

CAMBRIDGE STUDIES IN
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RANDOM FRAGMENTATION AND COAGULATION PROCESSES

Fragmentation and coagulation are two natural phenomena that can be observed in many sciences and at a great variety of scales. This book is the first comprehensive theoretical account of mathematical models for situations where either phenomenon occurs randomly and repeatedly as time passes.

The fragmentation and coalescent processes considered in this text describe the evolution of particle systems, where particles are characterized by their sizes. In a fragmentation process, each particle splits at a rate which depends on its size, and independently of the other particles. In a coalescent process, coagulations occur at rates which depend only on the particles involved in the merging, and not on the other particles in the system. The book starts by developing the theory of fragmentation chains, that is processes in which each fragment remains stable for some random time and then splits; it then turns to the general situation where each fragment may split instantaneously, using Kingman's theory of exchangeable random partitions. Then, two quite different types of coagulation process are considered: "exchangeable" coalescents, where rates of coagulation do not depend on the masses in the system and coagulations may occur simultaneously and involve an arbitrary number of components, and "stochastic" coalescents, where only binary coagulations are permitted and the rate of such coagulation may depend on the two fragments involved.

This self-contained treatment develops the models in a way that makes recent developments in the field accessible to readers with a solid background in probability. Each chapter ends with a comments section in which important aspects not discussed in the main part of the text (often because the discussion would have been too technical and/or lengthy) are addressed and precise references are given.

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Jean Bertoin

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