# COMBINATORICS AND PROBABILITY

# COMBINATORICS AND PROBABILITY

Celebrating Béla Bollobás's 60th birthday

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# Preface

This volume brings together research in Combinatorics, Probability and Analysis that was presented at the conference entitled "Combinatorics in Cambridge", held in Cambridge during August 4th–7th 2003. The meeting was organized to mark the 60th birthday of Béla Bollobás on August 3rd. The esteem in which Béla is held by mathematicians throughout the world is attested to both by the exceptional academic standard of the conference and by the high quality of the contributions to this volume.

The mathematical activities of the conference took place in the newly built Centre for Mathematical Sciences, home to the Department of Pure Mathematics and Mathematical Statistics in the University of Cambridge. Accommodation was provided in Trinity College, where the memorable conference banquet was held also. The generous support of the London Mathematical Society, the Heilbronn Fund of Trinity College, the British Combinatorial Committee and DPMMS is gratefully acknowledged. The smooth day-to-day running of the conference owed much to the serenity and good humour of Tricia Simmons, of the University of Memphis.

The editors warmly thank all those who have contributed to this volume, both authors and referees, and indeed we thank all those whose participation in the conference served to make it so successful. Furthermore, since each of us has known the privilege that it is to be one of Béla's research students, we take this opportunity to thank him warmly too.

The illustration on the front cover is of the Fountain in Great Court, Trinity College, Cambridge, and was drawn by Gabriella Bollobás.

### Foreword

The thirty-one papers in this volume describe recent research in combinatorics, convex geometry, probability theory, statistical physics and theoretical computer science, and they form the proceedings of the conference "Combinatorics in Cambridge", held in Cambridge during August 4th–7th 2003.

The conference was timed to celebrate the 60th birthday of Béla Bollobás. Speakers were invited with the aim of representing the many areas, both geographical and mathematical, in which Béla has worked. The conference format comprised thirty-five half-hour lectures, with no parallel sessions; this arrangement proved very effective and was well received by the participants, but the time constraints did mean that not all those whom the organizers would have wished to speak were able to do so. We were delighted that so many excellent mathematicians agreed to speak at the meeting: that they did so is a tribute to the very high regard in which Béla is held by the worldwide research community, and the quality of those who participated despite being unable to speak is an equal tribute. We are therefore very grateful to all those participants who have contributed to this volume (and, simultaneously, to the journal *Combinatorics, Probability and Computing*).

The lectures themselves took place in the newly built Centre for Mathematical Sciences, which offered a superb environment both for the lectures and for other mathematical discussions. Memorable non-mathematical features of the conference were the punting afternoon, the piano recital given by Lindesay Dixon and, most unforgettable of all, the conference banquet, of which more later.

It is necessary to say a few words here about Béla Bollobás himself, his life and his work. It is difficult, though, to do so without adopting a valedictory tone which would be completely out of place. Those who know Béla will see at once how a slap on the back and a "well done, old chap" would not fit the bill, and indeed might invite injury. At the age of sixty, Béla is able to look back on nearly five decades of mathematical research of the highest standard, on well over three hundred research papers (with more than a hundred co-authors), on half a dozen

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textbooks and on around forty research students. But it nevertheless remains the case that the conference was not about celebrating achievements that are safely in the past.

Béla is still, as he always has been, a man of enormous energy, and he remains as active as ever: with his army of research students and his countless visitors and collaborators, his enthusiasm is undiminished, and he still seeks out fresh avenues of research. Now is not the time to sum up a career, for who can say what is yet to come? As for the past, even though that, manifestly, was what lay behind the conference, no serious attempt will be made in this foreword to do it justice. The papers in this volume are testimony enough to a man who has, in Endre Szemerédi's words, "played a fundamental role in the development of modern discrete mathematics".

So the remarks that follow are in no way intended to set out Béla's research, nor indeed are they meant to encapsulate his life or his character. Béla is unquestionably a very distinctive man: he is immensely gifted, he has a very powerful personality, and he has an unusual history spanning several countries – it is then no surprise that so many legends, so many myths and stories seem to surround him. Rather than paint a rounded picture, we offer instead what are in effect a few snapshots of aspects that some might find new, or at least (we hope) interesting.

