1
Introduction

The beginning is the most important part of the work.

(Plato)

In this introduction, we set out directly to answer the many questions the reader may have in mind about this book, such as:

(1) *Why* is this book being written? Why study theory in the age of high performance computing and experimental observations in unparalleled detail? In what way does it usefully augment the existing literature? *Who* is the target readership?
(2) *What* does it cover? What is the logic behind our particular choice of topics? *Where* will a reader stand and *how* will he or she benefit after completing this book?
(3) *What* is *not* included and *why* was it omitted? What alternative sources are recommended to the reader?

We now proceed to answer these questions.

1.1 Why?

Surely the need for study of plasma turbulence requires no explanation.

Turbulence pervades the dynamics of both laboratory and astrophysical plasmas. Turbulent transport and its associated confinement degradation are *the* main obstacles to achieving ignition in magnetically confined plasma (i.e., for magnetic confinement fusion (MCF) research), while transport bifurcations and self-generated shear flows are the principal means for controlling such drift wave turbulence. Indeed, predictions of degradation of confinement by turbulence have been used to (unjustifiably) challenge plans for ITER (International Thermonuclear Experimental Reactor). In the case of inertial confinement fusion (ICF) research, turbulent mixing driven by Rayleigh–Taylor growth processes limit implosion performance for indirect drive systems, while the nonlinear evolution of laser–plasma instabilities (such as filamentation – note, these are examples of turbulence in disparate

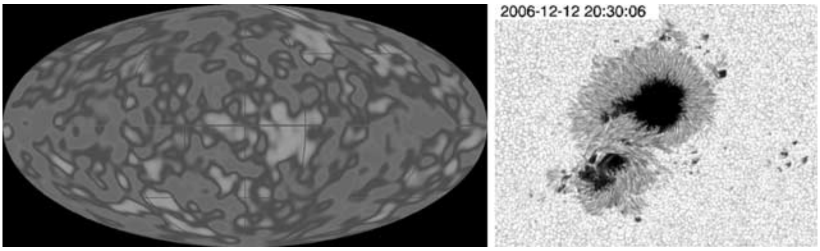

Fig. 1.1. The cosmic microwave background fluctuations (left). Fluctuations pervade the universe. The cosmic microwave background radiation is a remnant of the Big Bang and the fluctuations are the imprint of the density contrast in the early universe. [http://aether.lbl.gov/www/projects/cobe/COBE_Home/DMR_Images.html] Turbulent dynamics are observed in solar plasmas near the sunspot (right). [Observation by Hinode, courtesy NAOJ/JAXA.]

scale interaction) must be controlled in order to achieve fast ignition. In space and astrophysical plasma dynamics, turbulence is everywhere, i.e. it drives inter-stellar medium (ISM) scintillations, stirs the galactic and stellar dynamos, scatters particles to facilitate shock acceleration of cosmic rays, appears in strongly driven 3D magnetic reconnection, drives angular momentum transport to allow accretion in disks around protostars and active galactic nuclei (AGNs), helps form the solar tachocline, etc. – the list is indeed endless. (Some examples are illustrated in Figure 1.1.) Moreover, this large menu of MCF, ICF and astrophysical applications offers an immensely diverse assortment of turbulence from which to choose, i.e. strong turbulence, wave turbulence, collisional and very collisionless turbulence, strongly magnetized systems, weakly magnetized systems, multi-component systems with energetic particles, systems with sheared flow, etc., all are offered. Indeed, virtually *any possible* type of plasma turbulence finds some practical application in the realm of plasma physics.

Thus, while even the most hardened sceptic must surely grant the merits of plasma turbulence and its study, one might more plausibly ask, "Why study plasma turbulence *theory*, in the age of computation and detailed experimental observations? Can't we learn all we need from direct numerical simulation?" This question is best dealt with by considering the insights in the following set of quotations from notable individuals. Their collective wisdom speaks for itself.

"Theory gives meaning to our understanding of the empirical facts."

(John Lumley)

"Without simple models, you can't get anything out of numerical simulation."

