

Contents

<i>Preface</i>	<i>page</i>	ix
1 Basic concepts and results		1
2 Probabilistic tools		36
3 Bond percolation on \mathbb{Z}^2 – the Harris–Kesten Theorem		50
3.1 The Russo–Seymour–Welsh method		57
3.2 Harris’s Theorem		61
3.3 A sharp transition		63
3.4 Kesten’s Theorem		67
3.5 Dependent percolation and exponential decay		70
3.6 Sub-exponential decay		76
4 Exponential decay and critical probabilities – theorems of Menshikov and Aizenman & Barsky		78
4.1 The van den Berg–Kesten inequality and percolation		78
4.2 Oriented site percolation		80
4.3 Almost exponential decay of the radius – Menshikov’s Theorem		90
4.4 Exponential decay of the radius		104
4.5 Exponential decay of the volume – the Aizenman–Newman Theorem		107
5 Uniqueness of the infinite open cluster and critical probabilities		117
5.1 Uniqueness of the infinite open cluster – the Aizenman–Kesten–Newman Theorem		117
5.2 The Harris–Kesten Theorem revisited		124
5.3 Site percolation on the triangular and square lattices		129
5.4 Bond percolation on a lattice and its dual		136

viii	<i>Contents</i>	
	5.5 The star-delta transformation	148
6	Estimating critical probabilities	156
	6.1 The substitution method	156
	6.2 Comparison with dependent percolation	162
	6.3 Oriented percolation on \mathbb{Z}^2	167
	6.4 Non-rigorous bounds	175
7	Conformal invariance – Smirnov’s Theorem	178
	7.1 Crossing probabilities and conformal invariance	178
	7.2 Smirnov’s Theorem	187
	7.3 Critical exponents and Schramm–Loewner evolution	232
8	Continuum percolation	240
	8.1 The Gilbert disc model	241
	8.2 Finite random geometric graphs	254
	8.3 Random Voronoi percolation	263
	<i>Bibliography</i>	299
	<i>Index</i>	319
	<i>List of notation</i>	322