Béla is of Pecheneg ancestry on his father's side, and he combines Hungarian and Austrian roots on his mother's side. His paternal grandfather fell on hard times, but Béla's father was talented and determined to make a good living as a doctor. As a boy, he used to earn a little money by giving lessons to his classmates and later, when he wished to study medicine at university, Béla's grandfather moved the family to Budapest. Béla's father subsequently became a renowned ear, nose and throat surgeon. He set up his own home in Dunaharaszti just outside Budapest, and this was where Béla's family were living at the time of his birth on 3rd August 1943.

The years of Béla's boyhood were troubled times in Hungary, and many whose lives had been shipwrecked in the prevailing political wind found sanctuary in the Bollobás home. For Béla himself, the habitués provided a wonderful opportunity: from them he learnt languages, history, fencing (from a general), and how to head a ball (from Kocsis, inside right of the great Hungarian football team of the '50s). Not long afterwards these people were mostly banished from the capital, but the sense of an enlarged family remained, made yet larger by the Hungarian attitude to servants, who are seen as part of the extended family. In this home, with its ever open door and its welcoming table, can be traced the source of Béla's own hospitality, and especially of his treatment of his research students, whom he always regards as part of his family.

At the age of 14 Béla started to attend the Czukor utca school in Budapest, which was then considered to be the best in Hungary, and Béla's father moved the family

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into the capital. Béla was already developing a reputation for mathematical talent, enough to bring him an invitation to tea at the Golf Hotel to meet Paul Erdős and his mother. Of course, Erdős set Béla a problem, which Béla solved by the end of the meal, and so began a great collaboration and a great friendship. Erdős would always visit Béla's house when he was in Hungary.

The first International Olympiad took place in 1959, when Béla was still only 15. He competed with great success in each of the first three Olympiads, winning the last of them by a clear margin. At the age of 18 he entered the Eötvös Loránd University. His "junior" year, 1963–64, however, was spent in Cambridge, at Trinity College, on a visit arranged by Erdős with his friend Davenport, and here Béla enjoyed supervision by Cassels, Swinnerton-Dyer, C.T.C Wall and John Polkinghorne. In this context, supervision means the personal style of teaching involving two undergraduates bringing the work they have done in, say, Functional Analysis, to a supervisor for an hour's discussion: some years later, Béla was to become a supervisor himself, as we shall describe shortly.

On graduating in 1966 he was immediately given a job at the Mathematical Institute of the Hungarian Academy of Sciences (now the Rényi Institute). Whilst there he wrote a PhD in geometry, under the formal direction of Fejes Tóth but informally directed by Erdős. The academic year 1967-68 was spent at the Lomonosov University in Moscow with Gelfand. In February 1969 he arrived to spend half a year at Christchurch College, Oxford at the invitation of Coulson. Aside from mathematics, he found time to earn a Blue as a modern pentathlete and to get his oar rowing for his college. In the summer, facing the prospect of returning to Hungary and so losing the opportunity to travel in the West, Béla decided not to go back. Instead, he travelled to the buffer state of Yugoslavia, where he had arranged a tryst with Gabriella. They were married in a church in Novi Sad, and then returned, via Italy, to England. During his earlier visit to Trinity College, Béla had been told he could return to write a PhD there, and so in October 1969 he registered as a research student in topology under the supervision of Frank Adams. Within just a year, before Adams actually arrived in Cambridge from Manchester, Béla wrote a dissertation in functional analysis and won a Research Fellowship at Trinity (in his spare time picking up a fencing Blue for Cambridge).

This brings the story up to the modern era, so to speak, regarding which we shall be much briefer. In 1971 Béla joined the staff of the Department of Pure Mathematics and Mathematical Statistics in Cambridge (where he stayed till his resignation in 1996) and at the same time he became a Fellow of Trinity College (a position he still holds). From 1981 until 1995 he also held a Chair at Louisiana State University. In 1995 he was made Jabie Hardin Chair of Excellence in Graph Theory and Combinatorics at the University of Memphis. From 2006 he will hold a Senior Research Fellowship at Trinity College.

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The basis of "Combinatorics in Cambridge" was a celebration of Béla's research. Most conference participants would know little about his work as a teacher, but this side of his life has been equally effective and distinctive, and indeed it has had an impact on his research, insofar as many of his research students were first influenced by Béla through being taught by him as undergraduates. No Cambridge mathematics student who attended a Bollobás lecture will forget it, and those students who were in Béla's own college, Trinity, will be indelibly marked by his supervisions. The conference learnt a little of why this is true from the speeches made at the banquet by Tim Gowers and Imre Leader.

