

## Notation index

$TM$	tangent bundle of the manifold $M$ 2
$\ker \alpha$	kernel of the differential 1-form $\alpha$ 2
$-/-$	quotient of bundle by sub-bundle 2
	quotient of vector space by subspace 15
$(-)^{\perp}$	orthogonal complement of sub-bundle 2
	symplectic orthogonal complement of subspace 14
	symplectic orthogonal complement of sub-bundle 68
$\oplus$	Whitney sum of vector bundles 2
$\cong$	isomorphism of vector bundles 2
	isomorphism of groups 43
	homeo- or diffeomorphism of manifolds 45
$(-) _U$	restriction of vector bundle to the subset $U$
	of the base manifold 2
	restriction of map or linear form to the subspace $U$ 15
$C^{\infty}$	smooth or (infinitely often) differentiable 2
$\equiv$	identically equal to 2
	canonically identified with 58
$T^*M$	cotangent bundle of the manifold $M$ 3
$(-)^*$	dual bundle 3
	dual vector space, dual Lie algebra 14, 360
	adjoint of matrix 18
$T_p M$	tangent space of the manifold $M$ at the point $p$ 3
$\xi_p$	subspace defined by the distribution $\xi$ at the point $p$ 3
$\wedge$	wedge or exterior product 3
$[X, Y]$	Lie bracket of the vector fields $X$ and $Y$ 3
	homotopy classes of maps from the space $X$ to
	the space $Y$ 107
$X_p, X(p)$	tangent vector at the point $p$ 3, 166
$d\alpha$	exterior derivative of the differential form $\alpha$ 3

$(M, \xi)$	contact manifold	4
$\mathbb{R}$	field of real numbers	4
$\mathbb{R}^n$	Euclidean $n$ -space	4
$B \setminus A$	complement of the subset $A \subset B$ in $B$	4
$\partial_x$	velocity vector of the $x$ -curves; vector field given by regarding the partial derivative $\frac{\partial}{\partial x}$ as a derivation	5
$R_\alpha$	Reeb vector field of the contact form $\alpha$	5
$\alpha _{T_p M}, \alpha_p$	linear form on $T_p M$ defined by the differential form $\alpha$	5, 164
$\langle X \rangle$	line field spanned by the vector field $X$	5
$\mathbb{P}(-)$	projectivisation of a vector space or vector bundle	6
$Tf$	differential of a map $f$ between manifolds	7
$\times$	Cartesian product	7
	vector product in $\mathbb{R}^3$	39
$\mathbb{R}P^n$	real projective $n$ -space	7
$(p_0 : \dots : p_n)$	homogeneous coordinates on projective $n$ -space	7
$S^n$	$n$ -sphere	8
$T^n$	$n$ -torus $S^1 \times \dots \times S^1$ ( $n$ factors)	8
$:=, =:$	term on the left is defined by term on the right (respectively, the other way round)	8, 20
$z', z''$	first or second derivative, respectively, of the function $x \mapsto z(x)$ w.r.t. the variable $x$	9, 10
$f^*$	induced map on differential forms	9
	induced map on cohomology	106
<b>pq</b>	$\sum_j p_j q_j$	11
$A^t$	transpose of the matrix $A$	11
id	identity map	12
$(-, \omega)$	symplectic vector space	14
	symplectic manifold	20
	symplectic vector bundle	64
im, ker	image or kernel, respectively, of map or homomorphism	15
<b>0</b>	zero element or origin of vector space	15
$[-]$	equivalence class in quotient space	16
	homology class	106
	homotopy class	107
$\delta_{ij}$	Kronecker symbol (1 for $i = j$ ; 0 for $i \neq j$ )	16
$J$	complex (bundle) structure	17, 64
$\mathbb{C}$	field of complex numbers	17
$i$	$\sqrt{-1} \in \mathbb{C}$	17
$\mathcal{J}(\omega)$	space of $\omega$ -compatible complex structures	18

$\mathcal{G}$	space of inner products on a vector space	19
$[a, b]$	closed interval from $a$ to $b$	19
$\mathbf{p} \, d\mathbf{q}$	$\sum_j p_j \, dq_j$	20
$\mathbf{q} \, \partial_{\mathbf{q}}$	$\sum_j q_j \, \partial_{q_j}$	20
$C^k$	$k$ times continuously differentiable	21
$\dot{\mathbf{q}}$	derivative of the function $t \mapsto \mathbf{q}(t)$ w.r.t. the time variable $t$	21
$\partial H / \partial \mathbf{q}$	partial derivative w.r.t. a vector-valued coordinate $\mathbf{q}$	22
$X_H$	Hamiltonian vector field corresponding to the Hamiltonian function $H$	22
$\mathcal{L}$	Lie derivative	23
$i_Y \omega$	interior product of the differential form $\omega$ with the vector field $Y$	23
$e^z, \exp$	exponential function	24, 33
$\exp$	exponential map of Riemannian manifold	28
	exponential map of Lie group	362
$(-) _{TM}$	restriction of differential form to tangent spaces of a sub- manifold $M \subset W$ (i.e. pull-back under inclusion map)	25
$(B, g)$	Riemannian manifold	26
$\Gamma_{ij}^k$	Christoffel symbols	26
$SE$	unit sphere bundle of a Riemannian vector bundle $E$	27
$g_{ij}$	metric coefficients	28
$g_{ij,k}$	partial derivative of $g_{ij}$ w.r.t. the $k$ th coordinate function	28
$\ddot{\gamma}$	second derivative of the function $t \mapsto \gamma(t)$ w.r.t. the time variable $t$	28
$(g^{kl})$	inverse matrix of $(g_{ij})$	29
$ - $	absolute value of real number	34
	determinant	133
	modulus of complex number	154
$O$	Landau symbol	37
$\  - \ $	length of a vector	37
	norm of 2-plane field	216
$f_x, f_{xx}$	first or second partial derivative, respectively, of the function $f$ w.r.t. the variable $x$	38, 39
Diff	group of orientation-preserving diffeomorphisms	40
$D^n$	closed unit disc in Euclidean $\mathbb{R}^n$	40
$D_r^n$	disc of radius $r$	132
$\partial M$	boundary of the manifold $M$	40
$\Gamma_n$	group of diffeomorphisms of $S^{n-1}$ modulo those extending over $D^n$	41

$\text{Diff}_0$	group of diffeomorphisms isotopic to the identity	41
$[f, g]$	commutator of maps $f, g$	41
$\xi_{\text{st}}$	standard contact structure on $S^3$	42
	standard contact structure on $\mathbb{R}^{2n+1}$	52
	standard contact structure on $S^{2n+1}$	57
	standard contact structure on $S^2 \times S^1$	252
$\nu K$	closed tubular neighbourhood of the knot $K$	43
$H_k, H^k$	$k$ th homology or cohomology group, respectively (with integer coefficients, unless specified)	43, 106
$\mathbb{Z}$	ring of integers	43
$\mu, \lambda$	meridian and longitude	43, 110
$\text{GL}(n, R)$	general linear group of invertible $n \times n$ matrices over the ring $R$	44
$\mathbb{Q}$	field of rational numbers	44
$\text{lk}$	linking number	45, 111
$K_{p/q}$	3-manifold obtained from $S^3$ by $(p/q)$ -surgery along the knot $K$	46
$\mathbb{Z}_p$	cyclic group of order $p$	46
$\mathbb{Z}^\mu$	infinite cyclic group generated by $\mu$	46
$\pi_k$	$k$ th homotopy group	47
$\bar{U}$	closure of subspace in topological space	47
$\alpha_{\text{ot}}, \xi_{\text{ot}}$	standard overtwisted contact form/structure on $\mathbb{R}^3$	53
$\omega_{\text{st}}$	standard symplectic form on $\mathbb{R}^{2n}$	54
$U$	unitary group	55
$\mathbb{R}^+, \mathbb{R}_0^+$	positive or non-negative real numbers, respectively	62, 174
$\bigwedge^k$	$k$ th exterior power	64
$\text{End}(-)$	endomorphism bundle of a vector bundle	64
$\epsilon^k, \epsilon_{\mathbb{C}}^k$	trivial real or complex, respectively, vector bundle of rank $k$	65, 314
$B_{\text{st}}$	closed unit ball in $(\mathbb{R}^{2n+1}, \xi_{\text{st}})$	67
$\text{CSN}$	conformal symplectic normal bundle	69
$NL$	normal bundle of the submanifold $L \subset M$	69
$\mathcal{N}(-)$	neighbourhood of submanifold	71
$J^1(L)$	1-jet space of the manifold $L$	72
$\mathcal{E}(x)$	space of germs of differentiable functions at the point $x$	72
$:\Leftrightarrow$	statement on the left is defined by statement on the right	72
$j_x^1 f$	1-jet of the function $f$ at $x$	72
$\text{Sp}(2n)$	symplectic linear group on $\mathbb{R}^{2n}$	75

