

PROBABILITY AND RANDOM PROCESSES FOR ELECTRICAL AND COMPUTER ENGINEERS

The theory of probability is a powerful tool that helps electrical and computer engineers explain, model, analyze, and design the technology they develop. The text begins at the advanced undergraduate level, assuming only a modest knowledge of probability, and progresses through more complex topics mastered at the graduate level. The first five chapters cover the basics of probability and both discrete and continuous random variables. The later chapters have a more specialized coverage, including random vectors, Gaussian random vectors, random processes, Markov Chains, and convergence. Describing tools and results that are used extensively in the field, this is more than a textbook: it is also a reference for researchers working in communications, signal processing, and computer network traffic analysis. With over 300 worked examples, some 800 homework problems, and sections for exam preparation, this is an essential companion for advanced undergraduate and graduate students.

Further resources for this title, including solutions, are available online at www.cambridge.org/9780521864701.

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JOHN A. GUBNER

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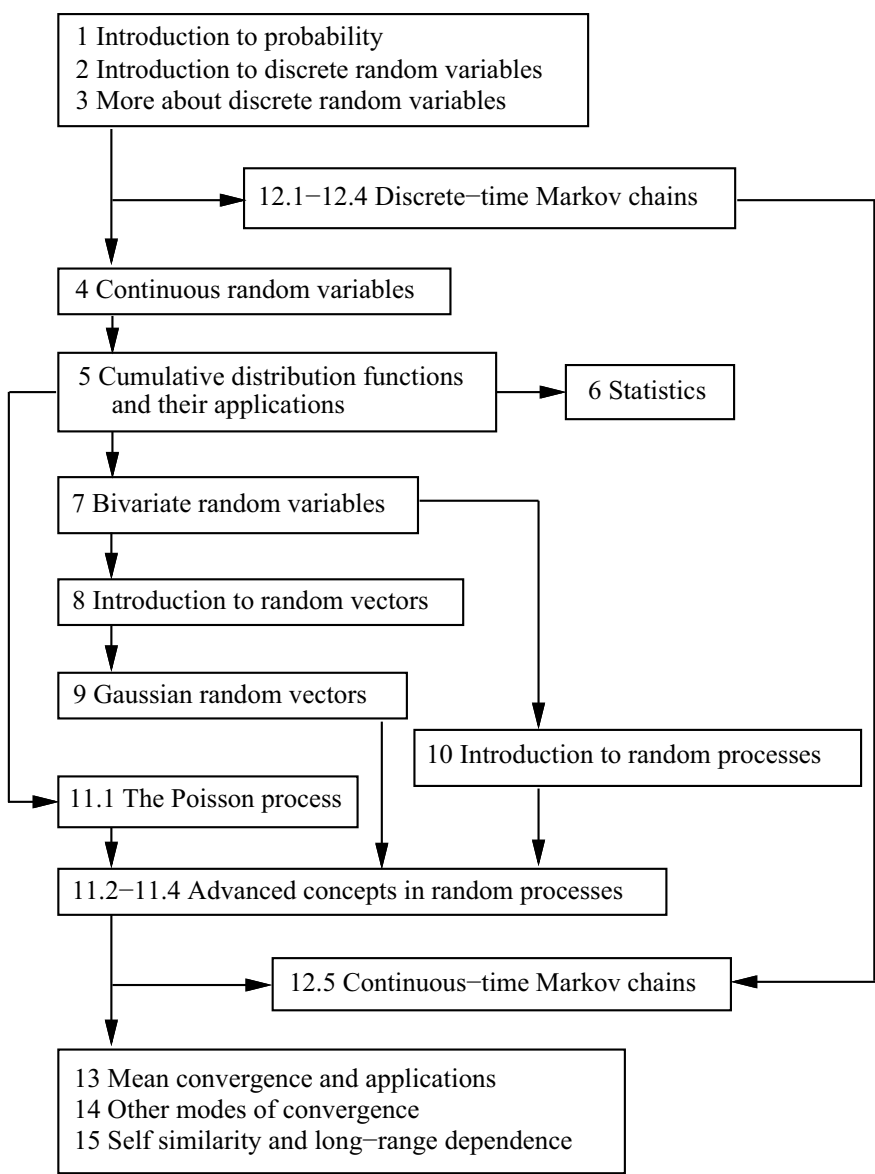
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Chapter dependencies



Preface

Intended audience

This book is a primary text for **graduate-level courses** in probability and random processes that are typically offered in electrical and computer engineering departments. The text starts from first principles and contains more than enough material for a two-semester sequence. The **level of the text** varies from advanced undergraduate to graduate as the material progresses. The principal **prerequisite** is the usual undergraduate electrical and computer engineering course on signals and systems, e.g., Haykin and Van Veen [25] or Oppenheim and Willsky [39] (see the Bibliography at the end of the book). However, later chapters that deal with random vectors assume some familiarity with linear algebra; e.g., determinants and matrix inverses.

How to use the book

A first course. In a course that assumes at most a modest background in probability, the core of the offering would include Chapters 1–5 and 7. These cover the basics of probability and discrete and continuous random variables. As the chapter dependencies graph on the preceding page indicates, there is considerable flexibility in the selection and ordering of additional material as the instructor sees fit.

A second course. In a course that assumes a solid background in the basics of probability and discrete and continuous random variables, the material in Chapters 1–5 and 7 can be reviewed quickly. In such a review, the instructor may want include **sections and problems marked with a ***, as these indicate more challenging material that might not be appropriate in a first course. Following the review, the core of the offering would include Chapters 8, 9, 10 (Sections 10.1–10.6), and Chapter 11. Additional material from Chapters 12–15 can be included to meet course goals and objectives.

Level of course offerings. In any course offering, the level can be adapted to the background of the class by omitting or including the **more advanced sections, remarks, and problems that are marked with a ***. In addition, discussions of a highly technical nature are placed in a Notes section at the end of the chapter in which they occur. Pointers to these discussions are indicated by **boldface numerical superscripts** in the text. These notes can be omitted or included as the instructor sees fit.

Chapter features

- Key equations are boxed:

$$P(A|B) := \frac{P(A \cap B)}{P(B)}.$$

- Important text passages are highlighted:

Two events A and B are said to be independent if $P(A \cap B) = P(A)P(B)$.

- Tables of discrete random variables and of Fourier transform pairs are found inside the front cover. A table of continuous random variables is found inside the back cover.
- The index was compiled as the book was written. Hence, there are many cross-references to related information. For example, see “chi-squared random variable.”
- When cumulative distribution functions or other functions are encountered that do not have a closed form, MATLAB commands are given for computing them; see “Matlab commands” in the index for a list. The use of many commands is illustrated in the examples and the problems throughout most of the text. Although some commands require the MATLAB Statistics Toolbox, alternative methods are also suggested; e.g., the use of `erf` and `erfinv` for `normcdf` and `norminv`.
- Each chapter contains a **Notes** section. Throughout each chapter, numerical superscripts refer to discussions in the Notes section. These notes are usually rather technical and address subtleties of the theory.
- Each chapter contains a **Problems** section. There are more than 800 problems throughout the book. Problems are grouped according to the section they are based on, and this is clearly indicated. This enables the student to refer to the appropriate part of the text for background relating to particular problems, and it enables the instructor to make up assignments more quickly. In chapters intended for a first course, **the more challenging problems are marked with a ***. Problems requiring MATLAB are indicated by the label **MATLAB**.
- Each chapter contains an **Exam preparation** section. This serves as a chapter summary, drawing attention to key concepts and formulas.

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