Tim explained, "In my first term at Cambridge I took four courses, one of which Béla gave. And while I cannot even remember who the other three lecturers were, I will never forget Béla's course, and in particular the first lecture. The course in question was an introduction to analysis, which, I need hardly say, he gave without notes. He began with a proof of the isoperimetric inequality in the plane. His argument was beautiful and satisfying, but just as we were congratulating ourselves on following it he suddenly asked us where he had gone wrong. We were all completely stumped. The answer was that he had taken for granted that there *was* a maximum area that could be achieved, a fact that he would not be able to prove until we had done some analysis. With this example, Béla had aroused our interest and got us thinking."

"Not unreasonably, he assumed that we would carry on thinking and being interested. Several people here will have desperately tried to make themselves inconspicuous when Béla decides to ask a question of his audience. If no answer is forthcoming from the audience in general, he turns to individuals he knows, and of course it doesn't occur to him that they might not be able to supply the answer ... I am told that on one occasion he said to an unfortunate undergraduate who was wearing a Pembroke T-shirt, 'Come on, even people from Pembroke should know that.'"

To accompany the lectures, example sheets, of the lecturer's own devising, are handed out, and it is the questions on these sheets that the students work on in their own time and discuss with the supervisor. Imre Leader recalls Béla's example sheets. "He has an inexhaustible supply of *nice* questions. They are not arbitrary or ugly. They are hard and interesting. They are the sort of question that, when you read them, you want to work on them. They are *unusual* questions, questions that make you sit up and say 'what?', questions that get to the heart of the matter. It is quite amazing that his questions are so often both hard and beautiful. Some he has collected; most he has invented."

"He did it in all subjects: not just Graph Theory, Combinatorics, Functional Analysis (his own areas), but also Metric Spaces, Topological Spaces, Ring Theory, Measure Theory, and so on. I once complained to him that there were no hard

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problems in Linear Algebra; the next day, there arrived in my pigeonhole a list of six gorgeous ones! And the same happened with Complex Variable. In most subjects, his questions are much better than those of the experts in the field. Actually, very often a hard question on another lecturer's sheet can be traced back to a Bollobás question!"

As a supervisor, Béla demanded high standards, and he would often set his own questions when he found the lecturer's too feeble. Imre continued: "Once, Keith Carne discovered that the problem that Béla had set him had only been solved (in a research paper) six months previously. He mentioned this to Béla, who said 'But you have a huge advantage: you know that the result is true'. In supervisions, Béla always lets you think. And, if needs be, sweat. You knew you were doing badly when he'd suddenly ask you what sports you played, or otherwise changed the subject. A sign that you were doing even worse was that he'd make himself a cup of coffee and offer you a dry biscuit."

Tim observed, "As we all know, Béla has views about everything, not always easy to guess in advance but always passionately held." Béla's ever forthcoming advice can sometimes be delivered indirectly, though, and Tim recalled the time in his third year when he had to choose which courses to do. "I had the idea that it would be good to do a mixture of pure and applied mathematics, so that I would not be too narrowly focused and would understand some of the great scientific theories of the early twentieth century. I put this to Béla, and he responded enthusiastically: 'That is an excellent idea. But of course you will do 90 percent pure.' I got the message, did 100 percent pure and to this day do not understand relativity or quantum mechanics."

When considering Béla's impact on the wider mathematical world, there are three aspects of his work that demand attention. The first is, of course, his own research. His creativity, energy and technical power have yielded a continuous stream of excellent results, meaning in particular that he has for decades been in constant demand as a conference speaker. His first paper, published with Erdős in 1962, gave two proofs of a conjecture of Pósa; they each found a proof independently (Béla while he was still at the Czukor utca school). In a 1965 paper he proved the fundamental theorem about strongly saturated hypergraphs, and for many years this remained one of the few non-trivial exact results about hypergraphs. Many significant papers appeared in the 1970s, principally about extremal graph and hypergraph theory – for example with Erdős he established the logarithmic speed of the Erdős-Stone theorem – but papers in functional analysis also form a major component of Béla's output during this period.

