Principles of Statistical Inference

In this important book, D. R. Cox develops the key concepts of the theory of statistical inference, in particular describing and comparing the main ideas and controversies over foundational issues that have rumbled on for more than 200 years. Continuing a 60-year career of contribution to statistical thought, Professor Cox is ideally placed to give the comprehensive, balanced account of the field that is now needed.

The careful comparison of frequentist and Bayesian approaches to inference allows readers to form their own opinion of the advantages and disadvantages. Two appendices give a brief historical overview and the author's more personal assessment of the merits of different ideas.

The content ranges from the traditional to the contemporary. While specific applications are not treated, the book is strongly motivated by applications across the sciences and associated technologies. The underlying mathematics is kept as elementary as feasible, though some previous knowledge of statistics is assumed. This book is for every serious user or student of statistics – in particular, for anyone wanting to understand the uncertainty inherent in conclusions from statistical analyses.

Principles of Statistical Inference

D.R. COX

Nuffield College, Oxford

CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 2RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521866736

First published 2006

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data
Copy to follow

ISBN-13 978-0-521-86673- 6 hardback
ISBN-10 0-521-86673-1 hardback

ISBN-13 978-0-521-68567- 2 paperback
ISBN-10 0-521-68567-2 paperback

Cambridge University Press
978-0-521-86673-6 - Principles of Statistical Inference
D. R. Cox
Frontmatter
More information

Contents

List of examples

Preface

Most statistical work is concerned directly with the provision and implementation of methods for study design and for the analysis and interpretation of data. The theory of statistics deals in principle with the general concepts underlying all aspects of such work and from this perspective the formal theory of statistical inference is but a part of that full theory. Indeed, from the viewpoint of individual applications, it may seem rather a small part. Concern is likely to be more concentrated on whether models have been reasonably formulated to address the most fruitful questions, on whether the data are subject to unappreciated errors or contamination and, especially, on the subject-matter interpretation of the analysis and its relation with other knowledge of the field.

Yet the formal theory is important for a number of reasons. Without some systematic structure statistical methods for the analysis of data become a collection of tricks that are hard to assimilate and interrelate to one another, or for that matter to teach. The development of new methods appropriate for new problems would become entirely a matter of ad hoc ingenuity. Of course such ingenuity is not to be undervalued and indeed one role of theory is to assimilate, generalize and perhaps modify and improve the fruits of such ingenuity.

Much of the theory is concerned with indicating the uncertainty involved in the conclusions of statistical analyses, and with assessing the relative merits of different methods of analysis, and it is important even at a very applied level to have some understanding of the strengths and limitations of such discussions. This is connected with somewhat more philosophical issues connected with the nature of probability. A final reason, and a very good one, for study of the theory is that it is interesting.

The object of the present book is to set out as compactly as possible the key ideas of the subject, in particular aiming to describe and compare the main ideas and controversies over more foundational issues that have rumbled on at varying levels of intensity for more than 200 years. I have tried to describe the various approaches in a dispassionate way but have added an appendix with a more personal assessment of the merits of different ideas.

Some previous knowledge of statistics is assumed and preferably some understanding of the role of statistical methods in applications; the latter understanding is important because many of the considerations involved are essentially conceptual rather than mathematical and relevant experience is necessary to appreciate what is involved.

The mathematical level has been kept as elementary as is feasible and is mostly that, for example, of a university undergraduate education in mathematics or, for example, physics or engineering or one of the more quantitative biological sciences. Further, as I think is appropriate for an introductory discussion of an essentially applied field, the mathematical style used here eschews specification of regularity conditions and theorem–proof style developments. Readers primarily interested in the qualitative concepts rather than their development should not spend too long on the more mathematical parts of the book.

The discussion is implicitly strongly motivated by the demands of applications, and indeed it can be claimed that virtually everything in the book has fruitful application somewhere across the many fields of study to which statistical ideas are applied. Nevertheless I have not included specific illustrations. This is partly to keep the book reasonably short, but, more importantly, to focus the discussion on general concepts without the distracting detail of specific applications, details which, however, are likely to be crucial for any kind of realism.

The subject has an enormous literature and to avoid overburdening the reader I have given, by notes at the end of each chapter, only a limited number of key references based on an admittedly selective judgement. Some of the references are intended to give an introduction to recent work whereas others point towards the history of a theme; sometimes early papers remain a useful introduction to a topic, especially to those that have become suffocated with detail. A brief historical perspective is given as an appendix.

The book is a much expanded version of lectures given to doctoral students of the Institute of Mathematics, Chalmers/Gothenburg University, and I am very grateful to Peter Jagers and Nanny Wermuth for their invitation and encouragement. It is a pleasure to thank Ruth Keogh, Nancy Reid and Rolf Sundberg for their very thoughtful detailed and constructive comments and advice on a preliminary version. It is a pleasure to thank also Anthony Edwards and Deborah Mayo for advice on more specific points. I am solely responsible for errors of fact and judgement that remain.

The book is in broadly three parts. The first three chapters are largely introductory, setting out the formulation of problems, outlining in a simple case the nature of frequentist and Bayesian analyses, and describing some special models of theoretical and practical importance. The discussion continues with the key ideas of likelihood, sufficiency and exponential families.

Chapter 4 develops some slightly more complicated applications. The long Chapter 5 is more conceptual, dealing, in particular, with the various meanings of probability as it is used in discussions of statistical inference. Most of the key concepts are in these chapters; the remaining chapters, especially Chapters 7 and 8, are more specialized.

Especially in the frequentist approach, many problems of realistic complexity require approximate methods based on asymptotic theory for their resolution and Chapter 6 sets out the main ideas. Chapters 7 and 8 discuss various complications and developments that are needed from time to time in applications. Chapter 9 deals with something almost completely different, the possibility of inference based not on a probability model for the data but rather on randomization used in the design of the experiment or sampling procedure.

I have written and talked about these issues for more years than it is comfortable to recall and am grateful to all with whom I have discussed the topics, especially, perhaps, to those with whom I disagree. I am grateful particularly to David Hinkley with whom I wrote an account of the subject 30 years ago. The emphasis in the present book is less on detail and more on concepts but the eclectic position of the earlier book has been kept.

I appreciate greatly the care devoted to this book by Diana Gillooly, Commissioning Editor, and Emma Pearce, Production Editor, Cambridge University Press.