(Mitchell J. Feigenbaum)

"When still photography was invented, it soon became so popular that it was expected to mark the end of drawing and painting. Instead, photography made artists honest, requiring more of them than mere representation."

(Peter B. Rhines)

In short, theory provides a necessary intellectual framework – a structure and a system from within which to derive meaning and/or a message from experiment, be it physical or digital. Theory defines the simple models used to *understand* simulations and experiments, and to extract more general lessons from them.

This process of extraction and distillation is a prerequisite for development of predictive capacity. Theory also forms the basis for both *verification* and *validation* of simulation codes. It defines exactly solvable mathematical models needed for verification and also provides the intellectual framework for a programme of validation. After all, any meaningful comparison of simulation and experiment requires specification of physically relevant questions or comparisons which must be addressed. It is unlikely this can be achieved in the absence of guidance from theory. It is surely the case that the rise of the computer has indeed made the task of the theorists more of a challenge. As suggested by Rhines, the advent of large-scale computation has forced theory to define ideas or to teach a conceptual lesson, rather than merely to crunch out numbers. Theory must constitute the knowledge necessary to make use of the raw information obtained from simulation and experiment. *Theory must then lead the scientist from knowledge to understanding*. It must identify, define and teach us a simple, compact lesson. As Rhines states, it must do more than merely represent. Indeed, the danger here is that in this data-rich age, without distillation of a message, a simulation or representation or experimental data-acquisition will grow as large and complex as the object being represented, as imagined in the following short fiction by the incomparable Jorge Luis Borges.

. . . In that Empire, the Art of Cartography attained such Perfection that the map of a single Province occupied the entirety of a City, and the map of the Empire, the entirety of a Province. In time, those Unconscionable Maps no longer satisfied, and the Cartographers Guilds struck a Map of the Empire whose size was that of the Empire, and which coincided point for point with it. The following Generations, who were not so fond of the Study of Cartography as their Forebears had been, saw that that vast Map was Useless, and not without some Pitilessness was it, that they delivered it up to the Inclemencies of Sun and Winters. In the Deserts of the West, still today, there are Tattered Ruins of that Map, inhabited by Animals and Beggars; in all the Land there is no other Relic of the Disciplines of Geography.

(Suarez Miranda, Viajes de varones pudentes,
Libro IV, Cap. XLV, Lerida, 1658.
Jorge Luis Borges)

Without theory, we are indeed doomed to a life amidst a useless pile of data and information.

1.2 The purpose of this book

With generalities now behind us, we proceed to state that this book has two principal motivations, which are:

(1) to serve as an up-to-date and advanced, yet accessible, monograph on the basic physics of plasma turbulence, from the perspective of the physical kinetics of quasi-particles,
(2) to stand as the first book in a three-volume series on the emerging science of structure formation and self-organization in turbulent plasma.

Our ultimate aim is not only to present developments in the *theory* but also to describe how these elements are applied to the understanding of structure formation phenomena, in real plasma, such as tokamaks, other confinement devices and in the universe, Thus, this series forces theory to confront reality! These dual motivations are best served by an approach in the spirit of Lifshitz and Pitaevski's *Physical Kinetics*, namely with an emphasis on quasi-particle descriptions and their associated kinetics. We feel this is the optimal philosophy within which to organize the concepts and theoretical methods needed for understanding *ongoing* research in structure formation in plasma, since it naturally unites resonant and non-resonant particle dynamics.

