## Contents

Preface to the third edition .............................. page xx  
Preface to the second edition .................................. xxiii  
Preface to the first edition .................................. xxv  

1 Preliminary algebra  ........................................ 1  
1.1 Simple functions and equations ....................... 1  
  Polynomial equations; factorisation; properties of roots  
1.2 Trigonometric identities .................................. 10  
  Single angle; compound angles; double- and half-angle identities  
1.3 Coordinate geometry ..................................... 15  
1.4 Partial fractions ......................................... 18  
  Complications and special cases  
1.5 Binomial expansion ...................................... 25  
1.6 Properties of binomial coefficients .................... 27  
1.7 Some particular methods of proof ...................... 30  
  Proof by induction; proof by contradiction; necessary and sufficient conditions  
1.8 Exercises .................................................. 36  
1.9 Hints and answers ....................................... 39  

2 Preliminary calculus ....................................... 41  
2.1 Differentiation ........................................... 41  
  Differentiation from first principles; products; the chain rule; quotients; \n  implicit differentiation; logarithmic differentiation; Leibnitz' theorem; special \n  points of a function; curvature; theorems of differentiation
## CONTENTS

2.2 Integration 59
Integration from first principles; the inverse of differentiation; by inspection; sinusoidal functions; logarithmic integration; using partial fractions; substitution method; integration by parts; reduction formulae; infinite and improper integrals; plane polar coordinates; integral inequalities; applications of integration

2.3 Exercises 76

2.4 Hints and answers 81

3 Complex numbers and hyperbolic functions 83
3.1 The need for complex numbers 83
3.2 Manipulation of complex numbers 85
Addition and subtraction; modulus and argument; multiplication; complex conjugate; division
3.3 Polar representation of complex numbers 92
Multiplication and division in polar form
3.4 de Moivre’s theorem 95
Trigonometric identities; finding the nth roots of unity; solving polynomial equations
3.5 Complex logarithms and complex powers 99
3.6 Applications to differentiation and integration 101
3.7 Hyperbolic functions 102
Definitions; hyperbolic–trigonometric analogies; identities of hyperbolic functions; solving hyperbolic equations; inverses of hyperbolic functions; calculus of hyperbolic functions
3.8 Exercises 109
3.9 Hints and answers 113

4 Series and limits 115
4.1 Series 115
4.2 Summation of series 116
Arithmetic series; geometric series; arithmetico-geometric series; the difference method; series involving natural numbers; transformation of series
4.3 Convergence of infinite series 124
Absolute and conditional convergence; series containing only real positive terms; alternating series test
4.4 Operations with series 131
4.5 Power series 131
Convergence of power series; operations with power series
4.6 Taylor series 136
Taylor’s theorem; approximation errors; standard Maclaurin series
4.7 Evaluation of limits 141
4.8 Exercises 144
4.9 Hints and answers 149
## Table of Contents

5 Partial differentiation 151
  5.1 Definition of the partial derivative 151
  5.2 The total differential and total derivative 153
  5.3 Exact and inexact differentials 155
  5.4 Useful theorems of partial differentiation 157
  5.5 The chain rule 157
  5.6 Change of variables 158
  5.7 Taylor’s theorem for many-variable functions 160
  5.8 Stationary values of many-variable functions 162
  5.9 Stationary values under constraints 167
  5.10 Envelopes 173
  5.11 Thermodynamic relations 176
  5.12 Differentiation of integrals 178
  5.13 Exercises 179
  5.14 Hints and answers 185

6 Multiple integrals 187
  6.1 Double integrals 187
  6.2 Triple integrals 190
  6.3 Applications of multiple integrals 191
    Areas and volumes; masses, centres of mass and centroids; Pappus’ theorems; moments of inertia; mean values of functions
  6.4 Change of variables in multiple integrals 199
    Change of variables in double integrals; evaluation of the integral \( \int_{-\infty}^{\infty} e^{-x^2} \, dx \); change of variables in triple integrals; general properties of Jacobians
  6.5 Exercises 207
  6.6 Hints and answers 211

