

## Contents

Contributors	ix
Foreword by Jeffrey C. Lagarias	xi
Preface	xix
Chapter 1. More Inflation Tilings	
DIRK FRETTLÖH	1
1.1. A simple inflation tiling without FLC	1
1.2. One-parameter families of inflation rules	4
1.3. A tiling with non-unique decomposition	5
1.4. Überpinwheel	6
1.5. Tile orientations with distinct frequencies	9
1.6. Tilings inspired by girih tiles	12
1.7. Cyclotomic rhombus tilings	12
1.8. Infinitely many prototiles	16
1.9. Inflations with an empty supertile	17
1.10. Overlapping tiles	19
1.11. Tiles from automorphisms of the free group	23
1.12. Mixed inflations	28
1.13. Fusion tilings	32
References	35
Chapter 2. Discrete Tomography of Model Sets:	
Reconstruction and Uniqueness	
UWE GRIMM, PETER GRITZMANN AND CHRISTIAN HUCK	39
2.1. Introduction	39
2.2. Basic notions of discrete tomography	41
2.3. Algorithmic issues in discrete tomography	42
2.4. Computational complexity of discrete tomography	46
2.5. Discrete tomography of model sets	51
2.6. Uniqueness in discrete tomography	57
References	69

Chapter 3. Geometric Enumeration Problems for Lattices and Embedded $\mathbb{Z}$ -Modules	
MICHAEL BAAKE AND PETER ZEINER	73
3.1. Introduction	73
3.2. Preliminaries on lattices	76
3.3. A hierarchy of planar lattice enumeration problems	78
3.4. Algebraic and analytic tools	83
3.5. Similar sublattices	87
3.6. Similar submodules	106
3.7. Coincidence site lattices and modules	112
3.8. (M)CSMs of planar modules with $N$ -fold symmetry	124
3.9. The cubic lattices	129
3.10. The four-dimensional hypercubic lattices	137
3.11. More on the icosian ring	148
3.12. Multiple CSLs of the cubic lattices	155
3.13. Results in higher dimensions	165
References	167
Chapter 4. Almost Periodic Measures and their Fourier Transforms	
ROBERT V. MOODY AND NICOLAE STRUNGARU	173
4.1. Introduction	173
4.2. Topological background	181
4.3. Almost periodic functions	184
4.4. Weak topologies and consequences	192
4.5. Means	217
4.6. The Eberlein convolution	224
4.7. $WAP = SAP \oplus WAP_0$	230
4.8. Fourier transform of finite measures	235
4.9. Fourier transformable measures	241
4.10. Almost periodic measures	256
4.11. Positive definite measures	262
References	268
Chapter 5. Almost Periodic Pure Point Measures	
NICOLAE STRUNGARU	271
5.1. Introduction	271
5.2. The Baake–Moody construction of a CPS	275
5.3. Almost periodic measures	286
5.4. Dense weighted model combs	291
5.5. Continuous weighted model combs	295
5.6. On $\varepsilon$ -dual characters	298
5.7. Almost lattices	305

5.8.	WAP measures with Meyer set support	312
5.9.	Diffraction of weighted Dirac combs on Meyer sets	316
5.10.	More on Bragg spectra of Meyer sets	332
5.11.	Concluding remarks	337
5.A.	Appendix. Harmonious sets	338
	References	340
Chapter 6. Averaging Almost Periodic Functions along Exponential Sequences		
	MICHAEL BAAKE, ALAN HAYNES AND DANIEL LENZ	343
6.1.	Introduction	343
6.2.	Preliminaries and general setting	344
6.3.	Averaging periodic functions	349
6.4.	Averaging almost periodic functions	352
6.5.	Further directions and extensions	359
	References	361
Epilogue. Gateways Towards Quasicrystals		
	PETER KRAMER	363
E.1.	Classical periodic crystallography	363
E.2.	Point symmetry: Das Pentagramma macht Dir Pein?	365
E.3.	Mathematical crystallography in higher dimensions	367
E.4.	Aperiodic tilings of the plane	367
E.5.	Cells and diffraction of the Penrose pattern	369
E.6.	Quasiperiodicity and Fourier modules	369
E.7.	Scaling and the square lattice	370
E.8.	Harmonious sets	373
E.9.	Incommensurate and modulated crystals	373
E.10.	The quasiperiodic Penrose pattern	374
E.11.	Icosahedral tilings in three dimensions	375
E.12.	Discovery of icosahedral quasicrystals	376
E.13.	Postscriptum	377
	References	378
	Index	381