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978-0-521-86300-1 - Markov Processes, Gaussian Processes, and Local Times

Michael B. Marcus and Jay Rosen

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MARKOV PROCESSES, GAUSSIAN PROCESSES, AND LOCAL TIMES

Written by two of the foremost researchers in the field, this book studies the local times of Markov processes by employing isomorphism theorems that relate them to certain associated Gaussian processes. It builds to this material through self-contained but harmonized “mini-courses” on the relevant ingredients, which assume only knowledge of measure-theoretic probability. The streamlined selection of topics creates an easy entrance for students and experts in related fields.

The book starts by developing the fundamentals of Markov process theory and then of Gaussian process theory, including sample path properties. It then proceeds to more advanced results, bringing the reader to the heart of contemporary research. It presents the remarkable isomorphism theorems of Dynkin and Eisenbaum and then shows how they can be applied to obtain new properties of Markov processes by using well-established techniques in Gaussian process theory. This original, readable book will appeal to both researchers and advanced graduate students.

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CAMBRIDGE UNIVERSITY PRESS
 Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
 32 Avenue of the Americas, New York, NY 10013-2473, USA
www.cambridge.org
 Information on this title: www.cambridge.org/9780521863001

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First published 2006

Printed in the United States of America

A catalog record for this publication is available from the British Library.

Library of Congress Cataloging in Publication Data

Marcus, Michael B.
 Markov processes, Gaussian processes, and local times / Michael B. Marcus, Jay Rosen.
 p. cm. – (Cambridge studies in advanced mathematics ; 100)

Includes bibliographical references and index.

ISBN-13: 978-0-521-86300-1 (hardback)

ISBN-10: 0-521-86300-7 (hardback)

1. Markov processes. 2. Gaussian processes. 3. Local times (Stochastic processes)

I. Rosen, Jay, 1948– II. Title. III. Series.

QA274.7.M35 2006

519.2'33–dc22 2006042511

ISBN-13 978-0-521-86300-1 hardback

ISBN-10 0-521-86300-7 hardback

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To our wives

Jane Marcus

and

Sara Rosen

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