
Index

- (X, \mathcal{E}) , 357, 374
 $B(0, r)$, 27
 $C(K)$, 2
 C^∞ vector, 201
 $C^n(X)$, 6, 23
 $C_0(X)$, 6
 $C_c^\infty(U)$, 54
 D^α , 67
 $L^\infty(X, dx)$, 37
 $L_0^\infty(\mathbf{R})$, 414
 $L^p(X, dx)$, 36
 $R(\lambda, a)$, 14
 $W^{n,2}(\mathbf{R}^N)$, 77
 Dom, 164
 EssSpec, 118, 330
 Gr, 164
 Hull(p, A), 277
 Hull $_n(A)$, 277
 Num(A), 264
 Num(p, A), 278
 Num $_n(A)$, 278
 Rad, 99
 Spec, 14
 \mathcal{B}^* , 6
 \mathcal{B}_+ , 359
 \mathcal{B}_c , 5
 \mathcal{C}_2 , 151
 \mathcal{C}_p , 153
 \mathcal{D}^∞ , 201
 $\mathcal{K}(\mathcal{B})$, 103, 118
 $\mathcal{L}(\mathcal{B})$, 12
 \mathcal{S}' , 72
 dist, 2
 $\langle \cdot, \cdot \rangle$, 3, 10, 36
 $\| \cdot \|_2$, 45, 111, 151
 $\| \cdot \|_{\text{HS}}$, 45, 151
 ω_0 , 178, 297
 \overline{A} , 286
 Conv, 108
 s-lim, 22
 supp, 36, 403
 $|A|$, 138
 w-lim, 19, 22
 $c_0(X)$, 11
 $f(H)$, 150
 $f(N)$, 144
 $f \vee g$ and $f \wedge g$, 359
 f_+ and f_- , 359
 k -tree, 374
 $l^p(X)$, 3, 36
 Abel-Lidskii basis, 82, 425
 adjoint operator, 13
 Airy operator, 250
 analytic
 functions, 25
 spaces of, 428
 vector, 203
 Anderson model, 134, 282, 284
 anharmonic oscillator, 426
 annihilator, 198
 anti-ferromagnetic, 369
 aperiodic, 369, 391
 approximate
 eigenvalues, 247
 point spectrum, 17
 approximation and regularization, 54
 approximation property, 103
 approximately normal, 150
 Arzela-Ascoli theorem, 106
 asymptotic
 dimension, 378
 stability, 298

- Banach
 algebra, 13, 61, 63
 lattice, 7
 space, 1
 Banach-Alaoglu theorem, 21
 Banach-Mazur theorem, 108
 Banach-Steinhaus theorem, 20
 Bargmann, 428
 base point, 376
 basis, 4, 80
 unconditional, 90
 Berezin, 431
 Bernstein's theorem, 66
 bilateral shift, 397
 billiards, 398
 biorthogonal, 80
 Böttcher, 124
 boundary conditions, 336
 bounded linear functional, 6
 Burke-Greenbaum theorem, 279
- Calkin, 153
 algebra, 118
 Carleson, 88
 Cauchy problem, 163, 171
 Cauchy process, 183, 186, 208
 Cauchy's integral formula, 26
 Cayley transform, 144, 310
 Chebyshev polynomials, 274
 circulant matrix, 369
 Clarkson inequalities, 41
 class \mathcal{P} , 341
 closed operator, 144, 164
 coherent state, 428
 cokernel, 116
 commutator bound, 161
 commuting operators, 176
 compact operator, 103, 123
 compatible Banach spaces, 49, 109
 complete, 80
 completely non-unitary, 314
 completion, 1
 complex scaling, 329, 423
 complexification, 357
 condition number, 258
 conditional basis, 80
 conjugate function, 303
 conjugate index, 38
 connected graph, 374
 consistent operators, 49, 104, 109, 177, 348
 continued fractions, xii
 contraction, dilation of, 307
 contraction semigroup, 230
 control theory, 262
 convection-diffusion, 248, 274, 275, 339, 395
 convex hull, 108
 convexity theory, 268
 convolution, 53, 63, 75, 129, 131, 274
 semigroup, 183, 205
 core, 172, 210
 coupling constant, 341
- Davies, 277
 Davies's theorem, 304, 407, 426
 Davies-Kuijlaars theorem, 425
 Davies-Martinez theorem, 282
 Davies-Simon, 288
 Davies-Simon theorem, 293
 deficiency indices, 145, 312
 degree, 374
 detailed balance, 366
 differentiability, 23, 201
 differential operator, 78, 188, 211, 335, 347
 dilation theorem, 309
 Dini's theorem, 85
 discrete Laplacian, 364
 dissipative, 231, 310
 distance, between subspaces, 140
 distribution, 72
 domain, 164, 210
 du Bois Reymond, 84
 dual
 operator, 13, 197
 space, 6, 72
 dynamical system, 398
- EigTool, 248
 Eisner-Zwart theorem, 324
 elliptic, 78, 79
 embedding, 1
 Embree, 248
 Enss, 423
 entire vector, 203
 equicontinuous, 106
 equilibrium state, 368
 equivalent norms, 6
 ergodic, 396
 essential range, 45, 143, 148, 165
 essentially bounded, 37
 exponential growth rate, 297
 extension, 164

