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0521865549 - Multiple Scattering: Interaction of Time-Harmonic Waves with N Obstacles

P. A. Martin

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Volume 107

Multiple Scattering

The interaction of waves with obstacles is an everyday phenomenon in science and engineering, cropping up for example in acoustics, electromagnetism, seismology and hydrodynamics. The mathematical theory and technology needed to understand the phenomenon is known as multiple scattering, and this book is the first devoted to the subject. The author covers a variety of techniques, for example separation of variables, T -matrix and integral equation methods, describing first the single-obstacle method and then extending it to the multiple-obstacle case. A key ingredient in many of these extensions is an appropriate addition theorem: a coherent, thorough exposition of these theorems is given, and computational and numerical issues around them are explored.

The application of these methods to different types of problems is also explained. In particular, sound waves, electromagnetic radiation, waves in solids and water waves. A comprehensive reference list of some 1400 items rounds off the book, which will be an essential reference on the topic for applied mathematicians, physicists and engineers.

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Interaction of Time-Harmonic Waves with N Obstacles

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To Ruth, Richard, Frances and, last but not least, Ann

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Preface

It has been said that ‘a true scholar publishes only one book in his lifetime and that posthumously’ [105, p. 435]. In fact, this book has not taken a lifetime to complete, although it has had a very long gestation period: there have been many distractions, most of which have not been unpleasant.

The book is concerned with the scattering of time-harmonic waves by obstacles; the words ‘multiple scattering’ in the title signify that there are at least two obstacles. The problems considered come from acoustics (sound waves, Helmholtz equation), electromagnetics (Maxwell’s equations), elastodynamics (waves in solids) and hydrodynamics (surface water waves). The book describes a variety of mathematical techniques for solving such problems: the main techniques involve separation of variables, integral equations and T -matrices. Most of the book is concerned with exact methods, although the last chapter discusses several effective approximate methods. There are also two chapters on addition theorems; these are useful in other contexts as well as for multiple scattering. For detailed information on the topics covered, see the Table of Contents and Section 1.7; for a list of topics *not* covered, see Section 1.2.

The mathematics used is classical: separation of variables, special functions, Green’s functions, Fourier methods, asymptotics. The reader should also have some familiarity with simple uses of boundary integral equations. Operator notation is used when convenient.

As far as I know, there is no other book that treats all four of the main physical domains: acoustics, electromagnetics, elastodynamics and hydrodynamics. The reader will see many connections between these domains. As far as I know, there is no other book that focuses on multiple scattering. (Of course, it is inevitable that we begin by considering scattering by one obstacle.) In addition, the book has an extensive reference list. This should be useful to future workers, and may help reduce duplication of effort.

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The book contains a sprinkling of quotations from the literature. These are intended to be illuminating, amusing or both. Some may help the reader to not lose heart when the calculations become complicated; some may hint that great men are human; and some may simply reflect the author's English sense of humour!

I cannot give thanks individually to everybody who has helped me over the years, perhaps by sending me reprints or answering my questions. However, I must thank four people for shaping my outlook and taste in applied mathematics: Ralph Kleinman, Frank Rizzo, Fritz Ursell and Gerry Wickham. I also thank Chris Linton for his detailed comments on an early draft. No doubt some errors remain, for which I must take sole responsibility. Please let me know if you find errors. In particular, if you think that I should have cited one of your papers, send me a copy!