

Contents

<i>Preface</i>	<i>page xv</i>
1 Equations of Motion	1
1.1. The fluid state	1
1.2. The material derivative	1
1.3. Conservation of mass: Equation of continuity	2
1.4. Momentum equation	3
1.4.1. Relative motion of neighbouring fluid elements	3
1.4.2. Viscous stress tensor	5
1.4.3. Navier–Stokes equation	7
1.4.4. The Reynolds equation and Reynolds stress	7
1.5. The energy equation	8
1.5.1. Alternative treatment of the energy equation	9
1.5.2. Energy equation for incompressible flow	10
1.6. Summary of governing equations	11
1.7. Boundary conditions	12
Problems 1	12
2 Potential Flow of an Incompressible Fluid	14
2.1. Ideal fluid	14
2.2. Kelvin’s circulation theorem	14
2.3. The velocity potential	16
2.3.1. Bernoulli’s equation	16
2.3.2. Impulsive pressure	18
2.3.3. Streamlines and intrinsic equations of motion	18
2.3.4. Bernoulli’s equation in steady flow	20
2.4. Motion produced by a pulsating sphere	21
2.5. The point source	22
2.6. Free-space Green’s function	24

2.7. Monopoles, dipoles, and quadrupoles	24
2.7.1. The vibrating sphere	26
2.7.2. Streamlines	28
2.7.3. Far field of a monopole distribution of zero strength	29
2.8. Green's formula	30
2.8.1. Volume and surface integrals	30
2.8.2. Green's formula	32
2.8.3. Sources adjacent to a plane wall	34
2.9. Determinancy of the motion	35
2.9.1. Fluid motion expressed in terms of monopole or dipole distributions	37
2.9.2. Determinancy of cyclic irrotational flow	39
2.9.3. Kinetic energy of cyclic irrotational flow	40
2.10. The kinetic energy	41
2.10.1. Converse of Kelvin's minimum-energy theorem	43
2.10.2. Energy of motion produced by a translating sphere	43
2.11. Problems with spherical boundaries	45
2.11.1. Legendre polynomials	45
2.11.2. Velocity potential of a point source in terms of Legendre polynomials	50
2.11.3. Interpretation in terms of images	52
2.12. The Stokes stream function	53
2.12.1. Stream function examples	55
2.12.2. Rankine solids	56
2.12.3. Rankine ovoid	58
2.12.4. Drag in ideal flow	58
2.12.5. Axisymmetric flow from a nozzle	60
2.12.6. Irrotational flow from a circular cylinder	63
2.12.7. Borda's mouthpiece	65
2.13. The incompressible far field	67
2.13.1. Deductions from Green's formula	68
2.13.2. Far field produced by motion of a rigid body	69
2.13.3. Inertia coefficients	70
2.13.4. Pressure in the far field	70
2.14. Force on a rigid body	71
2.14.1. Moment exerted on a rigid body	73
2.15. Sources near solid boundaries	75
2.15.1. The reciprocal theorem	76
2.16. Far-field Green's function	78
2.16.1. The Kirchhoff vector	80
2.16.2. Far-field Green's function for a sphere	80

CONTENTS

ix

2.17. Far-field Green's function for cylindrical bodies	84
2.17.1. The circular cylinder	85
2.17.2. The rigid strip	86
2.18. Symmetric far-field Green's function	89
2.18.1. Far field of an arbitrarily moving body	90
2.19. Far-field Green's function summary and special cases	91
2.19.1. General form	91
2.19.2. Airfoil of variable chord	92
2.19.3. Projection or cavity on a plane wall	93
2.19.4. Rankine ovoid	94
2.19.5. Circular aperture	95
2.19.6. Circular disc	96
Problems 2	96
 3 Ideal Flow in Two Dimensions	102
3.1. Complex representation of fluid motion	102
3.1.1. The stream function	102
3.1.2. The complex potential	104
3.1.3. Uniform flow	104
3.1.4. Flow past a cylindrical surface	105
3.2. The circular cylinder	106
3.2.1. Circle theorem	106
3.2.2. Uniform flow past a circular cylinder	106
3.2.3. The line vortex	109
3.2.4. Circular cylinder with circulation	110
3.2.5. Equation of motion of a cylinder with circulation	112
3.3. The Blasius force and moment formulae	115
3.3.1. Blasius's force formula for a stationary rigid body	116
3.3.2. Blasius's moment formula for a stationary rigid body	117
3.3.3. Kutta–Joukowski lift force	117
3.3.4. Leading-edge suction	118
3.4. Sources and line vortices	119
3.4.1. Line vortices	122
3.4.2. Motion of a line vortex	122
3.4.3. Kármán vortex street	127
3.4.4. Kinetic energy of a system of rectilinear vortices	127
3.5. Conformal transformations	128
3.5.1. Transformation of Laplace's equation	129
3.5.2. Equation of motion of a line vortex	132
3.5.3. Numerical integration of the vortex path equation	133
3.6. The Schwarz–Christoffel transformation	135
3.6.1. Irrotational flow from an infinite duct	138
3.6.2. Irrotational flow through a wall aperture	140