- Fejér's theorem, 273
 Feller, 228
 property, 407
 ferromagnetic, 369
 finite element method, 104, 266
 finite rank operator, 103
 first return probability, 371
 Fock space, 112
 Fourier series, 4, 80
 L^2 convergence, 57
 L^p convergence, 84
 absolutely convergent, 60
 pointwise divergence, 85
 Fourier transform, 67
 fractional powers, 29, 208
 Fréchet space, 19, 68, 203
 Fréchet-Riesz theorem, 7
 Fredholm operator, 116, 330
 Frobenius, 380, 405
 norm, 45, 151
 functional calculus, 144, 150
 holomorphic, 27

 Gabor transform, 428
 Gauss, xii
 Gaussian function, 69, 90, 185, 206, 241, 346
 Gel'fand, 61, 63, 203, 221
 Gel'fand's theorem, 100
 generator, 168
 classification of, 227
 Gesztesy-Tkachenko, 133
 Gibbs
 phenomenon, 58
 semigroup, 194
 state, 368
 Glauber dynamics, 367
 Gohberg, 125
 Goldsheid, 134
 Gram-Schmidt, 272
 graph, 131, 164, 357, 374, 388
 Green function, 157, 333, 408
 growth bounds
 basic, 177
 long time, 369

 Haar
 basis, 4
 measure, 53
 Haase's theorem, 320
 Hahn-Banach theorem, 6
 Hamiltonian, 368

 Hankel operator, 291
 Hardy space, 128
 harmonic oscillator, 329, 424
 Hartman, 125
 Hermite polynomials, 88, 274, 424
 higher order hull and range, 277
 Hilbert transform, 75
 Hilbert-Schmidt operator, 45, 110, 151
 Hille's theorem, 353
 Hille-Yosida theorem, 231
 Hölder
 continuous, 85
 inequality, 38
 holomorphic semigroup, 237, 345, 353
 homotopy invariant, 120
 Hörmander, 234
 hydrogen atom, 329
 hyperbolic tree, 374
 hypercontractive, 112

 ideal, of operators, 103, 151, 155
 image processing, 53
 incidence matrix, 374
 index, 116
 interpolation, 50, 177, 183, 219, 393
 invariant set, 361, 387, 389, 403
 invariant subspace, 18, 168, 173, 314
 inverse mapping theorem, 14
 inverse temperature, 368
 irreducible, 361, 365, 369, 390, 403
 isometric embedding, 1
 isometry
 partial, 140
 spectrum of, 398

 James's theorem, 41
 Jordan decomposition, 7
 Jordan matrix, 31, 34, 218, 222, 249, 269

 Kakeya, 271
 Keldysh, 251
 kernel, 116
 Kesten's theorem, 376
 Khoruzhenko, 134
 Koopman operator, 398
 Kortweg-de Vries equation, xii
 Krein's theorem, 126
 Krein-Šmulian theorem, 179

