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D. R. Cox
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Principles of Statistical Inference

In this important book, D. R. Cox develops the key concepts of the theory of statistical inference, in particular describing and comparing the main ideas and controversies over foundational issues that have rumbled on for more than 200 years. Continuing a 60-year career of contribution to statistical thought, Professor Cox is ideally placed to give the comprehensive, balanced account of the field that is now needed.

The careful comparison of frequentist and Bayesian approaches to inference allows readers to form their own opinion of the advantages and disadvantages. Two appendices give a brief historical overview and the author's more personal assessment of the merits of different ideas.

The content ranges from the traditional to the contemporary. While specific applications are not treated, the book is strongly motivated by applications across the sciences and associated technologies. The underlying mathematics is kept as elementary as feasible, though some previous knowledge of statistics is assumed. This book is for every serious user or student of statistics – in particular, for anyone wanting to understand the uncertainty inherent in conclusions from statistical analyses.

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D.R. COX
Nuffield College, Oxford



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Contents

	<i>List of examples</i>	ix
	<i>Preface</i>	xiii
1	Preliminaries	1
	Summary	1
	1.1 Starting point	1
	1.2 Role of formal theory of inference	3
	1.3 Some simple models	3
	1.4 Formulation of objectives	7
	1.5 Two broad approaches to statistical inference	7
	1.6 Some further discussion	10
	1.7 Parameters	13
	Notes 1	14
2	Some concepts and simple applications	17
	Summary	17
	2.1 Likelihood	17
	2.2 Sufficiency	18
	2.3 Exponential family	20
	2.4 Choice of priors for exponential family problems	23
	2.5 Simple frequentist discussion	24
	2.6 Pivots	25
	Notes 2	27
3	Significance tests	30
	Summary	30
	3.1 General remarks	30
	3.2 Simple significance test	31
	3.3 One- and two-sided tests	35

3.4	Relation with acceptance and rejection	36
3.5	Formulation of alternatives and test statistics	36
3.6	Relation with interval estimation	40
3.7	Interpretation of significance tests	41
3.8	Bayesian testing	42
	Notes 3	43
4	More complicated situations	45
	Summary	45
4.1	General remarks	45
4.2	General Bayesian formulation	45
4.3	Frequentist analysis	47
4.4	Some more general frequentist developments	50
4.5	Some further Bayesian examples	59
	Notes 4	62
5	Interpretations of uncertainty	64
	Summary	64
5.1	General remarks	64
5.2	Broad roles of probability	65
5.3	Frequentist interpretation of upper limits	66
5.4	Neyman–Pearson operational criteria	68
5.5	Some general aspects of the frequentist approach	68
5.6	Yet more on the frequentist approach	69
5.7	Personalistic probability	71
5.8	Impersonal degree of belief	73
5.9	Reference priors	76
5.10	Temporal coherency	78
5.11	Degree of belief and frequency	79
5.12	Statistical implementation of Bayesian analysis	79
5.13	Model uncertainty	84
5.14	Consistency of data and prior	85
5.15	Relevance of frequentist assessment	85
5.16	Sequential stopping	88
5.17	A simple classification problem	91
	Notes 5	93
6	Asymptotic theory	96
	Summary	96
6.1	General remarks	96
6.2	Scalar parameter	97

Contents

vii

6.3	Multidimensional parameter	107
6.4	Nuisance parameters	109
6.5	Tests and model reduction	114
6.6	Comparative discussion	117
6.7	Profile likelihood as an information summarizer	119
6.8	Constrained estimation	120
6.9	Semi-asymptotic arguments	124
6.10	Numerical-analytic aspects	125
6.11	Higher-order asymptotics	128
	Notes 6	130
7	Further aspects of maximum likelihood	133
	Summary	133
7.1	Multimodal likelihoods	133
7.2	Irregular form	135
7.3	Singular information matrix	139
7.4	Failure of model	141
7.5	Unusual parameter space	142
7.6	Modified likelihoods	144
	Notes 7	159
8	Additional objectives	161
	Summary	161
8.1	Prediction	161
8.2	Decision analysis	162
8.3	Point estimation	163
8.4	Non-likelihood-based methods	169
	Notes 8	175
9	Randomization-based analysis	178
	Summary	178
9.1	General remarks	178
9.2	Sampling a finite population	179
9.3	Design of experiments	184
	Notes 9	192
	<i>Appendix A: A brief history</i>	194
	<i>Appendix B: A personal view</i>	197
	<i>References</i>	201
	<i>Author index</i>	209
	<i>Subject index</i>	213