## Notation index

423

$S_\xi$	characteristic foliation on $S$ induced by $\xi$	77
$\Omega$	volume or area form	78
$\mathbb{R}_{\text{st}}^{2n+1}$	$(\mathbb{R}^{2n+1}, \xi_{\text{st}})$	86
$I_n$	$n \times n$ identity matrix	88
$\text{CSp}^+(2n)$	group of (positively) conformally symplectic $2n \times 2n$ matrices	88
$\mathbb{Z}^*$	non-zero integers	95
$\gamma_F, \gamma_L$	front or Lagrangian projection, respectively, of $\gamma$	96
$\mathbb{N}$	natural numbers $1, 2, \dots$	104
$[L]_M$	homology class in $H_*(M)$ represented by the submanifold $L \subset M$	105
$\cap$	cap product	106
$\tau_W$	Thom class of the disc bundle $W$	106
$K(\pi, n)$	Eilenberg–MacLane space	107
$PD$	Poincaré duality homomorphism from homology to cohomology	107
$\mathbb{C}P^n$	complex projective $n$ -space	107
rel	(homotopy) relative to	108
•	intersection product on homology	108
	intersection number	111
$\partial_*$	boundary homomorphism in homology	109
tb	Thurston–Bennequin invariant	115
$\#(-)$	number of	117
rot	rotation number	118, 123
$\overline{M}$	manifold $M$ with reversed orientation	139
$\langle -, - \rangle$	Kronecker product between cohomology and homology	119
	pairing between Lie algebra and its dual	360
$e$	Euler class	119
sl	self-linking number	125
$w_k$	Stiefel–Whitney class	134
$v_k$	Wu class	134
$\cup$	cup product	134
Sq	Steenrod squaring operation	134
$V_k(\mathbb{R}^n), V_k(TM)$	Stiefel manifold, frame bundle	135
O, SO	(special) orthogonal group	135
$\mathbb{H}$	algebra of quaternions	135
$i, j, k$	standard basis for imaginary quaternions	135
Int	interior of manifold with boundary	136
$f_\xi$	Gauß map of the 2-plane field $\xi$	137

$\sqcup$	disjoint union 139
$M^{(k)}$	$k$ -skeleton 141
$\xi^K$	contact structure obtained from $\xi$ by Lutz twist along the knot $K$ 143
$\Sigma(\phi), \Sigma_{\overline{\varphi}}(\phi)$	(generalised) mapping torus 148, 345
$M(\phi), M_{\overline{\varphi}}(\phi)$	open book 149, 345
$\operatorname{div}_{\Omega}(X)$	divergence 163
Re, Im	real or imaginary part, respectively 167, 279
$\chi$	Euler characteristic 175
$e_{\pm}, h_{\pm}$	number of positive or negative (respectively) elliptic or hyperbolic points, respectively 175
$\mathfrak{F}$	singular 1-dimensional foliation on surface 182
$\operatorname{Hom}(A, B)$	homomorphisms from $A$ to $B$ 194
Ext	'extension' functor 194
$\Xi^{\text{ot}}$	space of overtwisted contact structures 205
Distr	space of cooriented 2-plane distributions 205
$f_{\#}$	induced map on homotopy groups 208
$\mathfrak{F}^0$	singular 1-dimensional foliation on $S^2$ induced by $\xi_{\text{ot}}$ 210
$f$	leaf of $\mathfrak{F}$ 212
$\mathfrak{F}(h)$	almost horizontal foliation on $S^2$ with holonomy $h$ 212
$\#$	connected sum of manifolds 213 connected sum of simple foliations on $S^2$ 213 connected sum of tight contact structures 259
$\mathcal{C}$	simplicial complex 215
$\angle$	angle between lines or planes 216 angle between vectors 221
$B_{\text{ot}}$	standard overtwisted ball 225
$\#_{\text{b}}$	boundary connected sum 227
$\Gamma_S$	dividing set of the convex surface $S$ 229
$S^z$	$S \times \{z\} \subset S \times [-1, 1]$ 244
$\cup_{\partial}$	gluing along boundary 244
$\Delta$	Laplace operator 255
$(\widehat{M}, \widehat{\xi})$	closed tight contact 3-manifold obtained by capping off boundary 2-spheres 262
$T_p^{\mathbb{C}}\mathbb{C}^n, T_p^{\mathbb{C}}(\partial G)$	complex tangent space 278, 279
$\partial_z, \partial_{\bar{z}}$	holomorphic or antiholomorphic derivation, respectively 278
$\otimes_{\mathbb{R}}$	tensor product over $\mathbb{R}$ 278

$\partial, \bar{\partial}$	holomorphic or antiholomorphic differential, respectively 279
$L_p\rho$	Levi form 280
$+$	topological sum 291
SN	symplectic normal bundle 294
$T^{\mathbb{C}}f$	complexified differential 306
$\text{Imm}_{\xi}$	space of isotropic immersions 310
Mon	space of bundle monomorphisms 310
$\mathbb{L}$	Legendrian link 324
$V(\mathbf{a})$	$\{z_0^{a_0} + \cdots + z_n^{a_n} = 0\}$ 333
$\Sigma(\mathbf{a})$	$V(\mathbf{a}) \cap S^{2n+1}$ 333
$N^I$	subset of normal bundle defined by $\{r \in I\}$ , $I \subset \mathbb{R}_0^+$ 350
$\#_{\Phi}$	fibre connected sum 350
$\mathfrak{g}$	Lie algebra of the Lie group $G$ 360
$\mu$	momentum map 360
$\underline{X}$	vector field on manifold with $G$ -action generated by $X \in \mathfrak{g}$ 360
$W_k$	integral Stiefel–Whitney class 367
$\beta$	Bockstein homomorphism 367
$\rho_2$	homomorphism in cohomology induced by the coefficient homomorphism $\mathbb{Z} \xrightarrow{\text{mod } 2} \mathbb{Z}_2$ 367
$F_{2n+1}$	$\text{SO}(2n+1)/\text{U}(n)$ 368
$F_{2n}$	$\text{SO}(2n)/\text{U}(n)$ 369
$BG$	classifying space for the group $G$ 368
$EG$	total space of universal $G$ -bundle over $BG$ 371
$\ominus$	formal difference of vector bundles in $K$ -group 372
$\tilde{K}$	reduced $K$ -group of vector bundles 372
$\mathcal{J}^+(\mathbb{R}^{2n})$	space of complex structures on $\mathbb{R}^{2n}$ compatible with standard metric and orientation 373
$(\mathbb{C}^{2n})^{(1,0)}$	space of $(1,0)$ -vectors 375
$\text{Iso}^+(\mathbb{C}^{2n})$	manifold of positive maximal isotropic subspaces of $\mathbb{C}^{2n}$ 376
$TH_k, FH_k$	torsion or free part, respectively, of homology group 387
$\chi^*$	(Euler) semi-characteristic 390
$b_k$	$k$ th Betti number 391
$\langle u \rangle$	subgroup generated by $u$ 392
$X \vee Y$	one-point union of $X$ and $Y$ 397
$\Omega^k$	space of differential $k$ -forms 401