 Laguerre polynomials, 274
 Laplacian, 69, 185

- and Gaussian semigroup, 185
- L^2 spectrum, 79
- L^p spectrum, 241
- lasers, unstable, xi, 424
- Laurent operator, 53, 274
- Lebesgue measure, 36
- Legendre
 - polynomials, 273
 - transform, 303
- Levy process, 76
- Lieb-Thirring bound, 410
- linear sublattice, 359
- Liouville's theorem, 16, 26
- locally finite graph, 374
- log-concave envelope, 301
- log-convex, 37
- Lorch, 90
- Lumer-Phillips theorem, 231
- Markov
 - operator, 356
 - semigroup, 362, 399
 - reversible, 366
- Markov-Kakutani theorem, 317
- Martinez, 282
- maximal dissipative, 312
- Mazur's theorem, 108
- Maz'ya and Schmidt, 90
- measure space, 35, 395
- Mercer's theorem, 156
- Millennium Bridge, xii, 252
- minimal
 - complete, 81
 - polynomial, 272
- mixing, 404
- Miyadera, 228, 349
- Mockenhaupt classes, 89
- molecule, 368
- moment problem, 74
- momentum operator, 161
- multi-index, 67
- multiplication operator, 45, 76, 132, 143, 165, 172, 188
- Navier-Stokes equation, xii, 399
- Nelson's theorem, 173
- neutron diffusion equation, 386
- Nevanlinna's theorem, 280
- Newman's lemma, 61
- non-linear, 251
- non-negative operator, 135
- norm continuous semigroup, 190
- normal, 2, 143, 259
- normed space, 1
- null set, 36
- numerical range, 264, 268, 272, 409, 431
- obstacle scattering, 129
- one-parameter
 - group, 174, 315
 - semigroup, 167
 - generator of, 227
 - holomorphic, 237
 - long time bounds, 296
 - norm continuous, 190
 - on dual space, 197
 - short time bounds, 300
 - subordinated, 205
 - trace class, 194
- operator
 - bounded, 12
 - closed, 164
 - compact, 102
 - Fredholm, 116
 - Hilbert-Schmidt, 151
 - on a Hilbert space, 135
 - on an L^p space, 45
 - pencil, 251
 - positive, 380
 - trace class, 153
- order ideal, 387, 403
- ordered Banach space, 355
- ordering of operators, 136
- orthogonal
 - polynomials, 272, 288
 - projection, 140
- orthonormal, 4, 58, 80, 88, 93
- oscillation properties, 274
- partition, 35
 - function, 368
 - of the identity, 105
- path, 357, 371
- Pazy, 192
- pencils of operators, 252
- period of a semigroup, 390
- periodic operator, 132, 282, 422
- peripheral point spectrum, 287, 405
- Perron, 380, 405
- Perron-Frobenius operator, 398

