

ASTROPHYSICAL FLOWS

Almost all conventional matter in the Universe is fluid, and fluid dynamics plays a crucial role in astrophysics. This new graduate textbook provides a basic understanding of the fluid dynamical processes relevant to astrophysics. The mathematics used to describe these processes is simplified to bring out the underlying physics. The authors cover many topics, including wave propagation, shocks, spherical flows, stellar oscillations and the instabilities caused by effects such as magnetic fields, thermal driving, gravity and shear flows. They also discuss the basic concepts of compressible fluid dynamics and magnetohydrodynamics.

The authors are Directors of the UK Astrophysical Fluids Facility (UKAFF) at the University of Leicester, and Editors of the Cambridge Astrophysics Series. This book has been developed from a course in astrophysical fluid dynamics taught at the University of Cambridge. It is suitable for graduate students in astrophysics, physics and applied mathematics, and requires only a basic familiarity with fluid dynamics.

JIM PRINGLE is Professor of Theoretical Astronomy and a Fellow of Emmanuel College at the University of Cambridge, and Senior Visitor at the Space Telescope Science Institute, Baltimore.

ANDREW KING is Professor of Astrophysics at the University of Leicester and a Royal Society Wolfson Research Merit Award holder. He is co-author of *Accretion Power in Astrophysics* (Cambridge University Press, third edition, 2002).

ASTROPHYSICAL FLOWS

J. E. PRINGLE
University of Cambridge

A. R. KING
University of Leicester



Cambridge University Press & Assessment
978-0-521-86936-2 — Astrophysical Flows
James E. Pringle, Andrew King
Frontmatter
[More Information](#)



CAMBRIDGE
UNIVERSITY PRESS

Shaftesbury Road, Cambridge CB2 8EA, United Kingdom
One Liberty Plaza, 20th Floor, New York, NY 10006, USA
477 Williamstown Road, Port Melbourne, VIC 3207, Australia
314–321, 3rd Floor, Plot 3, Splendor Forum, Jasola District Centre, New Delhi – 110025, India
103 Penang Road, #05–06/07, Visioncrest Commercial, Singapore 238467

Cambridge University Press is part of Cambridge University Press & Assessment,
a department of the University of Cambridge.

We share the University's mission to contribute to society through the pursuit of
education, learning and research at the highest international levels of excellence.

www.cambridge.org

Information on this title: www.cambridge.org/9780521869362

© J. Pringle and A. King 2007

This publication is in copyright. Subject to statutory exception and to the provisions
of relevant collective licensing agreements, no reproduction of any part may take
place without the written permission of Cambridge University Press & Assessment.

First published 2007

First paperback edition 2014

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-86936-2 Hardback

ISBN 978-1-107-69340-1 Paperback

Cambridge University Press & Assessment has no responsibility for the persistence
or accuracy of URLs for external or third-party internet websites referred to in this
publication and does not guarantee that any content on such websites is, or will
remain, accurate or appropriate.

Contents

<i>Preface</i>	<i>page</i>	<i>ix</i>
1 The basic fluid equations		1
1.1 Conservation of mass and momentum		2
1.2 The Lagrangian derivative		4
1.3 Conservation of energy		5
1.4 The equation of state and useful approximations		6
1.5 The MHD approximation		8
1.6 Some basic implications		11
1.7 Conservation of energy		12
1.8 Further reading		14
1.9 Problems		15
2 Compressible media		17
2.1 Wave propagation in uniform media		18
2.2 Non-linear flow in one dimension		26
2.3 Further reading		38
2.4 Problems		38
3 Spherically symmetric flows		44
3.1 Steady inflow/outflow		44
3.2 Explosion in a uniform medium		50
3.3 Further reading		58
3.4 Problems		58
4 Stellar models and stellar oscillations		60
4.1 Models of stars		60
4.2 Perturbing the models		62
4.3 Eulerian and Lagrangian perturbations		63
4.4 Adiabatic perturbations – a variational principle		66
4.5 The Schwarzschild stability criterion		73

