Chapter 1

## Introduction

It is customary today to call a generator or amplifier of electromagnetic waves a maser (*Microwave Amplification* by the *Stimulated Emission* of *Radiation*) if its operation is based on the stimulated emission of distributed oscillators. Electrons and ions, rotating around an ambient magnetic field, are the oscillators in so-called cyclotron masers (CMs).

There are two types of CMs in space, which differ considerably from each other. In the Earth's magnetosphere the first type operates on open field lines in the auroral region at heights between  $10^3$  and  $10^4$  km, in plasma cavities where the plasma density is so low that the electron plasma frequency is much less than the electron cyclotron frequency. Here auroral kilometric radiation (AKR) is generated by energetic electrons in such a maser system (Fig. 1.1a). The eigenmodes of these auroral CMs are electromagnetic waves with frequencies close to the electron cyclotron frequency; the wave vector **k** is almost perpendicular to the geomagnetic field **B**. These auroral CMs are rather similar to a family of laboratory devices, termed gyrotrons (Fig. 1.1b). The operation of these devices is based on the cyclotron interaction of electrons, moving along a homogeneous magnetic field through an evacuated region inside a geometrical cavity resonator. A specific feature of a laboratory CM is the cyclotron interaction of a well-organized beam of electrons rotating around a homogeneous magnetic field with a monochromatic electromagnetic wave having a spatially fixed field structure. As a rule, such an electron beam has a very narrow spread of particle energies and pitch-angles. The investigation of such devices has a rich history, beginning with the pioneering paper of Twiss (1958) on the cyclotron instability. Very important contributions both to the theoretical development of the subject and to the design and construction of such devices have been made by the Gorky (now Nizhny Novgorod) group of scientists under the

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Figure 1.1 The three most typical magnetic field configurations for a cyclotron maser.

leadership of Gaponov-Grekhov. The first papers in this direction referred to the years 1959–1965 (Gaponov 1959, 1960; Petelin, 1961; Gaponov and Yulpatov, 1962; Yulpatov, 1965). The investigations of the cyclotron instability in a relativistic plasma by Zheleznyakov (1960a, b) were important for further applications to laboratory and space plasmas. It is necessary to mention as well the papers by Schneider (1959) and Bekefi *et al.* (1961). The gyrotrons and their modifications now find a wide range of applications in the fields of plasma experiments, controlled thermonuclear fusion, radar transmitters, and plasma and chemical technology. Details of these can be found in the review by Gaponov-Grekhov and Petelin (1980).

The specific feature of a space gyrotron where AKR is generated is the presence of cold electrons whose number density is comparable with, or slightly greater than, the number density of energetic electrons. A very important aspect of space gyrotrons is the inhomogeneity of both the magnetic field strength and the plasma parameters, the detailed investigation of which is an active field of research today.

Different aspects of space gyrotrons have been investigated when applied to AKR (Wu and Lee, 1979), Jupiter's decametric radiation and planetary radio emissions from the other outer planets (Zarka, 1992), and solar microwave bursts (Melrose and Dulk, 1982). A gyrotron-like radiation is also believed important for various stellar radiation bursts (Benz *et al.*, 1998; Trigilio *et al.*, 1998; Bingham *et al.*, 2001, 2004).

The second type of CMs in space, either electron or ion masers, operates along closed magnetic flux tubes (Fig. 1.1c) filled by a dense cold plasma. For such CMs the electron plasma frequency is greater than the electron gyrofrequency. In the Earth's magnetosphere these CMs function within the plasmasphere and in filaments of dense cold plasma outside (and sometimes attached to) the plasmasphere. Both types of CMs occur widely in natural plasmas – they exist in the magnetospheres of planets, in the solar corona and in the plasma envelopes of active stars.

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The main focus of this book is on CMs of the second type. They are large scale – yet unseen to the human eye – features of planetary magnetospheres. Via single or multi-hop wave propagation and amplification along geomagnetic flux tubes, they determine the population (energy spectrum and pitch-angle distribution) of energetic charged particles distributed in space around the Earth. In other cosmic plasmas with closed (dipole like) magnetic field lines, they can crucially influence the properties of energetic charged particles there too.