## Author index

- Abbena, Elsa, 373  
 Abe, Kinetsu, 334  
 Abraham, Ralph, 360–361  
 Abreu, Miguel, 285  
 A'Campo, Norbert, 309  
 Adachi, Jiro, 275  
 Adams, Douglas, 1  
 Aebischer, Beat, xii  
 Akbulut, Selman, 325  
 Albert, Claude, 361  
 Alexander, James Waddell, 149, 151  
 Andreotti, Aldo, 305  
 Apollonius of Perga, x, 11  
 Arnold, Vladimir, x, 1, 11, 19, 21, 37  
 Atiyah, Michael, 373
- Baird, Paul, 373  
 Banyaga, Augustin, 64  
 Barden, Dennis, 367, 383, 393, 401  
 Bartholomaeus Anglicus, 366  
 Bedford, Eric, 42, 256  
 Bennequin, Daniel, xi, 53, 129, 131, 203  
 Bing, R. H., 46  
 Bogomolov, Fedor, 285  
 Boothby, William, xi, 64, 84, 332, 339–341  
 Borer, M., xii  
 Bott, Raoul, 152, 300, 340, 351–352  
 Bourgeois, Frédéric, xv, 332, 338, 348  
 Boyer, Charles P., 338  
 Bredon, Glen, xii, 105–107, 110, 134, 137, 138, 145, 176, 205, 395  
 Brieskorn, Egbert, 334  
 Bröcker, Theodor, xii, 6, 45, 68, 135, 288, 289, 333, 357  
 Browder, William, 356
- Cannas da Silva, Ana, 14, 361  
 Cao, Huai-Dong, 46  
 do Carmo, Manfredo, 26  
 Cartan, Élie, 6, 369  
 Cerf, Jean, xii, 1, 42, 131
- Chekanov, Yuri, 125  
 Cieliebak, Kai, 306, 315  
 Colin, Vincent, 194, 257
- tom Dieck, Tammo, 135  
 Ding, Fan, xiv, 49, 125, 258, 272, 287, 320–324, 338, 357  
 Doyle, Arthur Conan, 130  
 Dubrovin, Boris A., 339, 340  
 Duchamp, Tom, 310  
 Dymara, Katarzyna, 318
- Easton, Sheena, 332  
 Edwards, C. H., Jr., 48  
 Ekholm, Tobias, 316  
 Eliashberg, Yakov, xi–xii, xiv, 1, 36, 42, 49–50, 62, 124–125, 127, 131, 157, 188, 193–194, 200, 204–205, 241, 253, 254, 256, 267, 268, 272, 285–287, 303, 305–306, 315, 318–319, 329, 338, 356, 367  
 Engel, Friedrich, 6  
 Erbacher, Joseph, 334  
 Erlandsson, Thomas, 55  
 Etnyre, John, xii–xiv, 49, 93, 117, 125, 149, 151, 193–194, 229, 269, 316, 318, 325, 331
- Fenn, Roger, 147  
 Fomenko, Anatoly T., 339, 340  
 Fraenkel, L. Edward, 255  
 Frankel, Theodore, 305  
 Fraser, Maia, 124–125, 318  
 Freudenthal, Hans, 105  
 Fuchs, Dmitry, 184, 188
- Gabai, David, 50  
 Galicki, Krzysztof, 338  
 Garbiero, Sergio, 373  
 Gaveau, Bernard, 42, 256



- Geiges, Hansjörg, xii, xiv, 2, 36, 49, 55, 72, 125, 131, 258, 272, 287, 303, 309, 312, 320–324, 338, 351–353, 357, 361, 366–367
- Ghiggini, Paolo, 114, 229, 285
- Gibbs, Josiah Willard, 37
- Ginzburg, Viktor L., 131
- Giroux, Emmanuel, xi–xiii, 79, 131, 156, 179, 184, 188, 194, 229, 244, 253, 321, 332, 344, 348, 367
- Givental, Alexander, 125
- Gompf, Robert, 300, 306, 327, 332, 356
- González-Acuña, Francisco, 46
- Gonzalo Pérez, Jesús, 37, 131, 148
- Gordon, Cameron, xii, 48–49
- Gottlieb, Daniel Henry, 106
- Goursat, Édouard, 6
- Grauert, Hans, 284
- Graustein, William Caspar, 311
- Gray, John W., xi, 60, 369
- Gromov, Mikhael, x, 36, 42, 256, 268, 272, 303, 329, 332, 338, 344, 351, 353
- Gunning, Robert C., 276, 284–285
- Gutierrez, Carlos, 172
- Haefliger, André, 367, 385
- Hartman, Philip, 167
- Hatakeyama, Yoji, 64, 84
- Hatcher, Allen, 107
- Hector, Gilbert, 211
- Helgason, Sigurdur, 369
- Hempel, John, 257–258, 263, 265
- Hilden, Hugh, 135, 147–148
- Hill, C. Denson, 284
- Hirsch, Morris, xii, 41, 42, 85, 94, 110, 135, 195–196, 206, 225, 288
- Hirsch, Ulrich, 211
- Hirzebruch, Friedrich, 334, 356
- Hobbes, Thomas, 51
- Hofer, Helmut, xiv, 125
- Honda, Ko, xiii, 125, 194, 229, 244, 257, 321, 325, 331
- Hopf, Heinz, 105
- Hsü, Chên Jung, 335
- Husemoller, Dale, 91, 314, 319, 383
- Huygens, Christiaan, x, 36–37
- Isherwood, Christopher, 286
- Jänich, Klaus, xii, 6, 45, 68, 288, 289, 333, 343, 357
- Jost, Jürgen, 21
- Kälén, Markus, xii
- Kanda, Yutaka, 114
- Kervaire, Michel, 42, 386, 390
- Kirby, Robion, 136, 289
- Kneser, Hellmuth, 257
- Kobayashi, Shoshichi, 339
- Kodaira, Kunihiko, 60
- van Koert, Otto, 338, 348, 367
- Kosinski, Antoni, 42, 89, 98, 138, 287–288, 308, 351, 386, 388, 394, 397
- Kotschick, Dieter, 309
- Kronheimer, Peter, xii, xiv, 1, 46, 50
- Kuiper, Nicolaas, 42
- Lanchester, John, x, 194
- Lefschetz, Solomon, 305
- Leibniz, Gottfried Wilhelm, 21
- Leuenberger, Christoph, xii
- Levy, Silvio, 40
- Lewis, Wyndham, 268
- Li-Jost, Xianqing, 21
- Lickorish, W. B. Raymond, 46, 130, 132
- Lie, Sophus, x, 6
- Lisca, Paolo, xiv, 194, 318, 322, 325
- Loose, Frank, 361
- Luecke, John, xii, 48–49
- Lutz, Robert, xi, 2, 130–131, 335, 348
- Lyon, Herbert C., 151
- Marsden, Jerrold E., 360–361
- Martin, Joseph M., 46
- Martinet, Jean, xi, 130–132
- Matić, Gordana, 325
- Mayer, Karl Heinz, 334, 356
- McDuff, Dusa, xii, 14, 21, 64, 76, 88, 255, 269–270, 272, 300, 340, 344, 360–361
- Meckert, Christiane, 286, 335
- de Melo, Wellington, 166–167, 172, 209, 214, 339
- Meyer, Kenneth R., 360
- Milligan, Spike, 226
- Millman, Richard S., 220
- Milnor, John, xiv, 42, 58, 124, 134, 137, 138, 167, 205, 257, 287, 290, 305, 370–371, 384, 386, 388, 390, 394, 398
- Mishachev, Nikolai, 36, 205, 303, 306
- Mohsen, Jean-Paul, 344, 348
- Moise, Edwin, 48
- Montesinos, José, 135, 147–148
- Moore, John C., 207
- Moser, Jürgen, xi, 60
- Mrowka, Tomasz, xii, xiv, 1, 46, 50
- Munkres, James, 42, 48–49
- Myers, Robert, 151
- Nacinovich, Mauro, 284
- Nakamaye, Michael, 338
- Niederkrüger, Klaus, 329, 338, 341, 348
- Novikov, Sergei P., 339, 340
- de Oliveira, Bruno, 285

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Index

[More information](#)

