- shift functor, 600, **631**, 654
- simple artinian ring, 184, 201, 395
- simple functor, 161, 166, 191, 231, 235, 240, 241, 254, 255, 435, 518–521, 544, 599
- simple module/object, **93**, 111, 330, 331, 334, 355, 357, 358, 364, 368, 371, 374, 392, 403, 429, 526, 534, 535, 538, 554, 616, 709
- simple ring, 73, 365, 367, **368**, 369, 392, 395–397, 517
- simple theory, 697
- simply reflexive, 271
- skeletally small category, xxv, 420, 444, 485, 530, 553, 604, 671, 675, **703**
- small category, xxv, **703**
- small over, 155
- socle, **xxvi**, 236, 255, 389, 391, 534, 587
- solution of pp-type, 89, 104, **126**, 148, 293, 314
- solution set of  
 pp condition, **4**, 126  
 pp-type, **20**, 25, 506  
 quantifier-free condition, **4**  
 sort, 168, 184, 433, **442**, 445, 473, 475, 485, 529, 556, 627, 658, 661, 663, 669, 681, 683, 685, 693, 699
- special biserial algebra, 407
- species, 335
- spectral space, **278**, 548
- spectrum, 276, 319, 647  
 of ring, 118, 569  
 based on simples, 571  
 Cohn, 201
- sphere, 647
- split epimorphism, 587, **704**
- split exact sequence, 497, 610
- split monomorphism, 44, 45, 147, 587, 676, **704**
- splittable system, 316, 322
- stability theory, 696, 697, 701
- stable module category, 205, 401, 500, **634**, 637, 640, 643, 645, 652
- stalk, 283, 555, 561, 562, 575, 620, **716**
- Stone space, **394**, 697
- strict representation embedding, **278**, 310
- strictly wild algebra, **202**, 321, 685, 698
- string, **343**, 407
- string algebra, 16, 318, 322, **343**, 349, 350, 387, 407, 540, 574, 653, 698
- string module, 7, 307, 329, 344, **345**, 389
- strongly indecomposable, 716
- strongly indecomposable module/object, 18, **29**, 37, 80, 81, 157, 162–164, 166, 179, 190, 208, 250, 273, 356, 439, 440, 501, 541, 586, 644
- strongly minimal, **697**
- strongly unbounded (representation type), **202**
- strongly uniform module, 393, **393**
- structure sheaf, xx, 279, 527, 550, 554, **561**, 569
- subfunctor, **423**
- subgenerated (category), **612**
- subgroup of finite definition; *see* pp-definable subgroup, **5**
- submodules/subobjects, class of, 112
- subquotient, 217, 221, 301, 313, 403, 406, 437, 483, 502–503, 513, 544, 594
- sum, direct, 19, 27, 48, 139, 172, 173, 177–178, 304, 342, 350, 457, 469, 598, 636, 642, 651
- summation map, **140**
- superatomic boolean algebra, **394**

- superdecomposable, 81, 172, **172**, 216,  
 226, 244, 246, 247, 258, 314–324, 351,  
 360, 374, 377, 385, 406, 521, 526, 579,  
**709**
- superstable, 699
- support, 112, 209, **215**, 235, 240, 258, 271, 282,  
 295, 314, 414, 544, 592, 664  
 of section, **626**
- support variety, 601, 653
- suspension, 647
- Sylow subgroup, 195, 323, 601, 640, 698
- Sylvester module rank function, **184**
- syntactic, 92, 415
- syzygy, 446, **600**, 646, 655
- $T_1$  space, 270, 392
- T-nilpotent, **116**
- tame (representation type), xxv, 55, **201**, 266,  
 307, 311, 322, 351, 387, 403, 406, 410,  
 574, 698
- tame hereditary algebra, 111, 122, 278, 327–343,  
 346, 376, 407, 540, 572, 590, 592, 689
- Tate cohomology ring, 653
- Telescope Conjecture, 690
- tensor product, 33–34, 37–40, 53–54, 59, 179,  
 184, 188, 274, 276, 447, 450, 456,  
 493–498, 500–501, 601, 605, 653, 673, 701
- term (of formal language), **411**
- theory, **412**, 697
- thick subcategory, 581, **651**
- tilted from tame, 689
- tilting, 687
- tilting module, **687**
- top of  
 endomorphism ring, 160, 202, 592, 599  
 module/object, 166, **388**, 390  
 ring, 269
- topological module, 131, **133**, 135
- topologically indistinguishable, 267, **270**, 278,  
 382, 386, 578
- topos, 460, 702
- topos theory, xxi, 657
- $Tor_1$ , 448, 688
- $Tor_n$ , 448, 457
- torsion (sub)class/(sub)category, 315, 401, **461**,  
 462–491, 516, 533, 535, 620
