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978-0-521-86920-1- Modern Plasma Physics, Volume 1: Physical Kinetics of Turbulent Plasmas

Patrick H. Diamond, Sanae-I. Itoh, Kimitaka Itoh

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MODERN PLASMA PHYSICS VOLUME 1: PHYSICAL KINETICS OF TURBULENT PLASMAS

This three-volume series presents the ideas, models and approaches essential to understanding plasma dynamics and self-organization for researchers and graduate students in plasma physics, controlled fusion and related fields such as plasma astrophysics.

Volume 1 develops the physical kinetics of plasma turbulence through a focus on quasi-particle models and dynamics. It discusses the essential physics concepts and theoretical methods for describing weak and strong fluid and phase space turbulence in plasma systems far from equilibrium. The book connects the traditionally “plasma” topic of weak or wave turbulence theory to more familiar fluid turbulence theory, and extends both to the realm of collisionless phase space turbulence. This gives readers a deeper understanding of these related fields, and builds a foundation for future applications to multi-scale processes of self-organization in tokamaks and other confined plasmas. This book emphasizes the conceptual foundations and physical intuition underpinnings of plasma turbulence theory.

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MODERN PLASMA PHYSICS

Volume 1: Physical Kinetics of Turbulent Plasmas

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Preface

The universe abounds with plasma turbulence. Most of the matter that we can observe directly is in the plasma state. Research on plasmas is an active scientific area, motivated by energy research, astrophysics and technology. In nuclear fusion research, studies of confinement of turbulent plasmas have lead to a new era, namely that of the international thermonuclear (fusion) experimental reactor, ITER. In space physics and in astrophysics, numerous data from measurements have been heavily analyzed. In addition, plasmas play important roles in the development of new materials with special industrial applications.

The plasmas that we encounter in research are often far from thermodynamic equilibrium: hence various dynamical behaviours and structures are generated because of that deviation. The deviation is often sufficient for observable mesoscale structures to be generated. Turbulence plays a key role in producing and defining observable structures. An important area of modern science has been recognized in this research field, namely, research on structure formation in turbulent plasmas associated with electromagnetic field evolution and its associated selection rules. Surrounded by increasing and detailed information on plasmas, some unified and distilled understanding of plasma dynamics is indeed necessary – “*Knowledge must be developed into understanding*”. The understanding of turbulent plasma is a goal for scientific research in plasma physics in the twenty-first century.

The objective of this series on modern plasma physics is to provide the viewpoint and methods which are essential to understanding the phenomena that researchers on plasmas have encountered (and may encounter), i.e., the mutually regulating interaction of strong turbulence and structure formation mechanisms in various strongly non-equilibrium circumstances. Recent explosive growth in the knowledge of plasmas (in nature as well as in the laboratory) requires a systematic explanation of the methods for studying turbulence and structure formation.

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The rapid growth of experimental and simulation data has far exceeded the evolution of published monographs and textbooks. In this series of books, we aim to provide systematic descriptions (1) for the theoretical methods for describing turbulence and turbulent structure formation, (2) for the construction of useful physics models of far-from-equilibrium plasmas and (3) for the experimental methods with which to study turbulence and structure formation in plasmas. This series will fulfil needs that are widely recognized and stimulated by discoveries of new astrophysical plasmas and through advancement of laboratory plasma experiments related to fusion research. For this purpose, the series constitutes three volumes: Volume 1: Physical kinetics of turbulent plasmas, Volume 2: Turbulence theory for structure formation in plasmas and Volume 3: Experimental methods for the study of turbulent plasmas. This series is designed as follows.

Volume 1: *Physical Kinetics of Turbulent Plasmas* The objective of this volume is to provide a systematic presentation of the theoretical methods for describing turbulence and turbulent transport in strongly non-equilibrium plasmas. We emphasize the explanation of the progress of theory for strong turbulence. A viewpoint, i.e., that of the “quasi-particle plasma” is chosen for this book. Thus we describe ‘plasmas of excitons, dressed by collective interaction’, which enable us to understand the evolution and balance of plasma turbulence.