428

*Author index*

- Özbağcı, Burak, xiv, 322, 324–325, 329–330  
 Ozsváth, Peter, xiv
- Palis, Jacob, Jr., 166–167, 172, 209, 214, 339  
 Parker, George D., 220  
 Paternain, Gabriel, 26  
 Peixoto, Maurício, 165, 172, 191, 199, 247–248  
 Perelman, Grigori, 46  
 Pires, Benito, 172  
 Poincaré, Henri, 6  
 Prasolov, Viktor, xii, 43, 123, 147–148, 355  
 Presas, Francisco, 344
- Range, R. Michael, 276, 280  
 Ranicki, Andrew, 150, 287  
 Reeb, Georges, 196  
 Reimann, Hans Martin, xii  
 Robinson, Clark, 166–168, 172  
 Rolfsen, Dale, xii, 43, 48, 147, 149, 151  
 Rosenberg, Jonathan, 287  
 Rossi, Hugo, 276, 284–285
- Salamon, Dietmar, xii, 14, 21, 64, 76, 88, 270, 300, 340, 344, 360–361  
 Salamon, Simon, 373  
 Saramago, José, 93  
 Sasaki, Shigeo, 335  
 Scheffers, Georg, 6  
 Schönenberger, Stephan, 229  
 Schopenhauer, Arthur, vi  
 Scott, G. Peter, 40, 377  
 Shakespeare, William, xiii, 237  
 Siegel, Carl Ludwig, 21  
 Smale, Stephen, 42, 257–258  
 Sossinsky, Alexei, xii, 43, 123, 147–148, 355  
 Spanier, Edwin H., 393  
 Spencer, Donald, 60  
 Stasheff, James, 58, 134, 137, 370–371, 384, 394, 398  
 Steenrod, Norman, 142, 368, 382–383  
 Stillwell, John, 243  
 Stipsicz, András, xiv, 300, 306, 322–324, 327, 329–330, 356  
 Sullivan, Michael G., 316  
 Szabó, Zoltán, xiv
- Tamura, Itiro, 3  
 Taubes, Clifford Henry, xiv  
 Thickstun, Thomas, 135, 147–148  
 Thom, René, 367, 385–386, 394  
 Thomas, Charles B., 338, 341, 344, 367, 401  
 Thurston, William, 40, 50, 130–131, 151, 193
- Tu, Loring W., 152, 300, 340, 351–352
- Ustilovskiy, Ilya, 335, 338
- Wall, C. T. C., 338, 386, 390, 393  
 Wallace, Andrew H., 46, 130, 132  
 Wang, Hsieu-chung, xi, 332, 339–341  
 Warner, Frank W., 3, 5, 23, 362, 368, 374  
 Weinstein, Alan, xiv, 69, 74, 270, 286–287, 300, 360–361, 367  
 Whitehead, George W., 370  
 Whitehead, J. H. C., 94  
 Whitney, Hassler, 311  
 Willett, Christopher, 361  
 Winkelkemper, H. Elmar, 130–131, 150, 151  
 Wood, John C., 373  
 Wysocki, Krzysztof, xiv
- Yokota, Ichiro, 377
- Zehmisch, Kai, 329  
 Zehnder, Eduard, xiv  
 Zhu, Xi-Ping, 46

## Subject index

- action
  - by Lie group, 361
  - by  $S^1$ , 361
  - coadjoint, 363
  - Hamiltonian, 360
  - isotropy group, 361
  - locally free, 361
  - strict contact, 361
- action functional, 21
- adjoint representation, 363
- adjunction inequality, 330
- (contact) Alexander trick, 89
- almost complex structure, 64
  - and section of principal  $F_{2n}$ -bundle associated with tangent bundle, 373
  - equivalent to reduction of structure group, 64–65
- almost contact structure, 66, 138
  - and section of principal  $F_{2n+1}$ -bundle associated with tangent bundle, 368, 373
  - and stable almost complex structure, 372
  - as lifting of classifying map, 370
  - equivalent to reduction of structure group, 66
  - existence and classification on 5-manifolds, 368
  - $W_3$  is the primary obstruction, 370
- almost regular contact form, 344
- $\alpha$ -limit set, 171
- attacking starlings with rice pudding fired from catapults, 226
- Baer's theorem, 243, 247
- Bennequin inequality
  - Legendrian, 203
  - transverse, 202
- binding, *see* open book
- blowing up, 344
- Bockstein homomorphism, 367
- Boothby–Wang theorem, 340, 341
- boundary
  - concave, 274
  - convex, 269, 274
  - differentiable, 276
  - of contact type, 23, 269
- boundary orientation, 43
- branch set, 353
  - downstairs vs. upstairs, 353
- branched covering, 135, 147, 353
  - and contact structures, 148, 354
  - description of 3-manifolds, 147
  - simple, 135
- branching index, 353
- Brieskorn manifold, 333–334
  - admits Stein fillable contact structure, 334
  - applications in contact geometry, 338
  - isotopic contact structures on, 336
- cancellation lemma, 322
- Cartan formula for Lie derivative, 23
  - for time-dependent vector fields, 406
- Cauchy–Riemann equation, 255
- Cerf's theorem ( $\Gamma_4 = 0$ ), 42, 254
- characteristic element in cohomology of Eilenberg–MacLane space, 107
- characteristic foliation, 77, 163
  - and Euler characteristic, 175–176, 178
  - and Euler class, 175–176
  - and holomorphic discs, 255
  - defining equation, 78, 163
  - divergence condition, 164
  - from homeomorphism to diffeomorphism, 241
  - in case of positive divergence, 182
  - is generically Morse–Smale, 172
  - modification near elliptic point, 182–184

- characteristic foliation (cont.)  
 on sphere, 209–214  
 on standard overtwisted disc, 158  
 on  $S^2 \subset (\mathbb{R}^3, \xi_{st})$ , 77–78  
 on  $S^2 \subset (S^2 \times S^1, \xi_{st})$ , 252  
 on  $S^2 \subset (S^3, \xi_{st})$ , 256  
 on  $T^2$ , 176–178, 237
- characteristic map, *see* sphere bundle over sphere
- Chern class  
 -es of complexified real vector bundle, 398  
 -es of symplectic vector bundle, 65  
 and cohomology of classifying space, 371  
 first – and classification of complex vector bundles over  $S^2$ , 371  
 first – as obstruction to homotopy of almost contact structures, 370  
 first – classifies almost contact structures on 5-manifolds, 368  
 top – is the Euler class, 352
- ‘classical’, usage of, 338
- classical invariants, 114, *see also*  
 Thurston–Bennequin invariant, rotation number, self-linking number  
 behaviour under connected sum, 317  
 classify Legendrian unknots, 125  
 classify transverse unknots, 127  
 do not, in general, classify Legendrian knots, 125  
 range for unknot, 124  
 relations between the, 123, 128, 203
- closed manifold = compact without boundary, 45
- coadjoint action, 363
- cobordism corresponding to surgery, 289, 291–293, 299–300, 351
- cochain homotopy, 401
- cogeodesic flow, 27
- collaring theorem, 288
- compatibility  
 almost contact structure with contact structure, 66  
 complex structure with contact structure, 66  
 complex structure with inner product, 17  
 complex structure with symplectic form, 17  
 complex structure with symplectic structure on vector bundle, 64  
 space of  $\omega$ -compatible complex structures is contractible, 18, 65  
 space of  $\xi$ -compatible almost contact structures is contractible, 66
- complex Hessian, *see* Levi form
- complex structure  
 $\omega$ -compatible, 17  
 on real vector space, 17  
 on vector bundle, 64  
 space of -s on  $\mathbb{R}^{2n}$  vs. manifold of isotropic subspaces of  $\mathbb{C}^{2n}$ , 376  
 space of -s on vector space as homogeneous space, 373
- complex tangent space  
 of smooth boundary, 279  
 description in terms of holomorphic differential, 279  
 of  $\mathbb{C}^n$ , 278
- concave boundary, *see* boundary
- conformal symplectic normal bundle  
 of contact submanifold, 75  
 determines neighbourhood, 75  
 of isotropic submanifold, 69  
 determines neighbourhood, 71
- connected sum, 288  
 as fibre connected sum, 351  
 as surgery, 288  
 fibre, 350  
 of contact manifolds, 299, 301–302  
 of knots, 317  
 of simple foliations, 213  
 of tight contact 3-manifolds, 259  
 is tight, 261
- connection 1-form on principal  $S^1$ -bundle, 340
- contact action, 361
- contact condition  
 and Levi pseudoconvexity, 54, 282  
 in  $\mathbb{R}$ -invariant situation, 180  
 in Dehn surgery along transverse knots, 133–134  
 in flow box, 214–215  
 in Lutz twist, 143  
 in open book, 154  
 in terms of defining 1-form, 4  
 in terms of Lie brackets (in dim. 3), 5  
 in terms of order of contact (in dim. 3), 37–40  
 in terms of  $(\xi_p, d\alpha_p)$  being a symplectic vector space, 17  
 independent of choice of defining 1-form, 4  
 near a hypersurface, 77, 163  
 on regions cut out by dividing set, 230  
 vs. integrability condition, 3–5
- contact Dehn surgery  
 along Legendrian knots, 320–324  
 along transverse knots, 132–134  
 and Lutz–Martinet theorem, 324  
 contact  $(-1)$ -surgery, 320  
 and Stein fillings, 320

