INTRODUCTION TO STRUCTURAL DYNAMICS

This textbook provides the student of aerospace, civil, or mechanical engineering with all the fundamentals of linear structural dynamics and scattered discussions of nonlinear structural dynamics. It is designed to be used primarily for a first-year graduate course. This textbook is a departure from the usual presentation of this material in two important respects. First, descriptions of system dynamics throughout are based on the simpler-to-use Lagrange equations of motion. Second, no organizational distinction is made between single and multiple degree of freedom systems. In support of those two choices, the first three chapters review the needed skills in dynamics and finite element structural analysis. The remainder of the textbook is organized mostly on the basis of first writing structural system equations of motion, and then solving those equations. The modal method of solution is emphasized, but other approaches are also considered. This textbook covers more material than can reasonably be taught in one semester. Topics that can be put off for later study are generally placed in sections designated by double asterisks or in endnotes. The final two chapters can also be deferred for later study. The textbook contains numerous example problems and end-of-chapter exercises.

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INTRODUCTION TO Structural Dynamics

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To Matteo, Olivia, and Bridget

Spiego, così imparo

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Preface for the Student

No actual structure is rigid. All structures deform under the action of applied loads. When the applied loads vary over time, so, too, do the deflections. The time-varying deflections impart accelerations to the structure. These accelerations result in body forces[1] called inertial loads. Since these inertia loads affect the deflections, there is a feedback loop tying together the deflections and at least the inertial load part of the total loads. When the applied loads result from the action of a surrounding liquid, then the deflections determine all the applied dynamic loads. Therefore, unlike static loads (i.e., slowly applied loads), differential equations based on Newton's laws are required to mathematically describe time-varying load–deflection interactions. Inertial loads can also have the importance of being the largest load set acting on parts of a structure, particularly if the structure is quite flexible.

In order to appreciate how significant time-varying forces can be, consider, for example, the time-varying loads that act on a typical large aircraft. After the aircraft starts its engines, it generally must taxi along taxiways to a runway and then travel along the runway during its takeoff run. Taxiways and runways are not perfectly flat. They have small alternating hills and valleys. As will be examined in a simplified form later in this book, these undulations cause the aircraft to move up and down and rock back and forth on its landing gear, that is, its suspension system. Since the aircraft structure is not rigid, this vibratory motion of the aircraft as a whole leads to the flexing of the major parts of the aircraft, particularly the wings. The relative deformations between various parts of the wing structure are, of course, conveniently described as strains. The strains go hand in hand with stresses, and these stresses can be the maximum stresses for the aircraft structure. For example, the maximum in-flight gross weight of many large aircraft is greater than the maximum takeoff gross weight. (In-flight refueling makes possible these different gross weights.) The up-and-down inertial loads induced by the design values for the anticipated waviness of the taxiway are often responsible for the lesser value of the takeoff gross weight.

[1] All structural engineering forces are either contact forces or body forces. Contact forces are simply the result of one mass system abutting another. Body forces are the result of a force field, such as a gravity or magnetic field.

Once the aircraft has taken off, it generally climbs to the desired altitude by using full power or at least higher values of engine thrust. This type of power plant operation often produces the worst-case acoustical (high-frequency) loading on the aircraft structure adjacent to the power plant. That noise, a high-frequency vibration, is of concern because it can induce acoustical fatigue, as well as be bothersome to passengers and crew. Each time the aircraft maneuvers during its flight, the control system alters the so-called g-loads (another name for inertia loads) distributed over the aircraft structure. For those types of aircraft, such as fighter aircraft, for which rapid maneuvers are important, it is easy to imagine that the maneuver loads could be, for the most part, the dominant load set. It is also possible that the critical loads occur when a large aircraft is flying straight and level if the aircraft is subjected to substantial vertically directed wind gusts. Such gusts can add considerable bounce to the flight, with considerable flexing of the aircraft's major components. If the flight goes well, eventually the aircraft will land, and that landing will create another set of important dynamic loads as a result of the impact of the aircraft's landing gear with the runway. A landing on an aircraft carrier in particular requires careful estimation of the distributed inertial loads along the wing as the wing tips bend toward the carrier flight deck immediately after the landing gear impacts on the flight deck. All the above-described situations generally result in an initial structural motion and a snap-back motion; that is, a *vibration* that is now defined as any back-and-forth motion of the structure. Again, those motions result in inertial loads that, when combined with other loads, can cause the critical stresses within the aircraft structure. Thus the importance of vibrations for aircraft structural engineers is clear. Similar scenarios are possible for other types of vehicles: land, sea, air, or space. The structure does not have to be that of a vehicle to be endangered by time-varying loads. Time-varying wind gust and earthquake loads must be considered in the design and analysis in many civil engineering structures.