3.7. Free-streamline theory	142
3.7.1. Coanda edge flow	142
3.7.2. Mapping from the w plane to the t plane	147
3.7.3. Separated flow through an aperture	147
3.7.4. The wake of a flat plate	151
3.7.5. Flow past a curved boundary	152
3.7.6. The hodograph transformation formula	158
3.7.7. Chaplygin's singular point method	159
3.7.8. Jet produced by a point source	160
3.7.9. Deflection of trailing-edge flow by a source	161
3.8. The Joukowski transformation	167
3.8.1. The flat-plate airfoil	170
3.8.2. Calculation of the lift	173
3.8.3. Lift calculated from the Kirchhoff vector force formula	173
3.8.4. Lift developed by a starting airfoil	174
3.9. The Joukowski airfoil	175
3.9.1. Streamline flow past an airfoil	176
3.10. Separation and stall	179
3.10.1. Linear theory of separation	180
3.11. Sedov's method	183
3.11.1. Boundary conditions	184
3.11.2. Sedov's formula	185
3.11.3. Tandem airfoils	187
3.11.4. High-lift devices	190
3.11.5. Plain flap or aileron	192
3.11.6. Point sources and vortices	192
3.11.7. Flow through a cascade	193
3.12. Unsteady thin-airfoil theory	195
3.12.1. The vortex sheet wake	195
3.12.2. Translational oscillations	197
3.12.3. The unsteady lift	198
3.12.4. Leading-edge suction force	199
3.12.5. Energy dissipated by vorticity production	201
3.12.6. Hankel function formulae	202
Problems 3	203
 4 Rotational Incompressible Flow	211
4.1. The vorticity equation	211
4.1.1. Vortex lines	212
4.1.2. Vortex tubes	212
4.1.3. Movement of vortex lines: Helmholtz's vortex theorem	213

CONTENTS

xi

4.1.4. Crocco's equation	214
4.1.5. Convection and diffusion of vorticity	215
4.1.6. Vortex sheets	218
4.2. The Biot–Savart law	221
4.2.1. The far field	223
4.2.2. Kinetic energy	227
4.2.3. The Biot–Savart formula in the presence of an internal boundary	228
4.2.4. The Biot–Savart formula for irrotational flow	229
4.3. Examples of axisymmetric vortical flow	232
4.3.1. Circular vortex filament	232
4.3.2. Rate of production of vorticity at a nozzle	233
4.3.3. Blowing out a candle	235
4.3.4. Axisymmetric steady flow of an ideal fluid	236
4.3.5. Hill's spherical vortex	237
4.4. Some viscous flows	239
4.4.1. Diffusion of vorticity from an impulsively started plane wall	239
4.4.2. Diffusion of vorticity from a line vortex	240
4.4.3. Creeping flow	242
4.4.4. Motion of a sphere at very small Reynolds number	242
4.4.5. The Oseen approximation	245
4.4.6. Laminar flow in a tube (Hagen–Poiseuille flow)	247
4.4.7. Boundary layer on a flat plate; Kármán momentum integral method	249
4.5. Force on a rigid body	253
4.5.1. Surface force in terms of the impulse	254
4.5.2. The Kirchhoff vector force formula	256
4.5.3. The Kirchhoff vector force formula for irrotational flow	258
4.5.4. Arbitrary motion in a viscous fluid	258
4.5.5. Body moving without rotation	259
4.5.6. Surface force in two dimensions	261
4.5.7. Bluff body drag at high Reynolds number	261
4.5.8. Modelling vortex shedding from a sphere	265
4.5.9. Force and impulse in fluid of non-uniform density	270
4.5.10. Integral identities	271
4.6. Surface moment	273
4.6.1. Moment for a non-rotating body	273
4.6.2. Airfoil lift, drag, and moments	274
4.7. Vortex–surface interactions	276
4.7.1. Pressure expressed in terms of the total enthalpy	276
4.7.2. Equation for B	277