- contact Dehn surgery (cont.)  
 contact (+1)-surgery is inverse of  
 contact (-1)-surgery  
 (cancellation lemma), 322  
 contact ( $\pm 1$ )-surgery is symplectic  
 handle surgery, 321–322  
 contact  $r$ -surgery, 320–321  
 is well defined for  $r = 1/k$ , 321  
 presentation of contact 3-manifolds,  
 322
- contact element, 6  
 space of -s, 6  
 equals the projectivised cotangent  
 bundle, 6  
 has natural contact structure, 7
- contact embedding, 90
- contact form, 4  
 -s are not stable, 61–62  
 almost regular, 344  
 on principal  $S^1$ -bundle over integral  
 symplectic manifold, 340  
 regular, 339  
 standard – on  $S^3$ , 24  
 standard – on  $S^{2n+1}$ , 53  
 is regular, 339
- contact framing, 114
- contact geometry is all geometry, 1
- contact homology, 338
- contact manifold, 4  
 -s are homogeneous spaces, 84  
 connected sum, *see* connected sum  
 fundamental groups of -s, 308  
 strict, 52
- contact orbifold, 362
- contact quotient  
 of  $G$ -action, 365  
 of  $S^1$ -action, 361
- contact reduction, *see* contact quotient
- contact structure, 3  
 -s are stable, 60  
 -s supported by the same open book  
 are isotopic, 157  
 and fundamental group, 308  
 classification on  $S^3$ , 254  
 every – is supported by an open  
 book, 348  
 existence on 3-manifolds, 132, *see*  
*also* Martinet's theorem  
 existence on 5-manifolds, 366  
 exotic -s on spheres, 338  
 extendibility over 3-ball, 213  
 homotopically standard – on  $S^{2n+1}$ ,  
 338  
 natural – on 1-jet space, 72  
 natural – on space of contact  
 elements, 7, 57  
 and Liouville form, 32  
 non-coorientable, 57–58  
 on boundary of plumbing, 357  
 on branched cover, 148, 354  
 on Brieskorn manifold, 334, 336  
 on fibre connected sum, 351  
 on hypersurface transverse to  
 Liouville vector field, 23  
 on 3-manifold can be obtained by  
 contact ( $\pm 1$ )-surgery, 322  
 on  $M^3 \times \Sigma^2$ , 352, 355  
 on open book, 151–153, 346  
 on principal  $S^1$ -bundle over integral  
 symplectic manifold, 340  
 on product of contact manifold  
 with  $T^2$ , 348  
 on  $T^{2n+1}$ , 348  
 on transverse intersection of complex  
 submanifold with unit sphere  
 in  $\mathbb{C}^{n+1}$ , 335  
 overtwisted, *see* overtwisted contact  
 structure  
 positive or negative, 5  
 standard – on  $\mathbb{R}^{2n+1}$ , 4, 51  
 equals standard – on  $S^{2n+1} \setminus \{p\}$ ,  
 54  
 standard – on  $S^3$ , 24  
 supported by open book, 154–156  
 standard – on  $S^{2n+1}$ , 53  
 as complex tangencies, 54  
 induced by Liouville vector field,  
 54  
 standard – on  $T^3$ , 8  
 standard overtwisted on  $\mathbb{R}^3$ , 53  
 supported by open book, 154  
 tight, *see* tight contact structure  
 tight vs. overtwisted, 159, 193  
 vertically invariant, 179
- contact submanifold, 59
- contact surgery, 293, 298, *see also* contact  
 Dehn surgery  
 and symplectic fillings, 272, 325  
 and the topology of Stein manifolds,  
 305–306  
 framing theorem, 304  
 3-dimensional case, 315  
 giving rise to a symplectic  
 cobordism, 297–298, 300  
 natural framing, 297–299  
 simplified picture without cobordism,  
 301–303  
 uniqueness up to isotopy, 299
- contact transformation, 8
- contact type hypersurface or boundary,  
 23, *see also* boundary
- contact vector field, 32, 179, *see also*  
 infinitesimal automorphism  
 strict, 32
- contactomorphism, 52  
 defined by Liouville flow, 52

- contactomorphism (cont.)  
 of  $(S^3, \xi_{\text{st}})$  extends to diffeomorphism of  $D^4$ , 254  
 strict, 52  
 preserves Reeb vector fields, 61  
 contactomorphism group acts transitively, 84  
 convex boundary, *see* boundary  
 convex surface, 179  
 and dividing set, 234  
 and vertically invariant contact form, 180  
 and vertically invariant contact structure, 179–180  
 any foliation divided by its dividing set can be realised as characteristic foliation, 238  
 condition is generic, 237  
 dividing set, *see* dividing set  
 Heegaard surface from open book, 233–234  
 implied by Morse–Smale characteristic foliation, 236  
 convexity  
 geometric, 277  
 geometric convexity implies Levi pseudoconvexity, 280  
 geometric vs. symplectic, 273  
 (strict) Levi pseudoconvexity, 280  
 and holomorphic filling, 283  
 implies contact condition, 282  
 is invariant under biholomorphisms, 280–281  
 symplectic, 269  
 corners can be smoothed, 289  
 correlation, 11  
 $\mathbb{C}P^3$   
 is the homogeneous space  $SO(6)/U(3)$ , 369  
 is the manifold of positive maximal isotropic subspaces of  $\mathbb{C}^6$ , 377  
 curvature form, 340  
 cusp, 96  
 left or right, 117  
 up or down, 121  
 Darboux’s theorem, 67  
 Dehn surgery, 43, 132–133, *see also* contact Dehn surgery  
 effect on homology, 327, 329  
 with integer coefficient is handle surgery, 45  
 dimension formula  
 for orthogonal complement with respect to symmetric inner product on  $\mathbb{C}^n$ , 375  
 for symplectic orthogonal complement, 15  
 disc theorem, 41, 85  
 contact, 85  
 distribution, 2, 137  
 of codim. 1 is the kernel of a global 1-form iff it is coorientable, 2  
 divergence of vector field, 163  
 behaviour under rescaling, 183, 187, 231  
 dividing set, 229, 232  
 and overtwistedness, 240  
 contains all the essential information about a convex surface, 238  
 creating additional dividing curves, 241  
 exits iff surface is convex, 234  
 is a submanifold, 230  
 is determined by characteristic foliation, 234  
 is non-empty, 230  
 is transverse to the characteristic foliation, 230  
 of convex surface, 229  
 of foliation, 232  
 parametric family, 251  
 domination, 269  
 vs. strong symplectic filling, 271  
 vs. weak symplectic filling, 271  
 Eilenberg–MacLane space, 107  
 elimination lemma, 187  
 used in tomography, 249–251  
 elimination position, 184  
 elliptic point, *see* singular point  
 embedding  
 contact, 90  
 isotropic, 82, *see also* isotropic embedding  
 Euler characteristic  
 and characteristic foliation, 175–176  
 and existence of vector fields, 137  
 negative part, 193  
 semi-characteristic, 390  
 Euler class, 119  
 -es realisable by contact structures, 145  
 -es realisable by tight contact structures, 194  
 and characteristic foliation, 175–176  
 as obstruction class, 144  
 dependence on orientation, 352  
 determines 2-plane bundle, 352  
 in Gysin sequence, 384  
 is Poincaré dual to self-intersection class of zero section, 145, 176  
 of principal  $S^1$ -bundle, 340  
 relation with obstruction class  $d_2$ , 146  
 Euler number, 119