In many laboratory experiments, magnetic mirrors are produced by additional magnetic fields due to current-carrying coils at the ends of the device. In a planetary magnetosphere or for a magnetic loop reaching into the solar corona, the trap is caused by the dipole-like geometry of the magnetic field. Such a magnetic trap, containing dense plasma, serves as a CM cavity; the high-energy fraction of charged particles is the active substance for the CM. The ends of the magnetic flux tube, immersed in the planetary ionosphere, serve as mirrors for the electromagnetic waves. The eigenmodes in space CMs are whistler-mode or Alfvén waves, which possess one very important property; they are guided by the magnetic field. The situation is similar to that in fibre optics when light is guided by dielectric filaments. Cyclotron resonance occurs when the electric field of a circularly polarized wave propagating through the plasma exactly matches the Doppler-shifted cyclotron motion of a charged particle. It takes place when the Doppler-shifted wave frequency coincides with the particle gyrofrequency, and can occur either for whistler-mode waves and energetic electrons, or for Alfvén (hydromagnetic) waves and energetic ions.

CM operation is based on the cyclotron instability (CI), which is due to the transverse anisotropy of the charged particle distribution function; this exists when the effective temperature transverse to the magnetic field direction,  $T_{\perp}$ , exceeds the longitudinal temperature,  $T_{\parallel}$  (the anisotropy factor  $\alpha = T_{\perp}/T_{\parallel}$  is greater than unity). The pioneering paper by Sagdeev and Shafranov (1960) on this instability played a crucial role in developing the concept of space cyclotron masers. Sagdeev and Shafranov introduced this anisotropy factor  $\alpha > 1$  as the universal quantitative measure for the momentum inversion of energetic charged particles, which serves as the necessary condition for space CM operation, just as the population inversion does for conventional lasers. It is important to recognize that this anisotropy, with  $\alpha > 1$ , is a natural feature of adiabatic traps because of the existence of the loss cone. This is the region in pitch-angle space with values near zero, where the energetic charged particles are lost through the ends of the magnetic trap by collision with atoms or molecules of the upper atmosphere. There are many other causes of the transverse anisotropy of energetic charged particles in space, such as magnetic compression, charged particle transport within planetary magnetospheres, acceleration by global electric fields, stochastic acceleration mechanisms, and so on. Most sources supply energetic particles with a wide spread of energies and pitch-angles. In space, as we shall see, the inhomogeneous magnetic field and broad range of energies and pitchangles radically change the wave-particle interaction process from the laboratory case with its well-organized beam and monochromatic wave.

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The second important step in the theoretical development of the anisotropic cyclotron instability (CI) was formulation of the quasi-linear (QL) theory of the CI by Vedenov *et al.* (1962) and Sagdeev and Galeev (1969). They introduced the idea of diffusion paths in velocity (phase) space, along which the charged particles resonant with cyclotron waves could reach the edge of the loss cone and be lost from a magnetic trap.

As applied to space CMs, operating via the geomagnetic trap, the CM theory was developed in relation to two geophysical phenomena, namely natural ELF (from 0.3 to 3 kHz) and VLF (from 3 to 30 kHz) electromagnetic emissions (Helliwell, 1965; Kimura, 1967) and the Earth's radiation belts (Van Allen *et al.*, 1958). Early estimates (Trakhtengerts, 1963) showed that the intensity of ELF/VLF emissions was very much (more than 50 dB) higher than the intensity of the thermal equilibrium emission from the Van Allen radiation belt electrons. The idea of quantitatively connecting these audio frequency radio emissions with the cyclotron instability (CI) of radiation belt electrons first appeared in 1963 (see below). The subject has been actively developed in the succeeding four decades.

An enormous quantity of valuable experimental data on energetic charged particles and ELF/VLF waves in the Earth's magnetosphere has been accumulated over the last four decades. Calculations have used these data to demonstrate the very important role of wave–particle interactions in determining the state of the radiation belts, and hence their lifetime. We concentrate our attention here on self-consistent models, which can give not only detailed and exhaustive information on the spectral and dynamical characteristics of waves generated in such CMs but also explain the dynamics of trapped and precipitated energetic charged particles.

