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List of notation

- $|A|$: cardinality, 4
 $A \Delta B$: symmetric difference, 38
 $A \square B, \mathcal{A} \square \mathcal{B}$: square/box product, 42
 $A \subset B$: subset (equality allowed), 38
 $A^{(\varepsilon)}$: closed ε -neighbourhood, 211
 A_i : boundary arc of a (discrete) domain, 179, 193
 A_i^+ : outer boundary arc of a discrete domain, 193
 $\text{Bi}(n, p)$: binomial distribution, 29
 $B_r(x)$: ball in a graph, 108
 $B_r(z)$: ball in \mathbb{R}^2 or \mathbb{C} , 183
 $B_r^+(x)$: out-ball in an oriented graph, 104
 \mathbb{C} : complex numbers, 178
 $C_x = \{y \in \Lambda : x \rightarrow y\}$: open cluster containing x , 4
 C_x^+ : open out-cluster, 25
 C_{Λ}^- : out-class graph, 84
 $D_{r,\lambda}$: occupied set in $G_{r,\lambda}$, 242
 \mathbb{E}_p etc: expectation associated to \mathbb{P}_p etc, 6
 $E(\Lambda)$: edge set of a graph Λ , 1
 $E_{\delta}^i(z)$: existence of open separating path, 201
 $E_{r,\lambda}$: vacant set in $G_{r,\lambda}$, 243
 $G(a)$: $G_{r,\lambda}$ with $\pi r^2 \lambda = a$, 244
 $G_A(W)$: generalized Gilbert model, 243
 G_{δ} : discrete domain, 191
 G_{δ}^{\pm} : discrete approximations to a domain, 212
 $G_{n,d}$: finite random geometric graph with degree d , 255
 $G_{r,\lambda}$: Gilbert model, 242
 G_r : $G_{r,1}$, 242
 $G_r(V_n), G_r(\mathbb{T}_{\ell}^2, \lambda)$: finite random geometric graphs, 254
 H : hexagonal lattice, 136
 $H(R) = H_b(R)$: R has a black horizontal crossing, 274
 $H_{n,k}$: k -nearest graph, 257
 I_k : there are k infinite open clusters, 118
 $\mathcal{P} = \mathcal{P}_{\lambda}$: Poisson process, 241
 \mathcal{P}^+ : open points of \mathcal{P} , 268
 \mathcal{P}^- : closed points of \mathcal{P} , 268
 $\mathbb{P}_{\Lambda,p}^b$: probability measure associated to bond percolation, 2
 $\mathbb{P}_{\Lambda,p}^{\text{ss}}$: probability measure associated to site percolation, 2
 Q_p^n, Q_p^n : weighted hypercube, 37
 \overline{Q} : quadrant of $\overline{\mathbb{Z}}^2$, 168
 $R_n(x) = \{x^+ \xrightarrow{n}\}$: there is an open path from $S_1^+(x)$ to $S_n^+(x)$, 87
 SLE_{κ} : Schramm–Loewner evolution, 235
 $S_r(x)$: sphere in a graph, 79
 $S_r^+(x)$: out-sphere in an oriented graph, 86
 T : triangular lattice, 129
 $\mathbb{T}_{\ell}^2, \mathbb{T}^2$: torus, 254
 $\mathbb{T}_n^2, \mathbb{T}^2$: discrete torus, 65
 $V(\Lambda)$: vertex set of a graph Λ , 1
 $V_b(R), V_w(R)$: R has a black/white vertical crossing, 275
 V_x : Voronoi cell of x , 263
 $\overline{\mathbb{Z}}^d$: natural orientation of \mathbb{Z}^d , 30
 $d(x)$: degree of a vertex/site, 21
 $d(x, y)$: graph distance, 61
 $d(x, y)$ in an oriented graph: distance from x to y , 86
 $\text{dist}(x, y)$: Euclidean distance, 211
 d_H : Hausdorff distance, 211
 $f_{\delta}^i(z)$: probability of $E_{\delta}^i(z)$, 207

List of notation

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- $f_p(\rho, s)$: ρ s by s crossing probability, 280
 g_δ^i : continuous extension of f_δ^i , 225
 $h_p(m, n)$: m by n crossing probability in \mathbb{Z}^2 , 59
 $h(m, n)$: $h_{1/2}(m, n)$, 59
 p_H : Hammersley critical probability, 5
 $p_H(d)$: critical probability for Voronoi percolation in \mathbb{R}^d , 269
 p_T : Temperley critical probability, 6
 p_c : p_H or p_T , 9
 $r(C_x)$: radius of open cluster, 61
 in an oriented graph, 86
 s : scale parameter, 279

 $\{x \rightarrow y\}$ or $\{x \rightarrow y\}$: there is an open path from x to y , 3
 $\{x \xrightarrow{n}\}$: there is a long open path from x , 79, 86
 $\{x^+ \rightarrow y\}$: an open path from $S_1^+(x)$ to y , 107
 $\{x^+ \xrightarrow{n}\} = R_n(x)$: there is an open path from $S_1^+(x)$ to $S_n^+(x)$, 87

 Λ : a (usually infinite) graph, 1
 Λ^* : dual graph, 12

 (Λ, \mathbf{p}) : a weighted graph, 138
 Λ_p^b : open subgraph in bond percolation, 2
 Λ_p^s : open subgraph in site percolation, 2
 $\overrightarrow{\Lambda}$: oriented graph, 25
 $\overleftarrow{\Lambda}_x^+$: out-subgraph, 25

 $\beta_i(A)$: influence, 46
 $\delta\Lambda$: rescaled lattice, 179
 δH : rescaled hexagonal lattice, 191
 δT : rescaled triangular lattice, 187
 $\theta(p) = \theta_x^b(p)$, etc: percolation probability, 4
 $\chi(p) = \chi_x^b(p)$ etc: expected cluster size, 6
 $\lambda(\mathbb{Z}^d)$: connective constant, 17
 $\rho_n(p)$: $\sup_x \rho_n(x, p)$, 88
 $\rho_n(x, p)$: probability of $R_n(x)$, 87

 $\partial^-(G)$: inner boundary, 192
 $\partial^+(G)$: outer boundary, 192
 ∂^∞ : external boundary, 13, 216
 \oint^D : discrete contour integral, 207

 a.s.: with probability 1, 240
 whp: with probability $1 - o(1)$, 240