- Euler–Lagrange equations, 21  
 exact symplectomorphism, 347  
 examples  
   almost horizontal foliation on  $S^2$ , 210  
   branched covering, 354  
   characteristic foliation of convex surface does not contain flow line from negative to positive singularity, 235  
   characteristic foliation on  $S^2 \subset (\mathbb{R}^3, \xi_{st})$ , 77  
   characteristic foliation on  $T^2$ , 176  
   characteristic foliations induced by parametric family of plane fields, 223  
   classical invariants for Legendrian unknots, 124  
   conformal symplectic normal bundle, 70  
   connected sum as surgery, 288  
   contact forms are not stable, 62  
   contact structure on  $M^3 \times \Sigma^2$   
     obtained by fibre connected sum, 352  
     via branched covering, 355  
   contact structure on  $M^3 \times T^2$   
     obtained by Bourgeois’s construction, 352  
   contact structure supported by open book, 155  
   contact surgery  
     along  $S^{n-1} \subset S^{2n-1}$ , 300  
   contactomorphism, 52, 57  
   convex surface, 179  
   deformation of surface into convex surface, 237  
   degeneracies in a parametric family of functions, 224  
   dividing set of convex surface, 230  
   dividing set of foliation, 232, 233, 237  
   dividing set of Heegaard surface  
     coming from open book, 233  
   every finitely presentable group is the fundamental group of a contact manifold, 308  
   foliation without dividing set, 232  
   front projection of Legendrian trefoil knot, 98  
   front projection of transverse trefoil knot, 100  
   hypersurface that is both  $\omega$ -convex and  $\omega$ -concave, 273  
   Lagrangian projection of Legendrian unknot, 99  
   Legendre transformation, 10  
   Legendrian submanifold, 35  
   Liouville vector field, 24  
   Moser trick in nbhd. thm. for isotropic submanifolds, 74  
   natural contact structure on space of contact elements of  $T^2$ , 8  
   natural framing of contact surgery along Legendrian knot, 299  
   neighbourhood of Legendrian knot, 72  
   neighbourhood of Legendrian submanifold, 72  
   neighbourhood of transverse knot, 76  
   non-convex surface, 181, 232  
   non-coorientable contact structure, 57  
   open book decomposition of  $S^3$ , 150, 155  
   overtwisted contact structure, 53  
   Poincaré return map, 170  
   Reeb vector field, 6  
   rotation number of standard Legendrian circle in  $S^3$ , 121  
   simple but not almost horizontal foliation on  $S^2$ , 210  
   singular points of vector fields on surfaces, 167  
   standard contact form on  $S^3$ , 24  
   standard contact structure  
     on  $\mathbb{R}^{2n+1}$ , 4, 51  
   standard contact structure  
     on  $S^{2n+1}$ , 53  
   strictly Levi pseudoconvex boundary, 282  
   strong symplectic filling, 269  
   surgery along  $S^k \subset S^n$ , 288–289  
   symplectic vector bundle, 64  
   symplectisation, 24  
   Thurston–Bennequin invariant  
     of ‘shark’, 116  
   Thurston–Bennequin invariant of standard Legendrian circle in  $S^3$ , 115  
   trefoil knot, 98  
   trivial symplectic cobordism, 274  
   unknot does not have Property P, 47  
 exhausting function, 283  
 fibre connected sum, 350  
   and contact structures, 351  
   and ordinary connected sum, 351  
   as a sequence of surgeries, 351  
   giving rise to a cobordism, 351  
 filling  
   holomorphic, 283  
     implies strong symplectic in dim. 3, 283  
   Stein, 284, 335  
     and contact  $(-1)$ -surgeries, 320  
     can be capped off, 325

- filling (cont.)  
 Stein (cont.)  
 implies holomorphic, 284–285  
 implies strong symplectic, 284–285  
 not implied by strong symplectic filling, 285  
 of Brieskorn manifold, 334  
 symplectic, *see* symplectic filling
- filling by holomorphic discs, *see* holomorphic disc
- flow  
 of time-dependent vector field, 59  
 topologically conjugate, 165
- flow box, 339  
 regular, 339
- flow box theorem, *see* tubular flow theorem
- foliation, 3  
 almost horizontal – on  $S^2$ , 210, *see also* simple foliation  
 induced by plane field of small norm, 220  
 is determined by its holonomy, 212  
 characteristic, *see* characteristic foliation  
 simple – on  $S^2$ , *see* simple foliation
- framing, 45, 288  
 contact vs. surface, 114–115  
 determined by choice of longitude, 45  
 natural – for contact surgery, 297–298  
 along Legendrian knot, 299  
 preferred longitude determines surface –, 115  
 theorem for contact surgery, 304  
 3-dimensional case, 315
- Frobenius integrability condition, 3
- front projection, 35, 96, 308  
 of Legendrian trefoil knot, 98  
 of transverse trefoil knot, 100  
 vs. Lagrangian projection, 99–100
- function  
 exhausting, 283  
 strictly plurisubharmonic, 283
- fundamental groups of contact manifolds, 308
- $\Gamma_n$ , 41  
 and differential topology, 42  
 is an abelian group, 41  
 is trivial for  $n = 2, 3, 4$ , 42
- Gauß map, 137, 216  
 and normal curvatures, 220–221
- generalised Poincaré lemma, 402–403
- genericity  
 for characteristic foliations, 172  
 for holonomy map, 224–225  
 for Legendrian curves, 96  
 for 1-parametric families of characteristic foliations, 247–248  
 for surfaces in 3-manifolds, 237  
 for vector fields, *see* Morse–Smale
- gentleman should never use a basis, 2
- genus bound in tight contact 3-mfd., 193, 243
- geodesic field, 26
- geodesic flow, 26
- Gioux criterion, 239
- global angular form, 351
- Gray stability theorem, 60
- Gysin sequence, 384
- Hadamard’s theorem on ovaloids, 220
- Haefliger’s theorem on embeddings, 385
- Hamiltonian action, 360
- Hamiltonian equations, 22  
 in coordinate-free notation, 22
- Hamiltonian flow, 27
- Hamiltonian function in classical mechanics, 21
- Hamiltonian vector field in symplectic geometry, 22
- handle, 289, 303  
 -body, 300  
 attaching map, 304  
 attaching sphere, 291  
 belt disc or cocore, 291  
 belt sphere, 291  
 core, 291  
 index, 289  
 lower boundary, 290, 304  
 model –, 290–291  
 symplectic, 296–297  
 upper boundary, 290
- handle surgery, *see* surgery
- handlebody, 300  
 and Heegaard splitting, 233  
 symplectic, 300
- Hartman–Grobman theorem, 167
- Heegaard splitting of contact 3-manifold  
 along convex surface, 233–234
- Hodge  $*$ -operator, 377
- holomorphic disc, 255, 256  
 and characteristic foliation, 255  
 and maximum principle, 255  
 cannot be tangent to pseudoconvex boundary, 255  
 filling by -s, 254, 256
- holomorphic filling, *see* filling
- holonomy, 212  
 can be made generic, 224–225
- homogeneous space, 368  
 contact manifold as, 84  
 $\mathbb{C}P^3$  as, 369  
 space of complex structures as, 373



- homology class
  - in  $H_1(M^3)$  is represented by a knot, 107
  - in  $H_2(M^3)$  is represented by a surface, 108
  - relative version, 109
  - version over  $\mathbb{Z}_2$ , 108
  - in  $H_{m-1}(W^m)$  or  $H_{m-2}(W^m)$  is represented by a submanifold, 386
- homology sphere, 327
- homotopically standard contact structure on  $S^{2n+1}$ , 338
- homotopy group, stability of -s, 372
- Hopf fibration, 24, 155
  - generalised, 369
- Hopf invariant, 142
- Hopf link, 155
- $h$ -principle, 36, 303
  - for 1-parametric families of transverse curves, 104
  - for isotropic embeddings, 307
  - for isotropic immersions, 306
    - parametric, 310
  - for Legendrian knots, 101
  - for transverse knots, 101
  - 1-parametric – for Legendrian immersions  $S^1 \rightarrow (\mathbb{R}^3, \xi_{st})$ , 311
- Hurewicz isomorphism, 371
- Huygens' principle, 36–37
- hyperbolic point, *see* singular point
- incidence of simplices, 215
- index
  - of handle, *see* handle
  - of singular point, *see* singular point
- infinitesimal automorphism
  - of contact form, 32
    - characterisation in terms of Lie derivative, 33
  - of contact structure, 32
    - characterisation in terms of Lie derivative, 33
    - correspondence with Hamiltonian functions, 62
- inner product
  - Hermitian, 18
  - $J$ -compatible, 17
- integrable hyperplane field, 3
- integral submanifold, 3
- integral symplectic form, 340
- integration along fibres, 401
- intersection number, 111
- intersection product, 108
  - and geometric intersection, 108–109
- isotopy extension theorem
  - for contact embeddings, 90
  - for isotropic embeddings, 82
    - for surfaces in 3-manifolds, 91
- isotopy of homeomorphisms vs. smooth isotopy, 242
- isotropic embedding, 82
  - $h$ -principle, 307
- isotropic immersion, 82
  - complexification of differential, 306
  - $h$ -principle, 306
    - parametric, 310
    - regular homotopy of -s, 310
- isotropic submanifold, 34, *see also* submanifold
- isotropy group, 361
- 1-jet, 72
  - and 1-prolongation, 224
- jet transversality theorem, 225
- Jordan curve theorem, 209
- knot, 43
  - classified by complement in  $S^3$ , 48
  - connected sum, 317
  - Legendrian, 94
    - classical invariants, *see* classical invariants
    - condition in terms of parametrisation, 96
    - Legendrian push-off, 95
    - stabilisation, 318
    - transverse push-off, 95, 128–129
  - loose, 318
  - Property P, *see* Property P for knots
  - transverse, 94
    - condition in terms of parametrisation, 96
    - impossible front projections, 100
    - Legendrian push-offs, 95
    - positively or negatively, 94
    - self-linking number, *see* self-linking number
  - trefoil, *see* trefoil knot
- knot or link diagram, 112
  - sign of crossings, 113
  - writhe of, *see* writhe of an oriented knot diagram
- Lagrangian function, 21
  - Legendre condition for, 21
- Lagrangian projection, 96
  - of Legendrian unknot, 99
- Lagrangian submanifold, 300
- Lagrangian subspace, 15
  - s are the isotropic subspaces of middle dimension, 17
- Landau symbol, 37
- Legendre condition for Lagrangian function, 21
- Legendre transformation, 11