ELF/VLF radio signals observed on the ground and in space alike include both broadband noise-like electromagnetic emissions, such as hiss and quasiperiodic noise bursts (Fig. 1.2), and narrowband, or discrete, emissions having a



**Figure 1.2** Examples of the dynamical spectrum of noise-like ELF/VLF emissions: (a) hiss band (taken from Helliwell, 1965); (b) quasi-periodic emissions (Sato and Fukunishi, 1981). (Copyright American Geophysical Union, reproduced with permission.)

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**Figure 1.3** Examples of the dynamical spectrum of discrete ELF/VLF signals: (a) chorus observed on the satellite GEOS-1 (Hattori *et al.*, 1991); (b) chorus observed on the ground (Manninen *et al.*, 1996); (c) whistlers from a lightning discharge (Ohta *et al.*, 1997). (Copyright American Geophysical Union, reproduced with permission); (d) triggered VLF emissions (Helliwell and Katsufrakis, 1974). (Copyright American Geophysical Union, reproduced with permission.)

quasi-monochromatic structure. Among these are natural signals, such as successions of rising frequency tones named chorus, and whistlers generated by lightning discharges, as well as man-made signals from VLF transmitters propagating in the whistler-mode, and triggered VLF emissions from both natural and man-made signals (Fig. 1.3).

The CM theory to explain such observations developed in two practically independent directions. The first of these is based on a quasi-linear (QL) theory of gyroresonant wave–particle interactions, which is valid for noise-like emissions when the electromagnetic field can be considered to be the sum of independent wave packets with stochastic phases and a broad frequency spectrum. This radiation interacts with the energetic particles as it does with particles in many energy level systems. In such a situation the mathematical description of CM operation is very

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similar to the balance approach of quantum generators (Khanin, 1995). In particular, as with quantum generators, the fundamental generation regime in CMs is a relaxation oscillation, when both the wave intensity and charged particle flux are modulated with a period depending on the properties of both the particle source and the wave and particle sinks. The bandwidth of the oscillation, and hence the quality factor, also depend on these source and sink properties. This is the basis of many other wave generation regimes of CMs (e.g. periodic, spike-like and stochastic). It leads to an explanation of the different modulation phenomena in natural electromagnetic emissions and energetic charged particle dynamics.

Trakhtengerts (1963), Brice (1963, 1964), Andronov and Trakhtengerts (1964), and Kennel and Petschek (1966) published the first fundamental papers on the QL theory of magnetospheric CMs. These studies were based on the classical plasma physics of the above mentioned papers by Sagdeev and Shafranov (1960) and Vedenov *et al.* (1962). Important contributions to the development of this theory in the Earth's magnetosphere were made by Trakhtengerts (1966, 1967), Kennel (1969), Gendrin (1968), Tverskoy (1968), Cornwall *et al.* (1970), Coroniti and Kennel (1970), Roux and Solomon (1971), Lyons *et al.* (1972), and Schulz and Lanzerotti (1974). The monograph by Bespalov and Trakhtengerts (1986a) partly summarizes the state of QL theory.

The second direction develops a nonlinear theory of monochromatic waveparticle interactions, generalizing it for the case of an inhomogeneous magnetic field. Here the first paper specifically considering space (magnetospheric) plasmas was that of Dungey (1963). New experimental results from the VLF transmitters in Russia and at Siple Station, Antarctica, stimulating development of the nonlinear theory, were published over a period of more than 25 years from 1967; for a review, see Molchanov (1985) and Helliwell (1993). Helliwell (1967, 1970) first formulated the key idea about second-order cyclotron resonance as a generation mechanism for discrete VLF emissions in the magnetosphere. Important and original contributions to the quantitative development of this nonlinear theory were made by Dysthe (1971), Nunn (1971, 1974), Sudan and Ott (1971), Budko *et al.* (1972), Helliwell and Crystal (1973), Karpman *et al.* (1974a, b), Roux and Pellat (1978), and others; more detailed references are given throughout the book. These theories are somewhat similar to those of laboratory CMs in the case of a homogeneous magnetic field, but have very important novel features when the magnetic field is inhomogeneous.