If the possibility of dynamic loads providing the maximum stresses is not enough of a reason for structural engineers to study vibrations, then there is the matter of the dynamic instabilities that are possible. In bridges and aircraft, these critical instabilities are grouped mostly under the heading "flutter." The general concept of flutter is familiar to anyone who has ever watched a ribbon tied to a fan or observed Venetian blinds lowered over an open window in a mild breeze. There are two possible sources of difficulty. One type of problem is where, perhaps because of a nonlinearity, the vibration amplitude is limited but nevertheless maintained at large deflection amplitudes. In such circumstances, there is the threat of a rapid fatigue failure. The second type of problem is where the combination of aerodynamic, inertial, and elastic loads produces vibrations whose amplitudes continue to increase. When the amplitudes of the vibration steadily increase, the strains and stresses also steadily increase until structural failure occurs. These dynamic instabilities generally result from the same combination of elastic, inertial, and applied loads that are present in any structural dynamics problem. Moreover, there is also the aircraft phenomenon called *propeller whirl* that, in addition to depending on aerodynamic, inertial, and elastic forces, depends on the gyroscopic forces of the rotating propeller.

The above brief discussion is intended to support two facts of engineering practice. The first fact is that particularly for land, sea, air, and space vehicles, the dynamics of the structure are important, often critically important. The second fact is that for an engineering analyst to prepare an adequate mathematical description of a structural dynamics problem, that analyst needs a certain understanding of dynamics as well as of structural analysis. This textbook first focuses on providing the student with all the information on the dynamics of solids that the student needs for such analyses. The textbook then explains how to use the commonplace finite element stiffness method to create those matrix differential equations that adequately describe the structural dynamics problem. The remainder of the textbook discusses solution techniques, principally the technique called the modal method.

For a student to succeed in using this book, he or she should have already studied some applications of Newton's laws and have studied structural mechanics to the point of being reasonably comfortable with elementary beam theory. Chapter 3 provides a sufficient and self-contained explanation of structural modeling using the finite element method to the extent of structures composed of such structural elements as beams, bars, and springs. An attempt has been made to illustrate all aspects of the presented theory by providing numerous example problems and exercises at the end of each chapter. The answers to the exercises are found in Appendix I.

Preface for the Instructor

This textbook is designed to be the basis for a one-semester course in structural dynamics at the graduate level, with some extra material for later self-study. Using this text for senior undergraduates is possible also if those students have had more than one semester of exposure to rigid body dynamics and are well versed in the basics of the linear, stiffness finite element method. This textbook is suitable for structural dynamics courses in aerospace engineering and mechanical engineering. It also can be used in civil engineering at the graduate level when the course focus is on analysis rather than earthquake design. The first two chapters on dynamics should be particularly helpful to civil engineers.

This textbook is a departure from the usual presentation of this material in two important ways. First, from the very beginning, descriptions of system dynamics are based on the simpler-to-use Lagrange equations. To this end, the Lagrange equations are derived from Newton's laws in the first chapter. Second, no organizational distinctions are made between multidegree of freedom systems and single degree of freedom systems. Instead, the textbook is organized on the basis of first writing structural system equations of motion and then solving those equations mostly by means of a modal transformation. Beam and spring stiffness finite elements are used extensively to describe the structural system's linearly elastic forces. If the students are not already confident assemblers of element stiffness matrices, Chapter 3 provides a brief explanation of that material. One of the advantages of this textbook is that it provides practice in the hand assembly of system stiffness matrices. Otherwise the student is expected only to bring to this study topic the usual calculus and differential equation skills developed in an accredited undergraduate curriculum. The one exception with respect to math skills occurs in Chapter 8, a wholly optional chapter, which deals with continuous mass models. There a couple of Bessel equations are used to describe nonuniform, vibratory systems. These tapered-beam examples just push that topic to its limits and thus easily can be skipped.

The traditional textbook and course material organization starts with an exhaustive study of single degree of freedom systems and only then proceeds to multidegree of freedom systems. The author's departure from this customary organization is prompted by his experience that this usual material organization leaves little time at the end of the semester for students to obtain a comfort level with the use of the modal transformation. Furthermore, the present organization provides more and better opportunities to apply the superior Lagrange equations and beam stiffness matrices to multidegree of freedom systems. This advantage in turn allows consideration of example structural systems that actually look like models for small structures as opposed to collections of rigid masses on wheels connected by elastic springs. Thus the vital link between structural analysis and structural dynamics is both maintained and evident.