- Legendrian immersions  $S^1 \rightarrow (\mathbb{R}^3, \xi_{st})$ ,  
 310–314  
 are classified by the rotation number, 311  
 regular homotopy of, 311  
 rotation number vs. complexified differential, 310
- Legendrian polygon, 196  
 in proof of genus bound, 195–199  
 with all vertices of the same sign, 200
- Legendrian push-off, *see* knot
- Legendrian submanifold, 35
- Levi form, 280  
 and strictly plurisubharmonic functions, 283
- Lie derivative  
 Cartan formula, 23  
 for time-dependent vector fields, 406  
 of time-dependent differential form, 59–60  
 with respect to time-dependent vector field, 59–60, 406
- limit cycle, 191
- line element, 9
- linear symplectic reduction, 360
- link, 45  
 (framed) cobordism class, 139–140  
 framed, 138, 140
- linking number, 111  
 additivity, 111  
 as intersection number, 111  
 computed from crossings in link diagram, 113  
 dependence on orientation, 111  
 symmetry, 112
- Liouville form, 19  
 and natural contact structure on space of contact elements, 32
- Liouville vector field, 23  
 flow defines contactomorphism, 52  
 induces contact structure on transverse hypersurface, 23
- longitude, 110  
 preferred, 43  
 characterised by linking number, 45  
 characterised in terms of Seifert surface, 45  
 homological characterisation, 43
- Lutz twist, 143  
 and contact Dehn surgery, 324  
 as topologically trivial Dehn surgery, 143  
 creates overtwisted disc, 161–162  
 effect on obstruction classes, 145–147  
 full, 160  
 does not change homotopy class, 160–161
- Lutz–Martinet theorem, 144  
 via contact Dehn surgery, 324
- manifold  
 differentiable vs. differential, 2  
 has the homotopy type of a  $CW$ -complex, 135
- 3-manifold  
 can be obtained by surgery, 46  
 irreducible, 263  
 is a branched cover of  $S^3$ , 147  
 is an open book, 149  
 is parallelisable, 134–137  
 non-trivial, 257  
 prime, 257  
 prime decomposition, 257
- 5-manifold  
 characterisation of  $S^3$ -bundles over  $S^2$ , 383  
 structure theorem for simply connected -s with  $W_3 = 0$ , 386
- mapping torus, 149  
 generalised, 345
- Marsden–Weinstein–Meyer quotient, 360
- Martinet’s theorem, 132  
 via branched coverings, 148  
 via contact Dehn surgery, 324  
 via open books, 151–153  
 via surgery, 132–134
- maximum principle  
 boundary point lemma, 255  
 strong – for subharmonic functions, 255
- meridian, 43
- momentum map  
 of contact  $G$ -action, 362  
 is equivariant, 363  
 of contact  $S^1$ -action, 361  
 of symplectic  $G$ -action, 360
- monodromy, *see* open book
- Morse, lemma of, 89, 98
- Morse–Smale  
 flow, 171  
 implies convexity, 236  
 vector field, 171  
 -s are structurally stable, 172  
 -s form open and dense subset, 172
- Moser trick, 60  
 for contact forms, 68  
 for symplectic forms, 346
- natural numbers, 104
- neighbourhood theorem  
 for contact submanifolds, 75  
 for isotropic submanifolds, 71  
 in *strict* contact manifolds, 294

*Subject index*

437

- neighbourhood theorem (cont.)  
 for Legendrian submanifolds, 72  
 for surfaces in 3-manifolds, 79  
 for transverse knots, 76
- non-degenerate  
 periodic orbit, *see* periodic orbit  
 singular point, *see* singular point
- norm of 2-plane field, 216  
 and almost horizontal foliation, 220
- normal bundle, 69  
 of an immersion, 352
- normal coordinates, 28
- obstruction theory  
 and parallelisability of 3-manifolds,  
 134–135  
 for 2-plane fields on 3-manifolds,  
 138, 140–142  
 for almost contact structures on  
 5-manifolds, 368–373
- $\omega$ -limit set, 171
- open book  
 abstract, 149  
 abstract – vs. – decomposition,  
 149–150  
 and contact structures, 151–153, 346  
 decomposition, 149  
 binding, 149  
 pages, 149  
 description of 3-manifolds, 149  
 gives rise to Heegaard splitting along  
 convex surface, 233–234  
 monodromy, 150, 156  
 from symplectic to exact  
 symplectic, 347  
 isotopic -ies give diffeomorphic  
 open books, 344  
 of 3-dimensional open book may  
 be assumed symplectic, 345  
 on  $S^3$ , 150–151, 154–156  
 supporting standard contact  
 structure, 154–156  
 stabilisation, 157  
 supporting contact structure, 154,  
 348
- order  
 of contact, 37  
 of vanishing, 37, 403
- orientation  
 boundary –, 43  
 induced by contact structure, 5  
 induced by symplectic form, 24, 49
- ovaloid, 220  
 Hadamard's theorem, 220
- 'overtwisted along a disc', 204
- overtwisted ball, 225
- overtwisted contact structure, 159  
 classification, 205–206  
 on  $S^3$ , 254  
 existence on 3-manifolds, 162  
 sufficient criterion, 191, 200, 240
- overtwisted disc, 158–159  
 characterisation in terms of framings,  
 158–159  
 found by elimination, 191–193
- page, *see* open book
- parallelisability, 134–137  
 explicit parallelisation of  $S^3$ , 135
- parity condition for **tb** and **rot**, 123, 316
- periodic orbit  
 non-degenerate, 170  
 attracting, 170  
 repelling, 170
- plumbing, 356  
 and contact structures, 357
- Poincaré return map, 170
- Poincaré–Bendixson theorem, 209, 212,  
 222, 248, 249
- Poincaré–Hopf index theorem, 176, 191,  
 209, 221
- polarity, 12
- Pontrjagin–Thom construction, 138
- prime decomposition theorem, 257  
 for tight contact 3-manifolds, 258
- principal  $S^1$ -bundle  
 connection 1-form, 340  
 curvature form, 340  
 Euler class, 340
- Property P for knots, 46  
 equivalent characterisation, 47  
 every non-trivial knot has it, 46  
 unknot does not have it, 47
- real structure on complex vector space,  
 377
- reconstruction lemma, 244
- reduction of structure group, 65
- Reeb flow  
 and cogeodesic flow, 27  
 and Hamiltonian flow, 25, 27  
 on  $(S^3, \alpha_{st})$  defines Hopf fibration,  
 24–25  
 on Brieskorn manifold, 335
- Reeb vector field, 5  
 of  $(S^3, \alpha_{st})$ , 24  
 of contact form on principal  
 $S^1$ -bundle, 340  
 regular – has constant period  
 function, 343
- regular contact form, 339
- regular flow box, 339
- regular homotopy, 310  
 Legendrian, 311
- regular vector field, 339  
 has smooth period function, 342