Both directions – quasi-linear theory for broadband waves and nonlinear theory for monochromatic waves – lead to the explanations of certain phenomena, noise-like emissions in the first case and triggered emissions in the second. Natural electromagnetic phenomena, produced by CM operation and observed on the ground or in space, reveal the very interesting phenomenon that noise-like emissions may sometimes become quasi-monochromatic signals, or vice versa. Thus, in some sense, disorder produces order. Some early observations here are those of Helliwell (1969), Reeve and Rycroft (1971) and Burtis and Helliwell (1976). The modern theory of a CM enables such a change to be understood. This transition

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can be due to the appearance of a step deformation on the distribution function at a certain velocity, when energetic charged particles interact with a noise-like emission. So, starting from noise-like emissions, we come to the situation when it is necessary to use the theory of monochromatic wave interaction with a specific particle distribution, taking phase effects into account. The very interesting backward wave oscillator (BWO) generation regime can then be realized; wave reflection and positive feedback are organized in a BWO generation regime via a charged particle beam whose specific properties are prepared by the noise-like emission.

In writing this book we have pursued the objective of describing analytically, and from a unified viewpoint, both directions of CM theory and also their interrelation. We start with the analyses of wave eigenmodes and their excitation conditions in CMs. Then the nonlinear equation of motion of a single charged particle moving in a homogeneous magnetic field and in the field of a monochromatic wave is investigated. This approach is generalized, successively, for the case of an inhomogeneous magnetic field, of two waves and of many waves. Collective effects are taken into account using a collisionless kinetic (Boltzmann) equation for the distribution function of charged particles. In such an approach the quasi-linear description is obtained as a generalization of the case of many waves with random phases. The kinetic equation for the particle distribution function and the electromagnetic wave equation with a current of energetic particles are solved self-consistently. Thus, the first nine chapters of this book cover the fundamentals, which gradually become more and more complex. The treatment is developed in a logical manner; so as not to disrupt the flow, key references are given only at the beginning and end of each chapter.

The latter part of this book (Chapters 10–13) discusses several applications of CM theory to different electromagnetic emissions, to the loss of energetic charged particles from the Earth's magnetosphere and to their precipitation into the upper atmosphere. Both noise-like and discrete emissions with frequencies ranging from 0.1 up to  $\sim$  10 Hz interacting with ions, or from 0.3 up to  $\sim$  10 kHz interacting with electrons, are explained. The analysis of particular cases, such as pulsating aurora or the interaction involving detached cold plasma regions, are examples of a rather sophisticated theory and a detailed comparison with experimental data. Applications of CM theory to Jupiter's magnetosphere and to the solar corona seem to be promising. Laboratory experiments and their quantitative interpretation give additional confidence in the CM theory.

In summary, space plasma experiments are a source of information on this fascinating and important physical phenomenon of cyclotron maser operation. The first steps have been taken in the application of this theory to different space plasma situations, and the dynamical phenomena discussed deserve further investigations. We hope that our book may stimulate these, whether they be experimental researches, theoretical studies or computational simulations. **Chapter 2** 

# Basic theory of cyclotron masers (CMs)

The purpose of this chapter is to formulate the self-consistent set of equations (expressed in cgs units) which describe the theoretical behaviour of cyclotron masers operating in a plasma constrained by a dipole magnetic field. Thus the equation of motion of a non-relativistic charged particle is considered; the first adiabatic invariant of motion is the magnetic moment, and the second is the bounce integral. The charged particle distribution function approach is then presented.

The general dispersion relation for electromagnetic waves propagating in a plasma is derived. From this expressions are obtained for the refractive index and polarization parameters of whistler-mode waves and of Alfvén waves. These lead to the wave eigenmodes (natural, or resonant, oscillations) in an operating cyclotron maser. Due to the current of hot particles in gyroresonance with the wave, a monochromatic wave is an evolving wave packet. Its amplitude changes slowly with time; the wave packet propagates at the group velocity.

The theory is derived first for a homogeneous plasma for which the plasma density and the ambient magnetic field do not vary with any spatial coordinate. Secondly it is extended to an inhomogeneous plasma, specifically to plasma and energetic charged particles confined by a dipole magnetic field such as the Earth's.

The necessary references to books and papers will be given at the start of each chapter. Much reference material has been presented in the book by Akasofu and Chapman (1972).

# 2.1 Attributes of a CM. The Earth's magnetic field in space

To construct CM theory we begin with a description of the basic elements of cyclotron masers – a cavity, an active substance and electromagnetic eigenmodes.

