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List of notation

$|A|$: cardinality, 4
$A \triangle B$: symmetric difference, 38
$A \,\square\, B$, $\mathcal{A} \,\square\, \mathcal{B}$: square/box product, 42
$A \subset B$: subset (equality allowed), 38
$A^{(\varepsilon)}$: closed ε-neighbourhood, 211
A_i: boundary arc of a (discrete) domain, 179, 193
A_i^+: outer boundary arc of a discrete domain, 193
$\mathrm{Bi}(n,p)$: binomial distribution, 29
$B_r(x)$: ball in a graph, 108
$B_r(z)$: ball in $\mathbb{R}^2$ or $\mathbb{C}$, 183
$B_r^+(x)$: out-ball in an oriented graph, 104
$\mathbb{C}$: complex numbers, 178
$C_x = \{y \in \Lambda : x \to y\}$: open cluster containing x, 4
C_x^+: open out-cluster, 25
$C_{\overrightarrow{\Lambda}}$: out-class graph, 84
$D_{r,\lambda}$: occupied set in $G_{r,\lambda}$, 242
$\mathbb{E}_p$ etc: expectation associated to $\mathbb{P}_p$ etc, 6
$E(\Lambda)$: edge set of a graph Λ, 1
$E_\delta^i(z)$: existence of open separating path, 201
$E_{r,\lambda}$: vacant set in $G_{r,\lambda}$, 243
$G(a)$: $G_{r,\lambda}$ with $\pi r^2 \lambda = a$, 244
$G_A(W)$: generalized Gilbert model, 243
G_δ: discrete domain, 191
$G_\delta^\pm$: discrete approximations to a domain, 212
$G_{n,d}$: finite random geometric graph with degree d, 255
$G_{r,\lambda}$: Gilbert model, 242
G_r: $G_{r,1}$, 242
$G_r(V_n)$, $G_r(\mathbb{T}_\ell^2, \lambda)$: finite random geometric graphs, 254
H: hexagonal lattice, 136
$H(R) = H_{\mathrm{b}}(R)$: R has a black horizontal crossing, 274
$H_{n,k}$: k-nearest graph, 257
I_k: there are k infinite open clusters, 118
$\mathcal{P} = \mathcal{P}_\lambda$: Poisson process, 241
$\mathcal{P}^+$: open points of $\mathcal{P}$, 268
$\mathcal{P}^-$: closed points of $\mathcal{P}$, 268
$\mathbb{P}^{\mathrm{b}}_{\Lambda,p}$: probability measure associated to bond percolation, 2
$\mathbb{P}^{\mathrm{s}}_{\Lambda,p}$: probability measure associated to site percolation, 2
Q_p^n, $Q_{\mathbf{p}}^n$: weighted hypercube, 37
$\overrightarrow{Q}$: quadrant of $\overrightarrow{\mathbb{Z}}^2$, 168
$R_n(x) = \{x^+ \xrightarrow{n}\}$: there is an open path from $S_1^+(x)$ to $S_n^+(x)$, 87
SLE_κ: Schramm–Loewner evolution, 235
$S_r(x)$: sphere in a graph, 79
$S_r^+(x)$: out-sphere in an oriented graph, 86
T: triangular lattice, 129
$\mathbb{T}_\ell^2$, $\mathbb{T}^2$: torus, 254
$\mathbb{T}_n^2$, $\mathbb{T}^2$: discrete torus, 65
$V(\Lambda)$: vertex set of a graph Λ, 1
$V_{\mathrm{b}}(R)$, $V_{\mathrm{w}}(R)$: R has a black/white vertical crossing, 275
V_x: Voronoi cell of x, 263
$\overrightarrow{\mathbb{Z}}^d$: natural orientation of $\mathbb{Z}^d$, 30

$d(x)$: degree of a vertex/site, 21
$d(x,y)$: graph distance, 61
$d(x,y)$ in an oriented graph: distance from x to y, 86
$\mathrm{dist}(x,y)$: Euclidean distance, 211
d_{H}: Hausdorff distance, 211
$f_\delta^i(z)$: probability of $E_\delta^i(z)$, 207

$f_p(\rho, s)$: ρs by s crossing probability, 280
g_δ^i: continuous extension of f_δ^i, 225
$h_p(m,n)$: m by n crossing probability in $\mathbb{Z}^2$, 59
$h(m,n)$: $h_{1/2}(m,n)$, 59
p_{H}: Hammersley critical probability, 5
$p_{\mathrm{H}}(d)$: critical probability for Voronoi percolation in $\mathbb{R}^d$, 269
p_{T}: Temperley critical probability, 6
p_{c}: p_{H} or p_{T}, 9
$r(C_x)$: radius of open cluster, 61
in an oriented graph, 86
s: scale parameter, 279

'$x \to y$' or $\{x \to y\}$: there is an open path from x to y, 3
$\{x \xrightarrow{n}\}$: there is a long open path from x, 79, 86
$\{x^+ \to y\}$: an open path from $S_1^+(x)$ to y, 107
$\{x^+ \xrightarrow{n}\} = R_n(x)$: there is an open path from $S_1^+(x)$ to $S_n^+(x)$, 87

Λ: a (usually infinite) graph, 1
$\Lambda^\star$: dual graph, 12
$(\Lambda, \mathbf{p})$: a weighted graph, 138
Λ_p^{b}: open subgraph in bond percolation, 2
Λ_p^{s}: open subgraph in site percolation, 2
$\overrightarrow{\Lambda}$: oriented graph, 25
$\overrightarrow{\Lambda}_x^+$: out-subgraph, 25

$\beta_i(A)$: influence, 46
$\delta\Lambda$: rescaled lattice, 179
δH: rescaled hexagonal lattice, 191
δT: rescaled triangular lattice, 187
$\theta(p) = \theta_x^{\mathrm{b}}(p)$, etc: percolation probability, 4
$\chi(p) = \chi_x^{\mathrm{b}}(p)$ etc: expected cluster size, 6
$\lambda(\mathbb{Z}^d)$: connective constant, 17
$\rho_n(p)$: $\sup_x \rho_n(x,p)$, 88
$\rho_n(x,p)$: probability of $R_n(x)$, 87

$\partial^-(G)$: inner boundary, 192
$\partial^+(G)$: outer boundary, 192
∂^∞: external boundary, 13, 216
$\oint^{\mathrm{D}}$: discrete contour integral, 207

a.s.: with probability 1, 240
whp: with probability $1 - o(1)$, 240