Up till this point random graphs feature little in his work, but the situation changes dramatically in the 1980s, with a flow of deep and seminal results on this subject. The most newsworthy was, no doubt, Béla's resolution of the old question

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of the chromatic number of random graphs, but there are many other memorable ones, such as for example the  $n^{2/3}$ -sized window illuminating the old 'double jump' of Erdős and Rényi, the threshold for the appearance of arbitrary subgraphs, the configuration model for graphs with a given degree sequence, and the existence of Hamiltonian cycles in regular graphs of high degree. The ideas impinge not just on other areas of mathematics such as analysis and probability, but are of significance in other disciplines, including computer science and physics. It is probably for his work on random graphs that Béla is most widely known.

Random combinatorial models continue to be a dominant theme in his subsequent work, with applications to percolation and to web graphs becoming prominent, but at the same time he has proved major results in extremal graph theory, isoperimetric theory and set systems. Some examples of these are the systematic study of discrete isoperimetric inequalities (with Leader), of judicious partitions (with Scott) and of hereditary properties (with several students and with Simonovits), the discovery of a general graph polynomial (with Riordan) and the interlace polynomial (with Arratia and Sorkin), hard results about scale-free graph models (with Janson and Riordan), progress on random geometric graphs (with Balister, Sarkar and Walters), and (with Balister, Riordan and Walters) developments in bootstrap and Voronoi percolation. Far from taking it easy as he enters his seventh decade, Béla appears to be as productive as ever, if not more so. The quantity, the breadth and the quality of his research output over the years is remarkable.

Béla's books make up the second of the three aspects of his work that must be mentioned. Here we mean principally his textbooks rather than the several volumes that he has edited, though not even Béla's academic editing life has been routine: he and Gabi befriended Littlewood in his later years, and Béla subsequently became Littlewood's literary executor, editing the famous Littlewood's Miscellany. Béla's own book, Extremal Graph Theory, published in 1978, was the first book to treat the subject in such depth and detail, and it remains an indispensible reference to this day, notwithstanding the developments that have been made since it was written. Graph Theory was published as a graduate text, although it was intended to accompany the undergraduate course Béla developed at Cambridge: as a student text it was unrivalled in depth, and in its recent rebirth as Modern Graph Theory it forms a fine handbook for even the professional combinatorialist. His Combinatorics and Linear Analysis are more compact, and very readable, treatments of these areas and are popular texts. In each of these books, the material Béla includes has been carefully selected and the proofs have all been polished (and, not infrequently, improved) from the original. When combined with the author's manifest command of his subject and enthusiasm for it, the result is a very appealing read. As for Random Graphs, it appeared at a critical moment in the mid-1980s, when this important subject was undergoing a huge surge in development. With its precise

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and thorough exposition of the fundamentals, including much due to Béla himself, the book achieved canonical status and has been hugely influential.

The third significant contribution that Béla has made to mathematics is through his supervision of research students. It was mentioned earlier that he has now had about forty students, and not all are combinatorialists: Keith Ball, Keith Carne, Tim Gowers, Jonathan Partington and Charles Read constitute a proud legacy in analysis. Whereas forty is not such a large number in disciplines where research is a team activity, or where supervision is delegated to subordinates, Béla has always given his students close personal attention, and lots of it. Though he demands much of his students he enjoys working with them, and most of them are familiar with that combination of fine hospitality and fine mathematics that is the life of a Bollobás student.

Both Tim Gowers and Imre Leader recounted some first-hand details at the banquet. Imre said, "Béla always takes a huge interest in his research students. Many of his students collaborate with him. He is *always* visitable. At his house, one works, with kitchen breaks for coffee with him and Gabi, or else Gabi may bring coffee and biscuits in to the room where we are working. It is an environment incredibly suited for work. Béla has amazing drive. He is quick to think up new questions, but always nice ones. He has an uncanny ability to see in advance if a method will work. And he has immense power: he can often force a solution through in a way that others cannot. He is also very good at knowing which example to look at (to get insight into a general problem). One might imagine that, when collaborating with Béla, he would be happy if you go away and work on the problem. But, if the stage we've reached is fun, he *wants* to do it together. 'You pig', he'll say, if you have gone away and done it yourself. That is the point – Béla is *fun* to do maths with."