List of examples

Example 1.1	The normal mean	3
Example 1.2	Linear regression	4
Example 1.3	Linear regression in semiparametric form	4
Example 1.4	Linear model	4
Example 1.5	Normal theory nonlinear regression	4
Example 1.6	Exponential distribution	5
Example 1.7	Comparison of binomial probabilities	5
Example 1.8	Location and related problems	5
Example 1.9	A component of variance model	11
Example 1.10	Markov models	12
Example 2.1	Exponential distribution (ctd)	19
Example 2.2	Linear model (ctd)	19
Example 2.3	Uniform distribution	20
Example 2.4	Binary fission	20
Example 2.5	Binomial distribution	21
Example 2.6	Fisher's hyperbola	22
Example 2.7	Binary fission (ctd)	23
Example 2.8	Binomial distribution (ctd)	23
Example 2.9	Mean of a multivariate normal distribution	27
Example 3.1	Test of a Poisson mean	32
Example 3.2	Adequacy of Poisson model	33
Example 3.3	More on the Poisson distribution	34
Example 3.4	Test of symmetry	38
Example 3.5	Nonparametric two-sample test	39
Example 3.6	Ratio of normal means	40
Example 3.7	Poisson-distributed signal with additive noise	41

Example 4.1	Uniform distribution of known range	47
Example 4.2	Two measuring instruments	48
Example 4.3	Linear model	49
Example 4.4	Two-by-two contingency table	51
Example 4.5	Mantel–Haenszel procedure	54
Example 4.6	Simple regression for binary data	55
Example 4.7	Normal mean, variance unknown	56
Example 4.8	Comparison of gamma distributions	56
Example 4.9	Unacceptable conditioning	56
Example 4.10	Location model	57
Example 4.11	Normal mean, variance unknown (ctd)	59
Example 4.12	Normal variance	59
Example 4.13	Normal mean, variance unknown (ctd)	60
Example 4.14	Components of variance	61
Example 5.1	Exchange paradox	67
Example 5.2	Two measuring instruments (ctd)	68
Example 5.3	Rainy days in Gothenburg	70
Example 5.4	The normal mean (ctd)	71
Example 5.5	The noncentral chi-squared distribution	74
Example 5.6	A set of binomial probabilities	74
Example 5.7	Exponential regression	75
Example 5.8	Components of variance (ctd)	80
Example 5.9	Bias assessment	82
Example 5.10	Selective reporting	86
Example 5.11	Precision-based choice of sample size	89
Example 5.12	Sampling the Poisson process	90
Example 5.13	Multivariate normal distributions	92
Example 6.1	Location model (ctd)	98
Example 6.2	Exponential family	98
Example 6.3	Transformation to near location form	99
Example 6.4	Mixed parameterization of the exponential family	112
Example 6.5	Proportional hazards Weibull model	113
Example 6.6	A right-censored normal distribution	118
Example 6.7	Random walk with an absorbing barrier	119
Example 6.8	Curved exponential family model	121
Example 6.9	Covariance selection model	123
Example 6.10	Poisson-distributed signal with estimated background	124
Example 7.1	An unbounded likelihood	134
Example 7.2	Uniform distribution	135
Example 7.3	Densities with power-law contact	136
Example 7.4	Model of hidden periodicity	138

List of examples

xi

Example 7.5	A special nonlinear regression	139
Example 7.6	Informative nonresponse	140
Example 7.7	Integer normal mean	143
Example 7.8	Mixture of two normal distributions	144
Example 7.9	Normal-theory linear model with many parameters	145
Example 7.10	A non-normal illustration	146
Example 7.11	Parametric model for right-censored failure data	149
Example 7.12	A fairly general stochastic process	151
Example 7.13	Semiparametric model for censored failure data	151
Example 7.14	Lag one correlation of a stationary Gaussian time series	153
Example 7.15	A long binary sequence	153
Example 7.16	Case-control study	154
Example 8.1	A new observation from a normal distribution	162
Example 8.2	Exponential family	165
Example 8.3	Correlation between different estimates	165
Example 8.4	The sign test	166
Example 8.5	Unbiased estimate of standard deviation	167
Example 8.6	Summarization of binary risk comparisons	171
Example 8.7	Brownian motion	174
Example 9.1	Two-by-two contingency table	190

Preface

Most statistical work is concerned directly with the provision and implementation of methods for study design and for the analysis and interpretation of data. The theory of statistics deals in principle with the general concepts underlying all aspects of such work and from this perspective the formal theory of statistical inference is but a part of that full theory. Indeed, from the viewpoint of individual applications, it may seem rather a small part. Concern is likely to be more concentrated on whether models have been reasonably formulated to address the most fruitful questions, on whether the data are subject to unappreciated errors or contamination and, especially, on the subject-matter interpretation of the analysis and its relation with other knowledge of the field.