- torsion module/object, 105, 356, 371, **462**, 604,  
 619
- torsion radical/functor, **462**, 462–491
- torsion theory, xxv, 264, 288, **461**, 462–491,  
 511, 525, 549, 555, 619, 633, 688, 708
- torsionfree (sub)class/(sub)category, 106, 117,  
 214, **461**, 462–491, 525, 580, 651
- torsionfree module/object, **61**, 76, 105, 113, 122,  
 158, 213, 227, 264, 289, 353, 375, **462**,  
 574, 620, 690, 698
- torsionfree, wrt torsion theory, 509, 510, 513,  
 519
- total (relation), **281**, 294
- totally disconnected space, 392
- totally transcendental, 173
- trace, **26**
- translation functor, *see* shift functor, **631**
- triangle, **631**
- triangulated category, xxi, xxiii, 258, 465, 581,  
 601, **631**, 630–656
- triangulated subcategory, **636**
- trivial lattice, **299**
- trivial module, **144**, 653, 654
- tube, 122, 328, 330, 331, 333–335, 339, 351,  
 372, 376, 377, 569, 573, 589–593  
 generalised, 111, 114, **591**
- tubular algebra; *see* canonical algebra, 351
- 2-category, 467, 675, 678
- type,  $e$ -, **378**
- type-definable, **264**
- types-over-formulas topology; *see* full support  
 topology, 264
- ultrafilter, **101**, 102, 137, 394, 414, 507,  
**715**
- ultrapower, **101**, 107, 507, 610, 676, 679,  
 701, **715**
- ultraproduct, **101**, 102, 136, 137, 414, 415, 507,  
 509, 664, 677, **715**
- undecidability, 311, 374, 687
- undefined, **534**
- unidimensional theory, 697
- uniform, 663
- uniform module/object, 29, 159, 170, 218, 238,  
 243, 245, 271, 393, 501, 505, 507, 551,  
 576, **708**
- uniformly bounded (pp condition), **361**
- unique to isomorphism, **709**
- uniserial module/object, **77**, 81, 247, 317, 372,  
 377, 383, **545**, 547
- uniserial ring, 16, 53, 78, **81**, 377, 382–387
- unit-regular ring, 393
- universal enveloping algebra, 19, 324, **358**, 367,  
 370, 699
- universal localisation, **275**, 334, 335, 337, 339,  
 549, 570, 633

- universally minimal element, 250  
 unseparated space, 568
- V-ring, **374**
- valuation domain, 320, 381, 541  
 valuation ring, 74, 77, 134, **318**, 323, 377, 383  
 variety (universal algebraic), 701  
 Vaught's Conjecture, **699**  
 Verma module, 6, **363**
- von Neumann regular ring, xvii, 6, 55, 58, **66**, 76, 102, 117, 139, 208, 228, 272, 286–287, 297, 306, 319, 359–360, 363, 365, 371, 391–701
- Warfield ring, **75**  
 weakly minimal, **697**  
 weight condition, **361**  
 weight space, 361  
 weight, type with, 697  
 weighted projective line, 328, 351  
 well-powered category, 534, **708**
- Weyl algebra,  $n$ -th, 73, 76  
 Weyl algebra, first, xvii, 6, 74, 75, 232, 253, 287, 324, 355, **367**, 574, 584, 616  
 Weyl algebra, generalised – *see* generalised Weyl algebra, **367**
- wide (poset), **312**, 321  
 width, 216, 244, 258, 312, **314**, 320, 322, 385, 521, 546, 700
- wild (representation type), 55, 56, **201**, 311, 321, 323, 374, 376, 540, 574, 698  
 word, **343**
- Yoneda embedding, **425**, 433, 442, 449, 493, 500, 555, 556, 605, 638, 644, 670, 673, 674  
 Yoneda Lemma, 166, **425**, 427, 430, 445, 497, 674, 706
- $\omega$ -stable, 699, 700  
 $\omega_1$ -incomplete, **137**
- Zariski spectrum, xx, 319, 395, **550**, 551, 553, 571, 575, 577, 580–581, 653–655  
 Zariski topology, 279  
 zero object, **706**  
 ZFC set theory, 55, 639
- Ziegler spectra, duality of, 185, 303, 562, 696  
 Ziegler spectrum, xv, xvii–xx, xxvi, 39, 111, 122, 161, 167, 186, 194, 197, 203, 205, **210**, 229, 250, 260, 268, 270, 277, 282, 303, 306, 325–398, 434, 443, 452, 457, 460, 479, 486, 493, 499, 502–503, 508, 510, 513, 514, 516, 526, 541, 544, 547, 556, 562, 563, 574, 577, 581, 591, 592, 599, 640, 650, 654, 664, 669, 689, 692, 698, 699, 702  
 compactness of, xviii, 252  
 finite, 252  
 of triangulated category, **648**, 648–651  
 Ziegler's Criterion, **159**, 212, 379, 693