The porcine epithet came up in Tim's recollections too, when it was applied to a friend of his. "Later, Béla apologised and explained that 'pig' was not a strong insult in Hungarian – one shudders to think what is." Tim gave a further non-apocryphal account. "One cannot really succeed as a mathematician without being obsessed with the subject to some extent. Of course, Béla shows magnificently that it is possible to combine that obsession with a rich and varied life, but there must be times when mathematics comes first. Several years ago he made this point very clearly to two of his students as he drove them down the A1 to London. Eager not to waste a valuable opportunity to talk mathematics, he began to explain to them a problem in graph theory. It was a little complicated, so he demonstrated what he was talking about on the windscreen – with both hands. History does not tell us, but knowing Béla it seems likely that his speed at the time was not entirely negligible. Apparently, even with the demonstration the two students found the problem hard to take in."

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Tim spoke of what he had himself had learned from Béla. Basic lessons included "... there are certain things that One Should Not Do. I have learnt, either directly or second-hand, that no civilized person wears shorts on an aeroplane, that it is illbred to answer the telephone without immediately identifying oneself, and that it is absolutely not done to supervise in the morning." More importantly, "... when I had just started as a research student, I mentioned to Béla an interesting problem I had just heard about. His immediate response was, 'Why don't you solve it?' I'd like to be able to report that I did, but unfortunately I can't. Nevertheless, in just five words Béla had got across two important messages: first, that one should not be afraid of unsolved problems, and second, that one should be greedy for opportunities to make mathematical progress." And on a more personal note, he concluded, "There is one other extraordinary side of Béla that makes him a very unusual research supervisor, and that is the close personal interest he takes in his students... it is the sort of love that a father has for his children. I don't think anybody can have been a student of Béla's without being very aware of this analogy."

Béla's affection for his research students lay behind a hugely enjoyable party given by Gabi in their garden on the afternoon of Saturday August 2nd, preceding the conference. On a superbly sunny afternoon, most of Béla's students from the last thirty years (plus a couple of "honorary students"), along with spouses and children, came together to wish him well on his birthday. It was enlightening for students from different generations, who in many cases had never met before, to compare notes: for example, whereas the earliest students lived in the understanding that Béla, as a pentathlete, might kill you in any of three different ways, it was felt by the latest generations that this nowadays represented a low probability event.

The climax of the afternoon came with the presentation of the gift. It was in line with Béla's great interest in historical dates, and it related to that most momentous decade of Hungarian history, the 1950s – to be precise, to that high water mark on 25th November 1953, when Béla was just ten years old. It was gratifying that Béla, who for years has grilled his students on whether they know this or that historical date, could not immediately recall the significance of this one. It was, of course, the day that Hungary beat England 6–3 at Wembley. For those who don't know about football, this event marked the first time England had been beaten at home since Trinity College invented the game a hundred years earlier (as Béla would have it). So we presented Béla with an original match-day programme, full of stories and pictures of his boyhood heroes, together with a copy of the next day's *Times*, written in praise of the "Magic Magyars". Such is the power of the Bollobás legend that we half expected to see Béla's own name in the list of those who played that day (it wasn't).

The conference banquet itself, as alluded to earlier, will live long in the memories of those who were there. To understand why, the reader needs to be aware that the

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Wednesday of that conference week was the hottest day of modern times; indeed, trains from Cambridge to London had been cancelled because the rails had buckled in the heat, and the air conditioning equipment in the conference lecture hall had died in the struggle. It was expected that the temperature would exceed one hundred degrees for the first time ever in England, though that in fact did not happen till the following Sunday. But on this sweltering evening, a banquet was held in Trinity College to celebrate Béla's birthday and, true to character, Béla insisted his friends dress for this formal occasion. It has already been said what a tribute it is to Béla that so many good mathematicians participated in the conference: now we saw the true measure of that tribute, for, somehow or other, scores of them managed to beg or borrow a black dinner jacket, and to wear it cheerfully in the steaming heat. Moved by compassion at the sight of such loyalty and noble fortitude, the presiding Fellow at the dinner, Professor Glynn, told the company that they might remove their jackets before sitting down to eat. But it cannot often have happened that two hundred people sat down to a sumptuous banquet and finished it weighing less than when they started.

