ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

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Combinatorial Matrix Classes

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ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS

Combinatorial Matrix Classes

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Preface

In the preface of the book *Combinatorial Matrix Theory*[1] (CMT) I discussed my plan to write a second volume entitled *Combinatorial Matrix Classes*. Here 15 years later (including 6, to my mind, wonderful years as Department of Mathematics Chair at UW-Madison), and to my great relief, is the finished product. What I proposed as topics to be covered in a second volume were, in retrospect, much too ambitious. Indeed, after some distance from the first volume, it now seems like a plan for a book series rather than for a second volume. I decided to concentrate on topics that I was most familiar with and that have been a source of much research inspiration for me. Having made this decision, there was more than enough basic material to be covered. Most of the material in the book has never appeared in book form, and as a result, I hope that it will be useful to both current researchers and aspirant researchers in the field. I have tried to be as complete as possible with those matrix classes that I have treated, and thus I also hope that the book will be a useful reference book.

I started the serious writing of this book in the summer of 2000 and continued, while on sabbatical, through the following semester. I made good progress during those six months. Thereafter, with my many teaching, research, editorial, and other professional and university responsibilities, I managed to work on the book only sporadically. But after 5 years, I was able to complete it or, if one considers the topics mentioned in the preface of CMT, one might say I simply stopped writing. But that is not the way I feel. I think, and I hope others will agree, that the collection of matrix classes developed in the book fit together nicely and indeed form a coherent whole with no glaring omissions. Except for a few reference to CMT, the book is self-contained.

My primary inspiration for combinatorial matrix classes has come from two important contributors, Herb Ryser and Ray Fulkerson. In a real sense, with their seminal and early research, they are the "fathers" of the subject. Herb Ryser was my thesis advisor and I first learned about the class $\mathcal{A}(R, S)$, which occupies a very prominent place in this book, in the fall of 1962 when I was a graduate student at Syracuse University (New York).

[1] Authored by Richard A. Brualdi and Herbert J. Ryser and published by Cambridge University Press in 1991.

In addition, some very famous mathematicians have made seminal contributions that have directly or indirectly impacted the study of matrix classes. With the great risk of offending someone, let me mention only Claude Berge, Garrett Birkhoff, David Gale, Alan Hoffman, D. König, Victor Klee, Donald Knuth, H.G. Landau, Leon Mirsky, and Bill Tutte. To these people, and all others who have contributed, I bow my head and say a heartfelt thank-you for your inspiration.

As I write this preface in the summer of 2005, I have just finished my 40th year as a member of the Department of Mathematics of the University of Wisconsin in Madison. I have been fortunate in my career to be a member of a very congenial department that, by virtue of its faculty and staff, provides such a wonderful atmosphere in which to work, and that takes teaching, research, and service all very seriously. It has also been my good fortune to have collaborated with my graduate students, and postdoctoral fellows, over the years, many of whom have contributed to one or more of the matrix classes treated in this book. I am indebted to Geir Dahl who read a good portion of this book and provided me with valuable comments.

My biggest source of support these last 10 years has been my wife Mona. Her encouragement and love have been so important to me.

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