The following are details of the textbook's organization. Chapters 1 and 2 provide a brief overview, or review, of only that portion of rigid body dynamics that is necessary to understand structural dynamics. Chapters 3 and 4 deal with writing the matrix equations of motion for undamped, discrete mass structural systems. Again, the elastic forces are described using mostly beam stiffness finite elements. Chapters 6 and 7 focus primarily on the modal method for solving those equations. Chapter 8 considers continuous mass structural systems as a means for providing further insight into discrete systems and as a means for demonstrating the serious difficulties often associated with continuous mass models. Numerical integration techniques, with or without modal transformations, are presented in Chapter 9.

Acknowledgments

The organization of this textbook mainly reflects the author's experience working at the Boeing Co., Beech Aircraft, and Lockheed Martin at various times. I acknowledge all the many people at those organizations from whom I have learned, as well as the helpful people at the places I have consulted and the universities where I was a student. I also thank my students, who, having endured earlier versions of this textbook, challenged me to prepare this material in a clear and concise manner.

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B.D.

List of Symbols

.	each dot placed above a symbol indicates one total differentiation with respect to time.
,	a comma as part of a subscript indicates partial differentiation with respect to all the variables that follow the comma.
′	each prime indicates one differentiation with respect to the single variable of the equation, usually a spatial variable.
[ ]	a square or rectangular matrix.
{ }	a column matrix.
⌊ ⌋	a row matrix; i.e., the transpose of a column matrix.
[\ \]	a diagonal (square) matrix.
a, b	with a single subscript, a coefficient of a power series expansion of (usually) a deflection function for a structural element.
a, b, c	general lengthwise dimensions or proportionality factors.
$\boldsymbol{a}$	general acceleration vector. A subscript indicates the acceleration of a particular mass particle.
c	a general damping coefficient such that the damping coefficient multiplied by the corresponding velocity produces a damping force. In brackets, the damping matrix.
c	a flexibility coefficient, which in general terms, is the inverse of a stiffness coefficient; with square brackets, a flexibility matrix; and with two subscripts, the row and column entry of that matrix identified by the subscripts.
c	an airfoil chord length; i.e., the streamwise distance between the airfoil leading edge and the airfoil trailing edge.
d	various distances.
e	subscript or superscript refers to an individual structural finite element.
e	eccentricity of an ellipse or an offset distance of a lumped mass from a finite element model node.

$\boldsymbol{e}_j$	position vector of the jth mass particle relative to the center of mass of the mass system.
f	the frequency of a vibration measured in hertz (Hz; cycles per second). This is not to be confused with the circular frequency, ω, which has units of radians per second.
f	a general mathematical function of engineering interest.
f	a force per unit length acting along the length of a beam or a friction force.
g	the acceleration of gravity.
g	with an argument, the step response function. See Section 7.5.
g	a fictitious material damping factor distinguished from the actual material damping factor symbolized by γ.
h	the vertical translation of a wing segment.
h	with an argument, the impulse response function. A subscript indicates the associated natural mode.
h, k	with subscripts, increments in deflection and velocity for various numerical methods of integrating differential equations.
$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$	fixed unit vectors aligned with the Cartesian coordinate system.
i, j, k	positive integer indices.
k	a stiffness coefficient for a single coiled spring or, more generally, an entry in the stiffness matrix of a spring or a more complicated structural element such as a beam or plate. In square brackets, a stiffness matrix.
l, ℓ	generally a beam segment length.
m	mass. A subscript indicates a particular mass particle or mass at the ith finite element node. In square brackets, a mass matrix.
m, n	positive integer indices.
p	in braces, the vector of modal deflections; with a subscript, an entry of such a vector.
$\boldsymbol{p}$, $\boldsymbol{q}$	orthogonal unit vectors in the z plane that, depending on the subscripts, rotate (positive counterclockwise) with either the center of mass of the mass system or a particular mass particle of the mass system. See Figure 1.4.
q	general symbol for a generalized coordinate (degree of freedom). In braces, a vector of generalized coordinates.
$\boldsymbol{r}$	a position vector; i.e., a vector that locates the position $[x(t), y(t), z(t)]$ of a mass or mass particle. In the latter case there is a subscript that indicates which mass particle.
r	a radial polar coordinate.
r	with a subscript, the ramp response function for the mode indicated by the subscript.