- perturbation
 - bounded, 339
 - of an operator, 31, 325
 - of spectrum, 31, 328
 - rank one, 32, 334
 - relatively compact, 330
 - resolvent based method, 350
 - semigroup based method, 339
- Phillips, 228
- Plancherel theorem, 69
- Poisson
 - distribution, 206
 - process, 364
- polar decomposition, 138, 285
- polarization identity, 13
- polynomial
 - convex hull, 280
 - growth bound, 83, 378
- population growth models, 386
- Pöschl-Teller potential, 413
- position operator, 161
- positive operator, 381
- power series, 26
- power-bounded, 100, 317
- principal part, 79
- probability distribution, 356, 395
- projection, 22, 140
- pseudo-resolvent, 214
- pseudospectra, 213, 245, 247, 426
 - generalized, 255
 - structured, 261
- quantization, 428
- quantum mechanics, 423
- quotient space, 10
- radiation condition, 254
- radius of convergence, 26
- Radon-Nikodym theorem, 43
- random
 - matrix, 134, 263
 - walk, 206
- range, 210
- rank, of an operator, 31
- Ransford, 100, 264
- recurrence equation, 101
- recurrent, 370, 372
- reflexive, 10, 41
- relative bound, 325
- relatively compact, 21, 331
- Rellich, 31
- Rellich's theorem, 328
- renormalization, 393
- reproducing kernel Hilbert space, 428
- resolvent, 14
 - bounds in Hilbert space, 321
 - operators, 210
 - set, 210
- resonance, 254
- reversible, 366
- Riemann-Lebesgue lemma, 58, 71
- Riesz, 7, 30
 - basis, 93
- Riesz's theorem, 123, 327
- Riesz-Kakutani theorem, 7
- Riesz-Thorin theorem, 50
- Rouche's theorem, 339
- sample space, 357
- scalar type operator, 133, 422
- scattering theory, 168, 422
- Schatten, 153
- Schauder basis, 80
- Schauder's theorem, 108
- Schrödinger operator, 163, 335, 346, 348, 395, 408, 413, 417
- Schur's theorem, 291, 293
- Schwartz space, 68
- second dual space, 10
- sector, 147, 237
- self-adjoint, 13, 143, 145
- semi-classical analysis, 427
- semigroup, 390
- seminorm, 1, 19
- separable, 36
- separation theorem, 268
- sequential compactness, 103
- shift operator, 18, 121
- Shkarin, 101
- shooting method for ODEs, 253
- Silbermann, 124
- similarity invariant, 297
- Simon, 272
- singular
 - integral operators, 75
 - potential, 348
 - values, 153
- smooth, 68
- Sobolev space, 77
- spectral
 - mapping theorem, 18, 215, 221, 223
 - pollution, 266

- projection, 30, 110
- radius, 99
- theorem, 143
- spectrum
 - L^p dependence, 49
 - definition, 14, 211
 - essential, 118, 124, 330, 420
 - of a k -tree, 374
 - of Airy operator, 247
 - of consistent operators, 109, 219
 - of convolution operators, 65
 - of multiplication operator, 45
 - of operator pencil, 251
 - of Schrödinger operator, 418
 - of Toeplitz operator, 125
 - of ultracontractive operators, 394
- peripheral, 287
- square root lemma, 138
- stability, 245
- statistical dynamics, 367
- Stone, 143
- Stone-Weierstrass theorem, 60
- strong operator limit, 22
- Sturm-Liouville, 157, 273, 333, 408
- subadditive, 99, 297
- subexponential growth, 378
- sublattice, 392
- subMarkov operator, 363
- subordinated semigroup, 205
- subspaces, two, 140
- support, 36, 397, 403
- supremum norm, 2
- symbol, 78, 124, 128, 211, 337
- symmetric, 144
- Sz.-Nagy dilation theorem, 307
- Sz.-Nagy theorem, 315

- three lines lemma, 49
- thresholds, 424
- Tietze extension theorem, 2
- Toeplitz operator, 124, 128
- Toeplitz-Hausdorff theorem, 265
- topological vector space, 19, 220
- totally bounded, 103
- trace class
 - operator, 154
 - semigroup, 194
- transient, 370, 372
- translation invariant, 63
- tree, 374
- Trefethen, 248
- triangle, 146, 168
- tridiagonal operator, 281
- trigonometric polynomial, 56
- truncation, 217, 265, 274

- ultracontractive, 394
- unconditional basis, 90
- undirected graph, 366, 374
- uniform boundedness theorem, 20
- uniform convexity, 41
- unitary, 13, 143
- unstable, 386
- Urysohn's lemma, 2

- Vandermonde determinant, 338
- Voigt, 349
- Volterra operator, 100, 249, 291
- von Neumann, 143, 153
- von Neumann's theorem, 286, 309

- wave equation, 252
- waveguides, 253
- wavelets, 5, 97
- weak
 - convergence, 165
 - derivative, 75
 - operator limit, 22
 - topology, 19
 - weak* topology, 21
 - weakly closed operator, 165
- Weierstrass's theorem, 59
- Wiener, 125
- Wiener's theorem, 62
- wild, 83
- winding number, 126
- Wintner, 125
- Wright, 248
- Wrobel, 299

- Zabczyk's example, 222