7 Vector algebra 212
  7.1 Scalars and vectors 212
  7.2 Addition and subtraction of vectors 213
  7.3 Multiplication by a scalar 214
  7.4 Basis vectors and components 217
  7.5 Magnitude of a vector 218
  7.6 Multiplication of vectors 219
    Scalar product; vector product; scalar triple product; vector triple product
## CONTENTS

7.7 Equations of lines, planes and spheres 226
7.8 Using vectors to find distances 229
Point to line; point to plane; line to line; line to plane
7.9 Reciprocal vectors 233
7.10 Exercises 234
7.11 Hints and answers 240

8 Matrices and vector spaces 241
8.1 Vector spaces 242
Basis vectors; inner product; some useful inequalities
8.2 Linear operators 247
8.3 Matrices 249
8.4 Basic matrix algebra 250
Matrix addition; multiplication by a scalar; matrix multiplication
8.5 Functions of matrices 255
8.6 The transpose of a matrix 255
8.7 The complex and Hermitian conjugates of a matrix 256
8.8 The trace of a matrix 258
8.9 The determinant of a matrix 259
Properties of determinants
8.10 The inverse of a matrix 263
8.11 The rank of a matrix 267
8.12 Special types of square matrix 268
Diagonal; triangular; symmetric and antisymmetric; orthogonal; Hermitian and anti-Hermitian; unitary; normal
8.13 Eigenvectors and eigenvalues 272
Of a normal matrix; of Hermitian and anti-Hermitian matrices; of a unitary matrix; of a general square matrix
8.14 Determination of eigenvalues and eigenvectors 280
Degenerate eigenvalues
8.15 Change of basis and similarity transformations 282
8.16 Diagonalisation of matrices 285
8.17 Quadratic and Hermitian forms 288
Stationary properties of the eigenvectors; quadratic surfaces
8.18 Simultaneous linear equations 292
Range; null space; N simultaneous linear equations in N unknowns; singular value decomposition
8.19 Exercises 307
8.20 Hints and answers 314

9 Normal modes 316
9.1 Typical oscillatory systems 317
9.2 Symmetry and normal modes 322
9.3 Rayleigh–Ritz method 327
9.4 Exercises 329
9.5 Hints and answers 332

10 Vector calculus 334
10.1 Differentiation of vectors 334
10.2 Integration of vectors 339
10.3 Space curves 340
10.4 Vector functions of several arguments 344
10.5 Surfaces 345
10.6 Scalar and vector fields 347
10.7 Vector operators 347
10.8 Vector operator formulae 354
10.9 Cylindrical and spherical polar coordinates 357
10.10 General curvilinear coordinates 364
10.11 Exercises 369
10.12 Hints and answers 375

11 Line, surface and volume integrals 377
11.1 Line integrals 377
11.2 Connectivity of regions 383
11.3 Green’s theorem in a plane 384
11.4 Conservative fields and potentials 387
11.5 Surface integrals 389
11.6 Volume integrals 396
11.7 Integral forms for grad, div and curl 398
11.8 Divergence theorem and related theorems 401
11.9 Stokes’ theorem and related theorems 406
11.10 Exercises 409
11.11 Hints and answers 414

12 Fourier series 415
12.1 The Dirichlet conditions 415
12.2 The Fourier coefficients 417
12.3 Symmetry considerations 419
12.4 Discontinuous functions 420
12.5 Non-periodic functions 422
12.6 Integration and differentiation 424
12.7 Complex Fourier series 424
12.8 Parseval's theorem 426
12.9 Exercises 427
12.10 Hints and answers 431

13 Integral transforms 433
13.1 Fourier transforms 433
   The uncertainty principle; Fraunhofer diffraction; the Dirac δ-function;
   relation of the δ-function to Fourier transforms; properties of Fourier
   transforms; odd and even functions; convolution and deconvolution; correlation
   functions and energy spectra; Parseval's theorem; Fourier transforms in higher
   dimensions
13.2 Laplace transforms 453
   Laplace transforms of derivatives and integrals; other properties of Laplace
   transforms
13.3 Concluding remarks 459
13.4 Exercises 460
13.5 Hints and answers 466

