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978-0-521-86559-3 - Learning Theory: An Approximation Theory Viewpoint

Felipe Cucker and Ding-Xuan Zhou

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