This long-term goal motivates much of the choice of topical content of the book, in particular:

(1) the discussion of dynamics in both real space and wave-number space; i.e. explanation of Prandtl's theory of turbulent boundary layers in parallel with Kolmogorov's cascade theory (K41 theory), in Chapter 2, where the basic notions of turbulence are surveyed. Prandtl mixing length theory is an important paradigm for profile stiffness, etc. and other commonplace ideas in MFE (magnetic fusion energy) research;
(2) the contrast between the zero spectral flux in "near equilibrium" theory (i.e. the dressed test particle model) and the large spectral flux inertial range theory (as by Kolmogorov), discussed in Chapter 2. These two cases bound the dynamically relevant limit of weak or moderate turbulence, which we usually encounter in the real world of confined plasmas;
(3) the treatment of quasi-linear theory in Chapter 3, which focuses on the energetics of the interaction of resonant particles with quasi-particles. This is, without a doubt, the most useful approach to mean field theory for collisionless relaxation;
(4) the renormalized or dressed resonant particles response, discussed at length in Chapter 4. In plasma, both the particle and collective responses are nonlinear and require detailed, individual treatment. The renormalized particle propagator defines a key, novel time-scale;

(5) the extensive discussion of disparate scale interaction, in Chapters 5–7:
  (a) from the viewpoint of nonlocal wave–wave interactions, such as induced diffusion, in Chapter 5,
  (b) from the perspective of Mori–Zwanzig theory in Chapter 6,
  (c) in the context of adiabatic theory for Langmuir turbulence (both mean field theory for random phase wave kinetics *and* the coherent Zakharov equations) in Chapter 7.

  We remark here that disparate scale interaction is fundamental to the dynamics of "negative viscosity phenomena" and so is extremely important to structure formation. Thus, it merits the very detailed description accorded to it here;
(6) the detailed and extensive discussion of phase space density granulation and its role in the description of mean field relaxation, which we present in Chapter 8. In this chapter, the notion of the "quasi-particle in turbulence" is expanded to encompass the screened "clump" or phase space vortex. An important consequence of this conceptual extension is the manifestation of *dynamical friction* in the mean field theory for the Vlasov plasma. Note that dynamical friction is *not* accounted for in standard quasi-linear theory, which is the traditional backbone of mean field methodology for plasma turbulence;
(7) the discussion of quenching of diffusion in 2D MHD (magnetohydrodynamics), presented in Chapter 9. Here, we encounter the principle of a quasi-particle with a dressing that confers a *memory* to the dynamics. This memory which follows from the familiar MHD freezing-in law, quenches the diffusion of fluid relative to the fluid, and so severely constrains relaxation.

All of (1)–(7) represent new approaches not discussed in existing texts on plasma turbulence.

Throughout this book, we have placed special emphasis on identifying and explaining the physics of key time and space scales. These are usually summarized in an offset table which is an essential and prominent part of the chapter in which they are developed and defined. Essential time-scale orderings are also clarified and tabulated. We also construct several tables which compare and contrast the contents of different problems. We deem these useful in demonstrating the relevance of lessons learned from simple problems to more complicated applications. More generally, understanding of the various nonlinear time scales and their interplay is essential to the process of construction of tractable simple models, such as spectral tranfer scalings, from more complicated frameworks such as wave kinetic formalisms. Thus, we place great emphasis on the physics of basic time scales.

The poet T. S. Eliot once wrote, "Dante and Shakespeare divide the world between them. There is no third." So it is with introductory books on plasma turbulence theory – the two classics of the late 1960s, namely R. Z. Sagdeev and A. A. Galeev's *Nonlinear Plasma Theory*[1] and B. B. Kadomtsev's *Plasma Turbulence* are the twin giants of this field and are still quite viable guides to the subject. Any new monograph must meet their standards. This is a challenge for all monographs prepared on the subject of plasma turbulence.

Nevertheless, we presumptuously argue that now there *is* indeed room for a 'third'. In particular (the following list does not intend to enumerate the shortcomings of these two classics but rather to observe the significant advancement of plasma physics in the past two decades) we aim to elucidate the following important issues:

(1) the smooth passage from one limit (weak turbulence theory – Sagdeev and Galeev) to the other (strong turbulence based on ideas from hydrodynamics – Kadomtsev), and the duality of these two approaches in collisionless regimes;
(2) the important relation between self-similarity in *space* (i.e. turbulent mixing), and self-similarity in *scale* (turbulent cascade), including the inverse cascades or MHD turbulence dynamics;
(3) the resonance broadening theory or Vlasov response renormalization, both of which extend the concept of eddy viscosity into phase space in an important way;
(4) the important subject of the theory of disparate scale interaction or "negative viscosity phenomena", which is crucial for describing self-organization and structure formation in turbulent plasma. This class of phenomena is *the* central focus of our series;
(5) the theory of phase space density granulation. This important topic is required for understanding and describing stationary phase space turbulence with resonant heating, etc., where dynamical friction is a *must*;
(6) the important problem of the structure of two-point correlation in turbulent plasma;
(7) applications to MHD turbulence or transport, or to the quasi-geostrophic/drift wave turbulence problem.