We stress (a) test field response (particles and waves, respectively), taking into account screening and dressing, as well as noise, (b) disparate scale interaction and (c) mean field evolution of the screened element gas. These three are essential building blocks with which to construct a physics picture of plasma turbulence in a strongly non-equilibrium state. In the past several decades, distinct progress has been made in this field, and verification and validation of nonlinear simulations are becoming more important and more intensively pursued. This is a good time to set forth a systematic explanation of the progress in methodology.

Volume 2: *Turbulence Theory for Structure Formation in Plasmas* This volume presents the description of the physics pictures and methods to understand the formation of structures in plasmas. The main theme has two aspects. The first is to present ways of viewing the system of turbulent plasmas (such as toroidal laboratory plasmas, etc.), in which the dynamics for both self-sustaining structure and turbulence coexist. The other is to illustrate key organizing principles and to explain appropriate methods for their utilization. The competition (e.g., global inhomogeneity, turbulent transport, quenching of turbulence, etc.) and self-sustaining mechanisms are described.

One particular emphasis is on a self-consistent description of the mechanisms of structure formation. The historical recognition of the proverb “*All things flow*” means that structures, which disappear within finite lifetimes, can also be, and are usually, continuously generated. Through the systematic description of plasma

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turbulence and structure formation mechanisms, this book illuminates principles that govern evolution of laboratory and astrophysical plasmas.

Volume 3: *Experimental Methods for the Study of Turbulent Plasmas* The main objective of this volume is to explain methods for the experimental study of turbulent plasmas. Basic methods to identify elementary processes in turbulent plasmas are explained. In addition, the design of experiments for the investigation of plasma turbulence is also discussed with the aim of future extension of experimental studies. This volume has a special feature. While many books and reviews have been published on plasma diagnostics, i.e., how to obtain experimental signals in high temperature plasmas, little has been published on how one analyzes the data in order to identify and extract the physics of nonlinear processes and nonlinear mechanisms. In addition, the experimental study of nonlinear phenomena requires a large amount of data processing. This volume explains the methods for performing quantitative studies of experiments on plasma turbulence.

Structure formation in turbulent media has been studied for a long time, and the proper methodology to model (and to formulate) has been elusive. This series of books will offer a perspective on how to understand plasma turbulence and structure formation processes, using advanced methods.

Regarding readership, this book series is aimed at the more advanced graduate student in plasma physics, fluid dynamics, astrophysics and astrophysical fluids, nonlinear dynamics, applied mathematics and statistical mechanics. Only minimal familiarity with elementary plasma physics at the level of a standard introductory text is presumed. Indeed, a significant part of this book is an outgrowth of advanced lectures given by the authors at the University of California, San Diego, at Kyushu University and at other institutions. We hope the book may be of interest and accessible to postdoctoral researchers, to experimentalists and to scientists in related fields who wish to learn more about this fascinating subject of plasma turbulence.

In preparing this manuscript, we owe much to our colleagues for our scientific understanding. For this, we express our sincere gratitude in the Acknowledgments. There, we also acknowledge the funding agencies that have supported our research. We wish to show our thanks to young researchers and students who have helped in preparing this book, by typing and formatting the manuscript while providing invaluable feedback: in particular, Dr. N. Kasuya of NIFS and Mr. S. Sugita of Kyushu University for their devotion, Dr. F. Otsuka, Dr. S. Nishimura, Mr. A. Froese, Dr. K. Kamataki and Mr. S. Tokunaga of Kyushu University also deserve mention. A significant part of the material for this book was developed in the Nonlinear Plasma Theory (Physics 235) course at UCSD in 2005. We thank the students in this class, O. Gurcan, S. Keating, C. McDevitt, H. Xu and A. Walczak for their penetrating questions and insights. We would like to express our gratitude

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