- regular vector field (cont.)  
 on closed manifold has periodic orbits only, 339
- rel = relative to, 108
- retrograde saddle–saddle connection, 237
- rotation number, 118, *see also* classical invariants  
 classifies Legendrian immersions  $S^1 \rightarrow (\mathbb{R}^3, \xi_{\text{st}})$ , 311  
 computation from front projection, 121–122  
 computation from Lagrangian projection, 122–123  
 dependence on orientations, 118  
 depends on the knot only, if Euler class is trivial, 120  
 in Legendrian Bennequin inequality, 203  
 independent of choices in the definition, 118–120  
 is also defined for Legendrian *immersions*, 123  
 is invariant under Legendrian isotopies, 118  
 of standard Legendrian circle in  $S^3$ , 121  
 parity condition with Thurston–Bennequin invariant, 123, 316  
 vs. complexified differential, 310
- rotation number of immersed circle in the plane, 123  
 determines regular homotopy class (Whitney–Graustein thm.), 311
- Seifert surface, 110  
 and linking number, 111
- self-intersection number, 329
- self-linking number, 125  
 computation from front projection, 127  
 depends on knot only, if Euler class is trivial, 127  
 in transverse Bennequin inequality, 202  
 independent of choices in the definition, 126  
 is invariant under transverse isotopies, 126  
 of transverse push-off, 128  
 parity, 128, 203
- semi-characteristic, 390
- separatrix, 169  
 retrograde saddle–saddle connection, 237  
 stable, 169  
 unstable, 169
- Serre fibration  
 evaluation map is, 207  
 restriction homomorphism is, 257
- shark, 116
- sign of singular point, *see* singular point
- simple foliation, 209  
 and extendibility of contact structure, 213  
 condition to be almost horizontal, 210  
 connected sum, 213  
 diagram of, 211  
 determines topological type, 211  
 north and south pole, 209
- simplex, special, 216
- singular point  
 degeneracies, 169–170  
 elliptic, 166  
 is essentially unique, 173–175  
 linear models, 167–168  
 hyperbolic, 166  
 differing usage, 166  
 is essentially unique, 173–175  
 linear models, 169  
 separatrix, *see* separatrix
- index, 166  
 non-degenerate, 166  
 linear model determines flow, 167
- sign, 167, 223  
 is a  $C^1$ –, not a  $C^0$ –invariant, 169, 176–178  
 of source or sink, 168
- sink, 167  
 source, 167  
 degenerate, 168–169
- sink, *see* singular point
- smoothing corners, 289
- source, *see* singular point
- special unitary basis, 378
- sphere bundle over sphere, 288–289  
 characteristic map, 289, 382  
 $S^3$ –bundles over  $S^2$ , 383
- sphere theorem, 196
- spin structure, 136
- stabilisation of Legendrian knot, 318
- stability of homotopy groups, 372
- Stein filling, *see* filling
- Stein manifold, 283  
 characterised by existence of exhausting plurisubharmonic function, 284  
 topological type, 305–306
- Stiefel–Whitney class  
 –es of total space of sphere bundle, 383  
 and Euler class, 145  
 first – and orientability, 135  
 integral, 367  
 relation with Wu classes, 134

- Stiefel–Whitney class (cont.)  
 second  
 and classification of vector bundles over  $S^2$ , 371  
 and Euler class, 136  
 and spin structures, 137  
 as obstruction class, 135, 384  
 of total space of disc bundle, 136  
 third integral – is primary  
 obstruction to existence of almost contact structure, 370
- straightening the angle, 289
- strictly plurisubharmonic function, 283  
 and Levi form, 283
- strong symplectic filling determines contact structure on boundary, 270
- structural stability of vector field, 172
- $SU(4)$  is double covering of  $SO(6)$ , 377
- submanifold  
 contact, 59  
 isotropic, 34  
 has dimension smaller than middle dimension, 34  
 Legendrian, 35
- subspace  
 coisotropic, 15  
 role in linear symplectic reduction, 360  
 isotropic, 15, 375  
 Lagrangian, 15  
 maximal isotropic, 375  
 positive, 375  
 symplectic, 15
- surface framing, 114–115  
 and preferred longitude, 115
- surgery, 45, 288  
 along  $S^k \subset S^n$ , 288–289  
 coefficient, 44  
 contact, *see* contact surgery  
 Dehn, *see* Dehn surgery  
 effect on Betti numbers, 391  
 giving rise to a cobordism, 289, 291–293, 299–300  
 presentation of 3–manifolds, 46  
 presentation of contact 3–manifolds, 322  
 via handle attaching, 289–293
- surgery coefficient, 132–133  
 and self-intersection number, 329
- symplectic cobordism, 273, 293  
 and symplectic fillings, 274  
 corresponding to contact surgery, 297–298, 300  
 defines a reflexive, but non-symmetric relation, 276  
 defines a transitive relation, 275  
 gluing, 275  
 trivial, 274
- symplectic filling  
 convex vs. concave, 274  
 semi-filling, 325  
 strong, 49, 269  
 as symplectic cobordism, 274  
 can be capped off, 325  
 does not imply Stein filling, 285  
 implied by holomorphic filling in dim. 3, 283  
 implied by Stein filling, 284–285  
 implies weak, 49  
 vs. domination, 271
- weak, 49, 269  
 also admits attaching of symplectic handles, 325  
 can be capped off, 49, 324–329  
 does not imply strong, 49, 272  
 implies tightness, 329  
 vs. domination, 271
- symplectic form, 20  
 integral, 340  
 standard – on  $\mathbb{R}^{2n}$ , 54, 88
- symplectic handle, 296  
 can be attached to weak filling, 325
- symplectic handlebody, 300
- symplectic linear form, 14
- symplectic manifold, 20
- symplectic normal bundle, 294  
 conformal, *see* conformal symplectic normal bundle  
 determines neighbourhood, 294
- symplectic orthogonal complement, 14  
 dimension formula, 15
- symplectic reduction  
 linear, 360  
 Marsden–Weinstein–Meyer quotient, 360–361
- symplectic vector bundle, 64  
 conformal class, 65, 269
- symplectic vector space, 14  
 has even real dimension, 16
- symplectisation of contact manifold, 24
- symplectomorphism, 270  
 exact, 347
- technical homotopy, 68
- thermodynamics is impossible to understand, 37
- Thom’s theorem about realising homology classes by submanifolds, 386
- Thurston–Bennequin invariant, 115, *see also* classical invariants  
 computation from front projection, 116–117  
 computation from Lagrangian projection, 117  
 dependence on orientations, 115, 118

- Thurston–Bennequin invariant (cont.)  
 in Legendrian Bennequin inequality, 203  
 is invariant under Legendrian isotopies, 115, 117  
 of ‘shark’, 116  
 of standard Legendrian circle in  $S^3$ , 115–116  
 parity condition with rotation number, 123, 316
- tight ball, 67, 227
- tight contact 3-manifold  
 connected sum, 259  
 is tight, 261  
 prime decomposition, 258  
 unique decomposition, 261
- tight contact structure, 159  
 and coverings, 331  
 and Euler class, 194  
 classification  
 on  $D^3$  rel boundary, 252  
 on  $\mathbb{R}^3$ , 252  
 on  $S^2 \times [-1, 1]$ , 246  
 on  $S^2 \times S^1$ , 252  
 on  $S^3$ , 252
- examples  
 on  $T^3$ , 331  
 standard structure on  $\mathbb{R}^3$  and  $S^3$ , 203–204, 331  
 standard structure on  $S^2 \times S^1$ , 331
- genus bound, 193, 243  
 implied by weak fillability, 329  
 3-manifold without, 194  
 need not be fillable, 331
- tomography, 244  
 reconstruction lemma, 244  
 role of elimination lemma, 249–251  
 uniqueness lemma, 245
- transfer map, *see*  
*Umkehrhomomorphismus*
- transitivity of contactomorphism group, 84
- transversal is a noun, 94
- transverse push-off, *see* knot
- trefoil knot, 98  
 left and right, 98  
 Legendrian, 98  
 transverse, 101
- tubular flow theorem, 214, 339
- tubular map, 73
- Umkehrhomomorphismus*, 105–106
- unique decomposition theorem, 257  
 does not apply to overtwisted contact 3-manifolds, 267  
 for tight contact 3-manifolds, 261
- uniqueness lemma, 245
- unit cotangent bundle, 27
- unit tangent bundle, 27
- universal bundle, 371
- vanishing  
 on a submanifold, 402  
 to second order, 403
- vector field  
 generated by Lie algebra element, 362  
 regular, *see* regular vector field
- (1, 0)- and (0, 1)-vectors, 375
- virtual bundle, 372
- volume form, 4
- Wang sequence, 393
- wave front, 34  
 projection, *see* front projection
- weak homotopy equivalence, 205
- Welt, unsere ist die bestmögliche, 21
- Whitney trick, 388
- Whitney–Graustein theorem, 311
- writhe of an oriented knot diagram, 116  
 is independent of the choice of orientation, 116
- Wu class, 134