Thus, even before we come to more advanced subjects such as zonal flow formation, phase-space density holes, solitons and collisionless shocks and transport barriers and bifurcations – all subjects for out next volume – it seems clear that a fresh look at the basics of plasma turbulence is indeed warranted. This book is our attempt to realize this vision.

1.3 Readership and background literature

We have consciously written this book so as to be accessible to more advanced graduate students in plasma physics, fluid dynamics, astrophysics and astrophysical fluids, nonlinear dynamics, applied mathematics and statistical mechanics. Only minimal familiarity with elementary plasma physics – at the level of a

[1] The longer version published in *Reviews of Plasma Physics*, Vol. VII is more complete and in many ways superior to the short monograph.

standard introductory text such as Kulsrud (2005), Sturrock (1994) or Miyamoto (1976) – is presumed.

This series of volumes is designed to provide a focused explanation of the physics of plasma turbulence. Introductions to many elementary processes in plasmas, such as the dynamics of particle motion, varieties of linear plasma eigenmodes, instabilities, the MHD dynamics of confined plasmas and the systems for plasma confinement, etc. are in the literature and are already widely available to readers. For instance, the basic properties of plasmas are explained in Krall and Trivelpiece (1973), Ichimaru (1973), Miyamoto (1976) and Goldston and Rutherford (1995); waves are thoroughly explained in Stix (1992); MHD equations, equilibrium and stability are explained in Freidberg (1989) and Hazeltine and Meiss (1992); an introduction to tokamaks is given in White (1989), Kadomtsev (1992), Wesson (1997) and Miyamoto (2007); drift wave instabilities are reviewed in Mikailovski (1992), Horton (1999) and Weiland (2000); issues in astrophysical plasmas are discussed in Sturrock (1994), Tajima and Shibata (2002) and Kulsrud (2005) and subjects of chaos are explained in Lichtenberg and Lieberman (1983) and Ott (1993). Further explanation is available by reference to the literature. The reader may also find it helpful to refer to books on plasma turbulence which precede this volume, e.g. Kadomtsev (1965), Galeev and Sagdeev (1965), Sagdeev and Galeev (1969), R. C. Davidson (1972), Itoh *et al.* (1999), Moiseev *et al.* (2000), Yoshizawa *et al.* (2003), Elskens and Escande (2003) and Balescu (2005). Advanced material on neutral fluid turbulence dynamics, which is related to the contents of this volume, can be found in Lighthill (1978), McComb (1990), Frisch (1995), Moiseev *et al.* (2000), Pope (2000), Yoshizawa *et al.* (2003) and P. A. Davidson (2004).

1.4 Contents and structure of this book

Having completed our discussion of motivation, we now turn to presenting the actual contents of this book.

Chapter 2 deals with foundations. Since most realizations of plasma turbulence are limits of "weak turbulence" or intermediate regime cases where the mode self-correlation time τ_c is longer than (weak) or comparable to (intermediate) the mode frequency $\omega (\tau_c \omega > 1)$, we address foundations by discussing the opposite extremes of:

(1) states of *zero* spectral flux – i.e. fluctuations at equilibrium, as described by the test particle model. In this limit, linear emission and absorption balance locally, at each **k**, to define the thermal equilibrium fluctuation spectrum. Moreover, the theory of dressed test particle dynamics is a simple, instructive example of the impact of collective screening effects on fluctuations. The related Lenard–Balescu theory, which we also discuss, defines *the* prototypical formal structure for a mean field theory of

transport and relaxation. These basic paradigms are fundamental to the subsequent discussions of quasi-linear theory in Chapter 3, nonlinear wave–particle interaction in Chapter 4, and the theory of phase space density granulations in Chapter 8;