2.1 Attributes of a CM. The Earth's magnetic field in space

As has been pointed out in Chapter 1, a CM cavity is formed by a magnetic trap, filled with plasma, which includes two components, a cold plasma which determines the wave eigenmodes and a fraction of energetic charged particles which serve as the active material. The magnetic trap is an adiabatic trap; this means that the spatial scale l of the trap is much larger than the gyroradius  $\rho_{\rm B}$  of the energetic particles contained in the trap:

$$l \gg \rho_{\rm B}.$$
 (2.1)

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This condition is readily fulfilled in CMs in space. Also the mean free path  $l_c$  between binary collisions of the energetic particles is much larger than the magnetic trap length *l*:

$$l_{\rm c} \gg l.$$
 (2.2)

We consider a single magnetic trap, which has just one minimum value of magnetic field B along the flux tube of interest. Such a trap possesses all the principal features of a CM.

The magnetic field in near-Earth space, in the magnetosphere, has a rather complex configuration. Except near the magnetopause, the magnetic field is essentially a dipolar magnetic field, expressed as

$$\vec{B} = \frac{M}{r^3} \left( -2\sin\lambda \,\vec{r}_0 + \cos\lambda \,\vec{\lambda}_0 \right) \tag{2.3}$$

or

$$B = \frac{M}{r^3} \left( 1 + 3\sin^2 \lambda \right)^{1/2}$$
(2.4)

where the vector  $\vec{B}$  is written in spherical polar coordinates whose origin is at the centre of the planet as shown in Fig. 2.1. Here, M is the magnetic moment, r is the radial distance, and  $\lambda$  is the latitude. The dipole magnetic field has axial symmetry.

In the case of the Earth

$$M = 0.311 R_0^3 \text{ G}, \quad R_0 = 6370 \text{ km}$$
 (2.5)

where  $R_0$  is the Earth's radius. The equation for a magnetic field line has the form

$$r = LR_0 \cos^2 \lambda \tag{2.6}$$

where L, the so-called L-shell, is McIlwain's (1961) parameter, as illustrated in Fig. 2.1.

The important magnetic trap parameter is the magnetic mirror ratio of the maximum value  $B_{\rm M}$  (on the planetary surface) to its lowest (minimum) value  $B_{\rm L}$  (in the equatorial plane):

$$\sigma = \frac{B_{\rm M}}{B_{\rm L}} = L^3 \left(4 - 3/L\right)^{1/2}.$$
(2.7)



Figure 2.1 The spherical polar coordinate system whose origin is at the centre of the planet; r is the radial distance,  $\lambda$  is the latitude,  $r_{eq}$  is the distance of the magnetic field line in the equatorial plane.

A good approximation for the magnetic field variation along a flux tube in the vicinity of the equatorial plane is a parabolic variation:

$$B = B_{\rm L}(1 + z^2/a^2) \tag{2.8}$$

where the coordinate z is measured from the central cross-section of the magnetic trap. For a dipole magnetic field (2.4), the scale length *a* is given by

$$a = LR_0/2.12. (2.9)$$

#### The motion of charged particles in a CM 2.2

The inequalities (2.1) and (2.2) permit us to use a guiding centre approach (Northrop, 1963) to describe charged particle motion in an inhomogeneous magnetic field. In this approximation a particle's cyclotron motion is expressed as the Larmor rotation around a guiding centre, which moves along a magnetic field line and drifts slowly (in longitude) across the magnetic field direction. Figure 2.2 shows this with respect to an origin which, for convenience, is also taken to be the centre of the planet

$$\vec{r} = \vec{R}_{\rm c} + \vec{\rho}_{B\alpha}.\tag{2.10}$$

Further, we are limited by non-relativistic considerations. In this case the equations of motion have the form:

$$\vec{\rho}_{Ba} = \rho_{B\alpha}(\vec{x}_0 \sin \varphi_\alpha, \vec{y}_0 \cos \varphi_\alpha), \quad \vec{v}_{\perp\alpha} = v_{\perp}(\vec{x}_0 \cos \varphi_\alpha, -\vec{y}_0 \sin \varphi_\alpha)$$
(2.11)

$$\frac{d\varphi}{dt} = \dot{\varphi}_{\alpha} = -\omega_{B\alpha}(\vec{R}_{c}), \quad \frac{m_{\alpha}v_{\perp}^{2}}{2B} = J_{\perp\alpha} = \text{const.}$$
(2.12)