r	in brackets, a rotation matrix that is part of the Jacobi method or a variation on the Jacobi method.
s	entries in the "sweeping" matrix that relates (1) the generalized coordinate vector constrained to be orthogonal to the lower numbered mode shapes to (2) the unconstrained generalized coordinate vector.
s	with a subscript, the sine response function for the mode indicated by the subscript.
sgn( )	a function that has the value positive 1.0 when the argument is positive and the value negative 1.0 when its argument is negative.
stp( )	the Heaviside step function. See Section 7.5.
t	time.
t	the thickness of a thin beam cross section.
u, v, w	translational deflections in the x, y, z directions, respectively. With subscripts, such translations at the nodes of a finite element model.
$\boldsymbol{v}$	general velocity vector.
x, y, z	Cartesian coordinates.
A	a beam cross-sectional area.
A	in braces or within row matrix symbols, an eigenvector of the amplitudes of the natural vibration.
A_0	an aerodynamic coefficient equal to $\frac{1}{2} C_{l\alpha} \rho S$.
A, B, C	constants of integration or unknown amplitudes of a vibratory motion.
B	a general matrix or a coefficient matrix for the column matrix of generalized coordinates that yields the strains appropriate to the finite element.
BCs	abbreviation for boundary conditions.
C_l	an airfoil lift coefficient. See Eq. (7.16).
$\mathcal{C}(\kappa)$	The Theodorsen function. See the explanation following Eq. (7.20).
CG	abbreviation for center of gravity, which here is the same as center of mass.
D	a plate stiffness factor. See Example 8.8.
D	with a single subscript, a term associated with an initial deflection vector.
D	in brackets, a material stiffness matrix for an elastic material; i.e., a coefficient matrix for strains that yields the corresponding stresses.
D	in brackets, the system dynamic matrix; i.e., the product of the inverse of the stiffness matrix premultiplying the mass matrix when it is nonsymmetrical or the result of a transformation using a Cholesky decomposition when it is symmetrical.
DOF	abbreviation for degrees of freedom.

E Young's modulus; i.e., the slope of the straight-line portion of the stress–strain curve for a structural material loaded in tension or compression.

E error term.

$\mathcal{E}$ total mechanical energy of a structural system.

$\boldsymbol{F}$ general force vector. An *ex* superscript indicates forces external to the mass system under consideration. An *in* superscript indicates forces internal to the mass system. A subscript indicates a force acting on a particular mass particle.

$\mathcal{F}$ the magnitude of an impulse; i.e., the integral of a short duration force over time.

F, G general mathematical functions of engineering interest.

G the shear modulus; i.e., the slope of the straight-line portion of the stress–strain curve for a structural material subjected to a shear loading.

G the universal gravitational constant.

H a general symbol for mass moment of inertia of a mass system about a point or an axis indicated, respectively, by the single or double subscript. As a "second moment," it is the sum of each mass particle or differential sized mass multiplied by the square of the distance from the point or axis indicated to the mass particle or differential mass.

H with a subscript, the complex frequency response function associated with the mode indicated by the subscript.

I a general symbol for the area moment of inertia of a beam cross section. Double subscripts indicate the centroidal axis about which the second moment of area is calculated. A p subscript indicates a polar moment of inertia.

I the value of an integral that is to be optimized or evaluated.

I in brackets, the identity matrix.

J for a beam cross section, the St. Venant constant for uniform torsion. It is equal to the cross-sectional area polar moment of inertia only in the case of circular and annular cross sections.

J with a subscript and an argument, a Bessel function of the first kind. A subscript indicates the order.

K in brackets, the stiffness matrix for an entire structural system that is composed of the compatible sum of the stiffness matrices of the individual structural elements. With subscripts, a submatrix of the total stiffness matrix.

K a torsional spring constant; i.e., the proportionality factor multiplying the twist in the spring that yields the moment necessary to achieve that twist.

L a general symbol for length, usually the length of a beam or beam segment.

L in brackets, the lower (or left) triangular matrix of a Cholesky decomposition.

$\boldsymbol{L}$ general angular momentum (moment of momentum) vector. Subscripts can indicate the point about which the moment arm is measured, or the mass particle under consideration.

$\mathcal{L}$ an aerodynamic lift force.

M an externally applied moment, or the internal moment stress resultant for a beam cross section.

M In brackets, a mass matrix.

$\mathcal{M}$ an aerodynamic moment; i.e., the moment acting on a wing segment because of the surrounding airflow.

$\boldsymbol{M}$ general moment vector.

N the axial force in a beam or a bar of a truss.

N with a single subscript, a "shape function" that, together with a generalized coordinate of a finite element, describes the deflections of the finite element associated with that generalized coordinate.

$O[\,]$ order of magnitude of the quantity within the brackets.