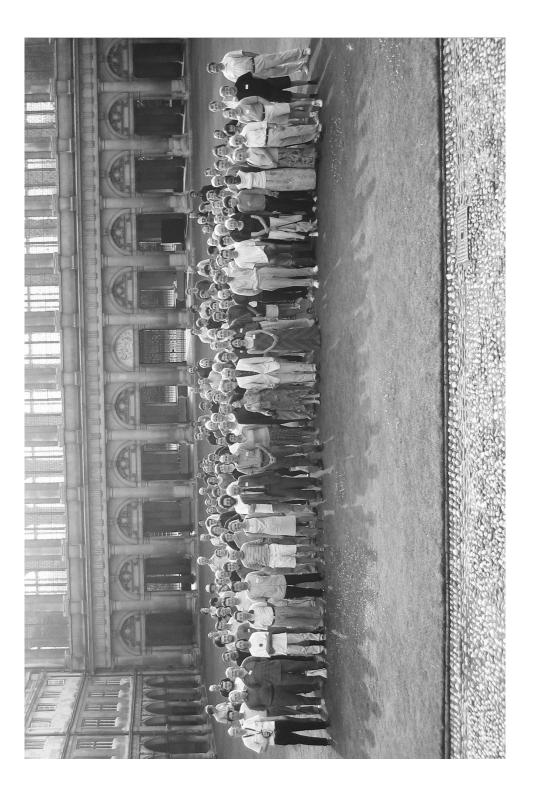
At the end of the dinner, short speeches were given by Tim Gowers, by Gyula Katona, who illuminated Béla's early years, and by Imre Leader. Some of what they said has already been mentioned. What has not yet been mentioned, though, is that each speaker paid tribute to Gabriella. In a style in keeping with the end-of-dinner spirit, Tim commented, "Of course, Gabi makes an enormous contribution to the family atmosphere. One of the great joys of knowing Béla is that one gets to know her as well, though she is so glamorous that it is not easy to think of her as a mother. Like Béla, she is free with her advice, which is not always the same as his but carries equal conviction. I have to confess that I have not yet followed her recent advice to me to develop the muscles in my upper body, but if in the future you notice changes of that kind, you will know why." The various tributes were brief and individual, but the response they received showed that no-one was in any doubt as to the profound importance of Gabriella to Béla's story.

That is enough by way of an introduction to this volume. What remains is the mathematics, which we trust will be found a fitting tribute to the first sixty years of Béla's life.

Andrew Thomason

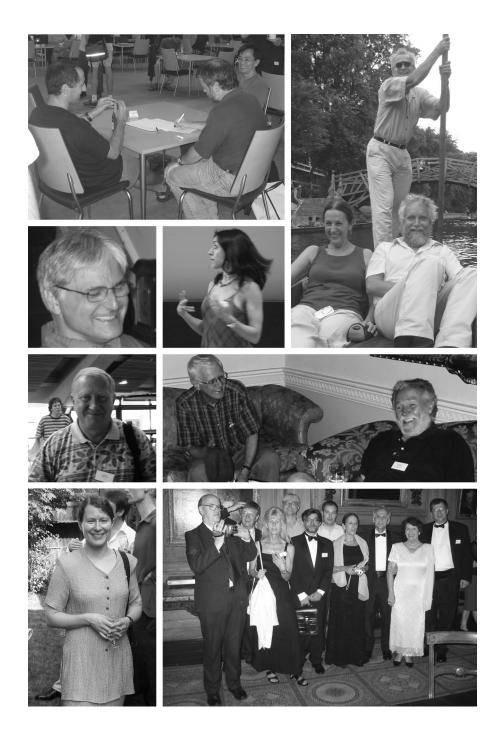
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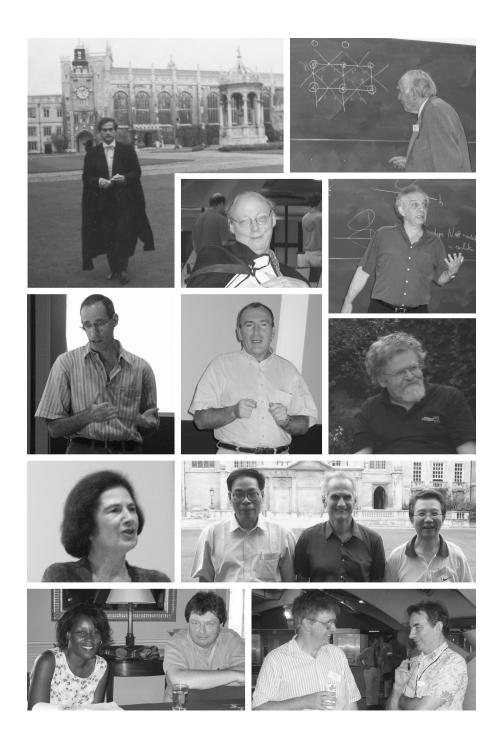
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