

Contents

<i>Preface</i>	<i>page vii</i>
1. Overview	1
1.1 Introduction to the Introduction	1
1.2 Erdős, Rényi, Molloy, and Reed	3
1.3 Six Degrees, Small Worlds	7
1.4 Power Laws, Preferential Attachment	11
1.5 Epidemics and Percolation	15
1.6 Potts Models and the Contact Process	18
1.7 Random Walks and Voter Models	20
1.8 CHKNS Model	21
2. Erdős–Rényi Random Graphs	27
2.1 Branching Processes	27
2.2 Cluster Growth as an Epidemic	34
2.3 Cluster Growth as a Random Walk	37
2.4 Diameter of the Giant Component	43
2.5 CLT for the Giant Component	46
2.6 Combinatorial Approach	50
2.7 Critical Regime	56
2.8 Threshold for Connectivity	62
3. Fixed Degree Distributions	70
3.1 Definitions and Heuristics	70
3.2 Proof of Phase Transition	75
3.3 Subcritical Estimates	82
3.4 Distances: Finite Variance	84
3.5 Epidemics	85
4. Power Laws	90
4.1 Barabási-Albert Model	90
4.2 Related Models	93
4.3 Martingales and Urns	99
4.4 Scale-Free Trees	105
4.5 Distances: Power Laws $2 < \beta < 3$	110

4.6	Diameter: Barabási-Albert Model	116
4.7	Percolation, Resilience	121
4.8	SIS Epidemic	125
5.	Small Worlds	132
5.1	Watts and Strogatz Model	132
5.2	Path Lengths	134
5.3	Epidemics	140
5.4	Ising and Potts Models	144
5.5	Contact Process	148
6.	Random Walks	153
6.1	Spectral Gap	153
6.2	Conductance	156
6.3	Fixed Degree Distribution	159
6.4	Preferential Attachment Graph	164
6.5	Connected Erdős-Rényi Graphs	169
6.6	Small Worlds	171
6.7	Only Degrees 2 and 3	175
6.8	Hitting Times	177
6.9	Voter Models	181
7.	CHKNS Model	187
7.1	Heuristic Arguments	187
7.2	Proof of the Phase Transition	190
7.3	Subcritical Estimates	193
7.4	Kosterlitz-Thouless Transition	197
7.5	Results at the Critical Value	200
	<i>References</i>	203
	<i>Index</i>	211