P in braces, the vector of applied modal forces; with a subscript, an entry of such a vector.

$\boldsymbol{P}$ general momentum vector equal to the scalar mass value multiplied by the velocity vector.

Q general symbol for a generalized force. Subscripts often indicate the corresponding generalized coordinate.

R general symbol for a support reaction, either a force reaction or a moment reaction.

R in brackets, a right triangular matrix of a Cholesky decomposition.

R in brackets, a coefficient matrix that relates one set of generalized coordinates to a second set of generalized coordinates that is rotated through one or more angles relative to the first set of generalized coordinates.

R a principal radius of curvature of a curved beam.

S the planform area of a wing segment.

S in brackets, a "sweeping" matrix that removes the presence of lower numbered modes from the system dynamic matrix.

T kinetic energy.

T the time period of one vibratory cycle.

T a matrix that transforms one set of generalized coordinates into another set of generalized coordinates.

U strain energy; i.e., the recoverable energy stored in an elastic system because of the deformation of that system.

U, V, W with an argument, a vibration amplitude function of a system with a continuous mass distribution.

V	a general potential energy other than strain energy.
V	the shearing force acting on a beam cross section. Subscripts indicate the direction and lengthwise position of the shearing force.
V	with a single subscript, a term associated with a system initial velocity vector.
V	the airspeed of the airfoil.
W	work, either the product of force acting through a translational displacement or moment acting through a rotational displacement. Superscripts and subscripts indicate the type of forces or moments doing the work, such as internal or external to the mass system, or energy conservative or nonconservative.
X	the x Cartesian coordinate nondimensionalized by division by the length of a beam segment.
X, Y	the horizontal and vertical components of an internal bar force.
Y	with a subscript and argument, a Bessel function of the second kind.
α	an alternate polar coordinate.
α, β	various angles, parameters, or proportionality factors.
γ	general symbol for angular or shearing strain, positive when the reference right angle decreases. Two differing Cartesian coordinate subscripts indicate the two coordinate axes forming the original right angle.
γ	nondimensional material damping factor.
γ	when used as a multiplier of an area term, an area correction factor that attempts to account for the variation of shearing stresses and strains over a beam cross section.
δ	the variational operator which (here) always precedes a function of deflections. When applied to quantities of engineering interest, such as work W, the resulting engineering interpretation is that of a "virtual" quantity, which in this case is virtual work.
δ	the Dirac delta function, which always has an associated spatial or temporal argument. The argument is always the difference between the variable and a related parameter. The latter is sometimes zero, in which case the argument contains only the variable. See Section 7.5.
ϵ	general symbol for normal strains (changes in length because of deformation divided by the original length). Two repeated subscripts indicate the direction in which the strain is measured. Elongations are positive.
ζ	a nondimensionalized value of the damping coefficient.
θ	various angles or an angular generalized coordinate at the node of a finite element model.
κ	the "reduced frequency"; a nondimensional frequency or nondimensional airspeed equal to $c\,\omega/(2\,V)$, where c is the airfoil chord length.

λ — a general symbol for an eigenvalue or, occasionally, a parameter having the same units as the system eigenvalues.

μ — coefficient of Coulomb friction, a mass ratio, or a coefficient of Duffing's equation.

ν — Poisson's ratio.

ρ — mass density; i.e., mass divided by volume.

ρ — an alternate polar coordinate.

σ — general symbol for both normal and shearing stresses. The same double subscripts indicate a normal stress in that Cartesian coordinate direction, whereas the first of two unlike subscripts indicates the plane on which the shearing stress acts, and the second of the two unlike subscripts indicates the direction in which the shearing stress acts.

τ — a value of the time variable different from another measure of time, t. Usually used as a parametric value.

ϕ, ψ — various angles or finite element nodal rotations.

ω — a given angular velocity, or more commonly here, a circular frequency of vibration having the units of radians per second, and therefore equal to $2\pi f$, where f is the frequency of the vibration measured in units of cycles per second.

Γ — with a subscript, the participation factor for the mode shape indicated by the subscript.

Δ — an operator that indicates a small increment in the quantity to which it is applied.

Θ — the fixed amplitude of the vibratory generalized coordinate θ.

Λ — a general symbol for the matrix of eigenvalues of another matrix or the vibratory system.

Π — the magnitude of an impulsive force expressed in modal coordinate terms.

Υ — an amplitude of a forced motion.

Φ — a matrix of all, or selected, eigenvectors.

Ω — the ratio of a forcing frequency to a natural frequency where the subscript indicates which natural frequency. Without a subscript, the natural frequency is the first natural frequency.