(2) states dominated by a large spectral flux, where nonlinear transfer exceeds all other elements of the dynamics. Such states correspond to turbulent cascades, in which nonlinear interactions couple sources and sinks at very different scales by a sequence of local transfer events. Indeed, the classic Kolmogorov cascade is defined in the limit where the dissipation rate ϵ is the *sole* relevant rate in the inertial range. Since confined plasmas are usually strongly magnetized, so that the parallel degrees of freedom are severely constrained, we discuss both the 3D forward cascade and the 2D inverse cascade in equal depth. We also discuss pertinent related topics such as Richardson's calculation of two-particle dispersion.

Taken together, (1) and (2) in a sense "bound" most plasma turbulence applications of practical relevance. However, given our motivations rooted in magnetic confinement fusion physics, we also devote substantial attention to *spatial transport* as well as spectral transfer. To this end, then, the Introduction also presents the Prandtl theory of pipe flow profiles in space on an equal footing with the Kolmogorov spectral cascade in scale. Indeed the Prandtl boundary layer theory is the prototype of familiar MFE concepts such as profile "stiffness", mixing length concepts, and dimensionless similarity. It is the natural example of self-similarity in space, with which to complement self-similarity in scale. For these and other reasons, it merits inclusion in the lead-in chapter on fundamentals, and is summarized in Table 2.4, at the conclusion of this chapter.

Chapter 3 presents quasi-linear theory, which is *the* practical, workhorse tool for mean field calculations of relaxation and transport for plasma turbulence. Despite quasi-linear theory's celebrated status and the fact that it appears in nearly every basic textbook on plasma physics, we were at a loss to find a satisfactory treatment of its foundations, and, in particular, one which does justice to their depth and subtlety. Quasi-linear theory is simple but *not* trivial. Thus, we have sought to rectify this situation in Chapter 3. In particular, we have devoted considerable effort to:

(1) a basic discussion of the origin of irreversibility – which underpins the coarse-graining intrinsic to quasi-linear theory – in particle stochasticity due to phase space island overlap;

(2) a careful introductory presentation of the many time scales in play in quasi-linear theory, and the orderings they must satisfy. The identification and ordering of pertinent time scales is one of the themes of this book. Special attention is devoted to the distinction between the wave–particle correlation time and the spectral auto-correlation time. This distinction is especially important for the case of the quasi-linear theory of 3D drift wave turbulence, which we discuss in detail. We also "locate" quasi-linear theory in the realm of possible Kubo number orderings;

(3) presentation of the multiple forms of conservation laws (i.e. resonant particles versus waves or particles versus fields) in quasi-linear theory, along with their physical meaning. These form the foundation for subsequent quasi-particle formulations of transport, stresses, etc. The concept of the plasma as coupled populations of resonant particles and quasi-particles (waves) is one of the most intriguing features of quasi-linear theory;
(4) an introduction to up-gradient transport (i.e. the idea of a thermodynamic inward flux or "pinch"), as it appears in the quasi-linear theory of transport. As part of this discussion, we address the entropy production constraint on the magnitude of up-gradient fluxes;
(5) an introduction to nonlinear Landau damping as 'higher-order quasi-linear theory', in which $\langle f \rangle$ relaxes via beat–wave resonances.

The aim of Chapter 3 is to give the reader a working introduction to mean field methods in plasma turbulence. The methodology of quasi-linear theory, developed in this chapter, is used throughout the rest of this book, especially in Chapter 7, 8 and 9. Specific applications of quasi-linear theory to advanced problems in tokamak confinement are deferred to Volume 2.

