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978-0-521-86813-6 - The Mathematical Foundations of Mixing: The Linked Twist Map as a Paradigm in Applications Micro to Macro, Fluids to Solids

Rob Sturman, Julio M. Ottino and Stephen Wiggins

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