vi	<i>Contents</i>	
4.6	Further reading	74
4.7	Problems	75
5	Stellar oscillations – waves in stratified media	78
5.1	Waves in a plane-parallel atmosphere	79
5.2	Vertical waves in a polytropic atmosphere	84
5.3	Further reading	87
5.4	Problems	87
6	Damping and excitation of stellar oscillations	90
6.1	A simple set of oscillations	91
6.2	Damping by conductivity	92
6.3	The effect of heating and cooling – the ϵ -mechanism	95
6.4	The effect of opacity – the κ -mechanism	97
6.5	Further reading	101
7	Magnetic instability in a static atmosphere	102
7.1	Magnetic buoyancy	102
7.2	The Parker instability	106
7.3	Further reading	111
7.4	Problems	111
8	Thermal instabilities	113
8.1	Linear perturbations and the Field criterion	114
8.2	Heating and cooling fronts	118
8.3	Further reading	120
8.4	Problems	120
9	Gravitational instability	123
9.1	The Jeans instability	123
9.2	Isothermal, self-gravitating plane layer	125
9.3	Stability of a thin slab	128
9.4	Further reading	130
9.5	Problems	131
10	Linear shear flows	134
10.1	Perturbation of a linear shear flow	135
10.2	Squire's theorem	136
10.3	Rayleigh's inflexion point theorem	136
10.4	Fjørtoft's theorem	138

Contents

vii

10.5	Physical interpretation	139
10.6	Co-moving phase	141
10.7	Stratified shear flow	142
10.8	The Richardson criterion	144
10.9	Further reading	145
10.10	Problems	145
11	Rotating flows	150
11.1	Rotating fluid equilibria	150
11.2	Making rotating stellar models	151
11.3	Meridional circulation	154
11.4	Rotation and magnetism	156
11.5	Further reading	157
11.6	Problems	157
12	Circular shear flow	158
12.1	Incompressible shear flow in a rigid cylinder	158
12.2	Axisymmetric stability of a compressible rotating flow	162
12.3	Circular shear flow with a magnetic field	167
12.4	Circular shear flow with self-gravity	172
12.5	Further reading	176
12.6	Problems	176
13	Modes in rotating stars	178
13.1	The non-rotating ‘star’	178
13.2	Uniform rotation	181
13.3	Further reading	187
13.4	Problems	187
14	Cylindrical shear flow–non-axisymmetric instability	191
14.1	Equilibrium configuration	191
14.2	The perturbation equations	193
14.3	The Papaloizou–Pringle instability	195
14.4	Further reading	197
14.5	Problems	197
	<i>References</i>	199
	<i>Index</i>	203

Preface

Almost all of the baryonic Universe is fluid, and the study of how these fluids move is central to astrophysics. This book originated in a 24-lecture course entitled ‘Astrophysical Fluids’ given by one of us (JEP) in Part III of the Mathematical Tripos at the University of Cambridge, comparable in level to a graduate course in the USA. The course was intended as a preparation for research, and the book reflects this. Preparing the lecture course and especially its booklist made it plain that there was a need to bring these ideas together in one place.

The book provides a brief coverage of basic concepts, but does assume some familiarity with undergraduate-level fluid dynamics, electromagnetic theory and thermodynamics. Our aim is to give a flavour of the fundamental fluid dynamical processes and concepts which an astrophysical theorist ought to know. To keep the book to a manageable size, we have had to be selective. In particular, we omit all discussion of dissipative fluid processes such as viscosity and magnetic diffusivity.

As well as covering a range of fluid dynamical concepts, we introduce some mathematical ideas and techniques. None of these is particularly deep or abstract, but some of the implementations do require some moderately heavy but straightforward algebra. Thus the reader will benefit from some familiarity with undergraduate-level mathematical methods, as well as some facility in mathematical manipulation. This takes practice and care, but more than anything it requires the ability to spot a mistake before proceeding too far.

Ideally, of course, one does not make mistakes, and some lecturers like to give their students the misleading impression that this is how research is done. In practice, errors occur all too frequently, and unfortunately some of these make their way into the research literature. The best method for finding errors is to understand the physical processes involved and how these processes are expressed in mathematical formulae. For this reason, this book emphasizes physical understanding and the extraction of relevant physical ideas from a mass of equations. To achieve this we often drastically simplify problems and keep only the physical processes of interest. For example, in the chapters on stellar oscillations we eliminate much of the heavy algebra which appears because real stars are spherical, and instead assume that stars are square (plane-parallel) or at worst (for rotating stars) cylindrical. This lets us get at the underlying physical processes without obscuring them with mathematics.

The problems at the ends of the chapters come both from the problem sheets associated with the course and from the examination questions set for it. They are intended to illustrate the course material further and also to introduce additional ideas. Thus they are an integral part of the book, and the determined reader will benefit from working through them.