Chapter 4 continues the thematic exploration of resonant particle dynamics by an introduction to *nonlinear* wave–particle interaction. Here we focus on selected topics, which are:

(1) resonance broadening theory, i.e. how finite fluctuation levels broaden the wave–particle resonance and define a nonlinear decorrelation time for the response δf. The characteristic scale of the broadened resonance width is identified and discussed. We present applications to 1D Vlasov dynamics, drift wave turbulence in a sheared magnetic field, and enhanced decorrelation of fluid elements in a sheared flow;
(2) perturbative or iterative renormalization of the 1D Vlasov response function. Together with (1), this discussion presents propagator renormalization or – in the language of field theory – "mass renormalization" in the context of Vlasov plasma dynamics. We discuss the role of background distribution counter-terms (absent in resonance broadening theory) and the physical significance of the non-Markovian character of the renormalization. The aim here is to connect the more intuitive approach of resonance broadening theory to the more formal and systematic approach of perturbative renormalization.
(3) The application of renormalization of the drift wave problem, at the level of drift kinetics. The analysis here aims to illustrate the role of energy conservation in constraining the structure of the renormalized response. This instructive example illustrates the hazards of naive application of resonance-broadening theory.

Further study of nonlinear wave–particle interaction is deferred until Chapter 8.

Chapter 5 introduces the important topic of nonlinear wave–wave interaction. Both the integrable dynamics of coherent interaction in discrete mode triads, as well as the stochastic, random phase interactions as occur for a broad spectrum of dispersive waves are discussed. This chapter is fundamental to all that follow. Specific attention is devoted to:

(1) the coherent, resonant interaction of three drift waves. Due to the dual constraints of conservation of energy and enstrophy (mean squared vorticity), this problem is demonstrated to be isomorphic to that of the motion of the free asymmetric top, and so can be integrated by the Poinsot construction. We also show that a variant of the Poinsot construction can be used to describe the coherent coupled motion of three modes which conserve energy and obey the Manley–Rowe relations. Characteristic time scales for parametric interaction are identified;
(2) the derivation of the random-phase spectral evolution equation (i.e. the *wave kinetic equation*) which is presented in detail. The stochastic nature of the wave population evolution is identified and traced to overlap of triad resonances. We explain the modification of the characteristic energy transfer time scales by stochastic scattering;
(3) basic concepts of wave cascades. Here, we discuss the cascade of energy in gravity wave interaction. In Chapter 9, we discuss the related application of the Alfvén wave cascade. The goal here is to demonstrate how a tractable scaling argument is constructed using the structure of the wave kinetic equation;
(4) non-local (in $\mathbf{k}$) wave coupling processes. Given our over-arching interest in the dynamics of structure formation, we naturally place a great deal of emphasis on non-local interactions in $\mathbf{k}$, especially the direct interactions of small scales with large, since these drive stresses, transport etc. (which are quadratic in fluctuation amplitude), which directly impact macro-structure. Indeed, significant parts of Chapters 5 and 6, and *all* of Chapter 7 deal with non-local, disparate scale interaction. In this chapter, we identify three types of non-local (in $\mathbf{k}$) interaction processes (induced diffusion, parametric subharmonic interaction and elastic scattering), which arise naturally in wave interaction theory. Of these, induced diffusion is especially important and is discussed at some length.

Chapter 6 presents renormalized turbulence closure theory for wave–wave interactions. Key concepts such as the nonlinear scrambling or self-coherence time, the interplay of nonlinear noise emission with nonlinear damping and the non-Markovian structure of the closure theory are discussed in detail. Non-standard aspects of this chapter include:

(1) a discussion of Kraichnan's random coupling model, which is the paradigm for understanding the essential physics content of the closure models, since it defines a physical realization of the closure theory equations;
(2) the development of the Mori–Zwanzig theory of problem reduction in parallel with the more familiar direct interaction approximation (DIA). The merits of this approach are two-fold. First, the Mori–Zwanzig memory function constitutes a well-defined limit of the DIA response function and so defines a critical benchmark for that closure method. Second the Mori–Zwanzig theory is a rigorous but technically challenging solution to the problem of disparate scale interaction. It goes further than the induced