Yet the formal theory is important for a number of reasons. Without some systematic structure statistical methods for the analysis of data become a collection of tricks that are hard to assimilate and interrelate to one another, or for that matter to teach. The development of new methods appropriate for new problems would become entirely a matter of ad hoc ingenuity. Of course such ingenuity is not to be undervalued and indeed one role of theory is to assimilate, generalize and perhaps modify and improve the fruits of such ingenuity.

Much of the theory is concerned with indicating the uncertainty involved in the conclusions of statistical analyses, and with assessing the relative merits of different methods of analysis, and it is important even at a very applied level to have some understanding of the strengths and limitations of such discussions. This is connected with somewhat more philosophical issues connected with the nature of probability. A final reason, and a very good one, for study of the theory is that it is interesting.

The object of the present book is to set out as compactly as possible the key ideas of the subject, in particular aiming to describe and compare the main ideas and controversies over more foundational issues that have rumbled on at varying levels of intensity for more than 200 years. I have tried to describe the

various approaches in a dispassionate way but have added an appendix with a more personal assessment of the merits of different ideas.

Some previous knowledge of statistics is assumed and preferably some understanding of the role of statistical methods in applications; the latter understanding is important because many of the considerations involved are essentially conceptual rather than mathematical and relevant experience is necessary to appreciate what is involved.

The mathematical level has been kept as elementary as is feasible and is mostly that, for example, of a university undergraduate education in mathematics or, for example, physics or engineering or one of the more quantitative biological sciences. Further, as I think is appropriate for an introductory discussion of an essentially applied field, the mathematical style used here eschews specification of regularity conditions and theorem–proof style developments. Readers primarily interested in the qualitative concepts rather than their development should not spend too long on the more mathematical parts of the book.

The discussion is implicitly strongly motivated by the demands of applications, and indeed it can be claimed that virtually everything in the book has fruitful application somewhere across the many fields of study to which statistical ideas are applied. Nevertheless I have not included specific illustrations. This is partly to keep the book reasonably short, but, more importantly, to focus the discussion on general concepts without the distracting detail of specific applications, details which, however, are likely to be crucial for any kind of realism.

The subject has an enormous literature and to avoid overburdening the reader I have given, by notes at the end of each chapter, only a limited number of key references based on an admittedly selective judgement. Some of the references are intended to give an introduction to recent work whereas others point towards the history of a theme; sometimes early papers remain a useful introduction to a topic, especially to those that have become suffocated with detail. A brief historical perspective is given as an appendix.

The book is a much expanded version of lectures given to doctoral students of the Institute of Mathematics, Chalmers/Gothenburg University, and I am very grateful to Peter Jagers and Nanny Wermuth for their invitation and encouragement. It is a pleasure to thank Ruth Keogh, Nancy Reid and Rolf Sundberg for their very thoughtful detailed and constructive comments and advice on a preliminary version. It is a pleasure to thank also Anthony Edwards and Deborah Mayo for advice on more specific points. I am solely responsible for errors of fact and judgement that remain.

The book is in broadly three parts. The first three chapters are largely introductory, setting out the formulation of problems, outlining in a simple case the nature of frequentist and Bayesian analyses, and describing some special models of theoretical and practical importance. The discussion continues with the key ideas of likelihood, sufficiency and exponential families.

Chapter 4 develops some slightly more complicated applications. The long Chapter 5 is more conceptual, dealing, in particular, with the various meanings of probability as it is used in discussions of statistical inference. Most of the key concepts are in these chapters; the remaining chapters, especially Chapters 7 and 8, are more specialized.

Especially in the frequentist approach, many problems of realistic complexity require approximate methods based on asymptotic theory for their resolution and Chapter 6 sets out the main ideas. Chapters 7 and 8 discuss various complications and developments that are needed from time to time in applications. Chapter 9 deals with something almost completely different, the possibility of inference based not on a probability model for the data but rather on randomization used in the design of the experiment or sampling procedure.

I have written and talked about these issues for more years than it is comfortable to recall and am grateful to all with whom I have discussed the topics, especially, perhaps, to those with whom I disagree. I am grateful particularly to David Hinkley with whom I wrote an account of the subject 30 years ago. The emphasis in the present book is less on detail and more on concepts but the eclectic position of the earlier book has been kept.

I appreciate greatly the care devoted to this book by Diana Gillooly, Commissioning Editor, and Emma Pearce, Production Editor, Cambridge University Press.