4.7.3. Solution of the B equation	278
4.7.4. The far field	279
Problems 4	281
5 Surface Gravity Waves	286
5.1. Introduction	286
5.1.1. Conditions at the free surface	286
5.1.2. Wave motion within the fluid	287
5.1.3. Linearised approximation	288
5.1.4. Time harmonic, plane waves on deep water	288
5.1.5. Water of finite depth	290
5.2. Surface wave energy	291
5.2.1. Wave-energy density	293
5.2.2. Wave-energy flux	294
5.2.3. Group velocity	295
5.3. Viscous damping of surface waves	297
5.3.1. The interior damping	297
5.3.2. Boundary-layer damping	298
5.3.3. Comparison of boundary-layer and internal damping for long waves	299
5.4. Shallow-water waves	299
5.4.1. Waves on water of variable depth	300
5.4.2. Shallow-water Green's function	301
5.4.3. Waves generated by a localised pressure rise	302
5.4.4. Waves approaching a sloping beach	307
5.5. Method of stationary phase	309
5.5.1. Formulation of initial-value dispersive-wave problems	309
5.5.2. Evaluation of Fourier integrals by the method of stationary phase	311
5.5.3. Numerical results for the surface displacement	313
5.5.4. Conservation of energy	315
5.5.5. Rayleigh's proof that energy propagates at the group velocity	317
5.5.6. Surface wave-energy equation	318
5.5.7. Waves generated by a submarine explosion	319
5.6. Initial-value problems in two surface dimensions	321
5.6.1. Waves generated by a surface elevation symmetric about the origin	322
5.6.2. The energy equation in two dimensions	324
5.7. Surface motion near a wavefront	325
5.7.1. One-dimensional waves	325
5.7.2. Waves generated by motion of the seabed	328

CONTENTS

xiii

5.7.3. Tsunami produced by an undersea earthquake	332
5.8. Periodic wave sources	333
5.8.1. One-dimensional waves	334
5.8.2. Periodic sources in two surface dimensions	336
5.8.3. The surface wave power	339
5.8.4. Surface wave amplitude	340
5.9. Ship waves	341
5.9.1. Moving line pressure source	342
5.9.2. Wave-making resistance	343
5.9.3. Moving point-like pressure source	345
5.9.4. Plotting the wave crests	349
5.9.5. Behaviour at the caustic	351
5.9.6. Wave-making power	352
5.9.7. Wave amplitude calculated from the power	354
5.10. Ray theory	354
5.10.1. Kinematic theory of wave crests	354
5.10.2. Ray tracing in an inhomogeneous medium	357
5.10.3. Refraction of waves at a sloping beach	357
5.11. Wave action	364
5.11.1. Variational description of a fully dispersed wave group	365
5.11.2. Fully dispersed waves in a non-uniformly moving medium	366
5.11.3. General wave-bearing media	369
5.12. Diffraction of surface waves by a breakwater	373
5.12.1. Diffraction by a long, straight breakwater	373
5.12.2. Solution of the diffraction problem	374
5.12.3. The surface wave pattern	377
5.12.4. Uniform asymptotic approximation: Method of steepest descents	379
Problems 5	384
 6 Introduction to Acoustics	390
6.1. The wave equation	390
6.1.1. The linear wave equation	391
6.1.2. Plane waves	392
6.1.3. Speed of sound	393
6.2. Acoustic Green's function	395
6.2.1. The impulsive point source	395
6.2.2. Green's function	396
6.2.3. Retarded potential	397
6.2.4. Sound from a vibrating sphere	397
6.2.5. Acoustic energy flux	399

6.2.6. Green's function in one space dimension: Method of descent	400
6.2.7. Waves generated by a one-dimensional volume source	401
6.3. Kirchhoff's formula	401
6.4. Compact Green's function	403
6.4.1. Generalized Kirchhoff formula	403
6.4.2. The time harmonic wave equation	404
6.4.3. The compact approximation	404
6.4.4. Rayleigh scattering: Scattering by a compact body	407
6.5. One-dimensional propagation through junctions	409
6.5.1. Continuity of volume velocity	410
6.5.2. Continuity of pressure	410
6.5.3. Reflection and transmission at a junction	411
6.6. Branching systems	413
6.6.1. Fundamental formula	414
6.6.2. Energy transmission	415
6.6.3. Acoustically compact cavity	416
6.6.4. The Helmholtz resonator	417
6.6.5. Acoustic filter	418
6.6.6. Admittance of a narrow constriction	419
6.7. Radiation from an open end	421
6.7.1. Rayleigh's method for low-frequency sound	421
6.7.2. The reflection coefficient	423
6.7.3. Admittance of the open end	423
6.7.4. Open-end input admittance	424
6.7.5. Flanged opening	426
6.7.6. Physical significance of the end correction	428
6.7.7. Admittance of a circular aperture	431
6.8. Webster's equation	432
6.9. Radiation into a semi-infinite duct	435
6.9.1. The compact Green's function	435
6.9.2. Wave generation by a train entering a tunnel	439
6.10. Damping of sound in a smooth-walled duct	445
6.10.1. Time harmonic propagation in a duct	446
6.10.2. The viscous contribution	447
6.10.3. The thermal contribution	449
6.10.4. The thermo-viscous damping coefficient	450
Problems 6	450
<i>Bibliography</i>	455
<i>Index</i>	457