Networks: Optimisation and Evolution

Point-to-point vs hub-and-spoke. Questions of network design are real and involve many billions of dollars. Yet little is known about the optimisation of design – nearly all work concerns the optimisation of flow for a given design. This foundational book tackles the optimisation of network structure itself, deriving comprehensible and realistic design principles.

With fixed material cost rates, a natural class of models implies the optimality of direct source–destination connections. However, considerations of variable load and environmental intrusion then enforce trunking in the optimal design, producing an arterial or hierarchical net. Its determination requires a continuum formulation, which can however be simplified once a discrete structure begins to emerge. Connections are made with the masterly work of Bendsøe and Sigmund on optimal mechanical structures and also with neural, processing and communication networks, including those of the Internet and the Worldwide Web. Technical appendices are provided on random graphs and polymer models and on the Klimov index.

PETER WHITTLE is a Professor Emeritus at the University of Cambridge. He is a Fellow of the Royal Society and the winner of several international prizes. This is his 11th book.

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To Käthe

Contents

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Lastly, I might well not have been able to finish this work had I not, after retirement, kindly been granted continued enjoyment of the facilities and activities of the Statistical Laboratory, University of Cambridge. The advantage is all the greater, in that the Laboratory is now housed with the rest of the Faculty in the resplendent new Centre for Mathematical Sciences.

References to the literature can be regarded as a continuing stream of formal acknowledgement, as well as of association. If I make no reference on a given piece of work, then this is an indication that I regard it as either standard or new, with the greater likelihood of misapprehension in the second case.

Conventions on notation

The fact that we cover a wide range of topics, each with its own established notation, makes it difficult to hold to uniform conventions, and we do not do so entirely. We do consistently use x to denote the state variable of a system, but this will be the set of flows in Part I and the set of node occupation numbers in Part III, for example. We are forced then to use ξ to denote Cartesian co-ordinates. In general we follow the mathematical programming literature in using y to denote the variable dual to x, but in Part II bow to the conventions of control theory and use λ to denote this dual variable, releasing y to denote the observations (i.e. the information input).

The treatment is in general mathematical, although scarcely rising above the sophistication of 'mathematical methods'. The use of the theorem/proof presentation is then simply the tidiest and most explicit way of summarising current conclusions, implying neither profundity nor the pretence of it. The three appendices collect the material that is densest technically.

Equations, theorems and figures are numbered consecutively through a chapter, and also carry a chapter label. Equation (5.4) is thus the fourth equation of the fifth chapter.