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978-0-521-86300-1 - Markov Processes, Gaussian Processes, and Local Times

Michael B. Marcus and Jay Rosen

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