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978-0-521-86991-1 - Encyclopedia of Mathematics and its Applications: Aperiodic Order: Volume 1:

A Mathematical Invitation

Michael Baake and Uwe Grimm

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