14 First-order ordinary differential equations 468
14.1 General form of solution 469
14.2 First-degree first-order equations 470
   Separable-variable equations; exact equations; inexact equations, integrat-
   ing factors; linear equations; homogeneous equations; isobaric equations;
   Bernoulli's equation; miscellaneous equations
14.3 Higher-degree first-order equations 480
   Equations soluble for p, for x; for y; Clairaut's equation
14.4 Exercises 484
14.5 Hints and answers 488

15 Higher-order ordinary differential equations 490
15.1 Linear equations with constant coefficients 492
   Finding the complementary function γ(x); finding the particular integral
   γ(x); constructing the general solution γ(x) + γ(x); linear recurrence
   relations; Laplace transform method
15.2 Linear equations with variable coefficients 503
   The Legendre and Euler linear equations; exact equations; partially known
   complementary function; variation of parameters; Green's functions; canonical
   form for second-order equations
## CONTENTS

15.3 General ordinary differential equations 518  
Dependent variable absent; independent variable absent; non-linear exact  
equations; isobaric or homogeneous equations; equations homogeneous in $x$  
or $y$ alone; equations having $y = Ae^x$ as a solution  
15.4 Exercises 523  
15.5 Hints and answers 529  

16 Series solutions of ordinary differential equations 531  
16.1 Second-order linear ordinary differential equations 531  
Ordinary and singular points  
16.2 Series solutions about an ordinary point 535  
16.3 Series solutions about a regular singular point 538  
Distinct roots not differing by an integer; repeated root of the indicial  
equation; distinct roots differing by an integer  
16.4 Obtaining a second solution 544  
The Wronskian method; the derivative method; series form of the second  
solution  
16.5 Polynomial solutions 548  
16.6 Exercises 550  
16.7 Hints and answers 553  

17 Eigenfunction methods for differential equations 554  
17.1 Sets of functions 556  
Some useful inequalities  
17.2 Adjoint, self-adjoint and Hermitian operators 559  
17.3 Properties of Hermitian operators 561  
Reality of the eigenvalues; orthogonality of the eigenfunctions; construction  
of real eigenfunctions  
17.4 Sturm–Liouville equations 564  
Valid boundary conditions; putting an equation into Sturm–Liouville form  
17.5 Superposition of eigenfunctions: Green’s functions 569  
17.6 A useful generalisation 572  
17.7 Exercises 573  
17.8 Hints and answers 576  

18 Special functions 577  
18.1 Legendre functions 577  
General solution for integer $\ell$; properties of Legendre polynomials  
18.2 Associated Legendre functions 587  
18.3 Spherical harmonics 593  
18.4 Chebyshev functions 595  
18.5 Bessel functions 602  
General solution for non-integer $\nu$; general solution for integer $\nu$; properties  
of Bessel functions
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.6</td>
<td>Spherical Bessel functions</td>
<td>614</td>
</tr>
<tr>
<td>18.7</td>
<td>Laguerre functions</td>
<td>616</td>
</tr>
<tr>
<td>18.8</td>
<td>Associated Laguerre functions</td>
<td>621</td>
</tr>
<tr>
<td>18.9</td>
<td>Hermite functions</td>
<td>624</td>
</tr>
<tr>
<td>18.10</td>
<td>Hypergeometric functions</td>
<td>628</td>
</tr>
<tr>
<td>18.11</td>
<td>Confluent hypergeometric functions</td>
<td>633</td>
</tr>
<tr>
<td>18.12</td>
<td>The gamma function and related functions</td>
<td>635</td>
</tr>
<tr>
<td>18.13</td>
<td>Exercises</td>
<td>640</td>
</tr>
<tr>
<td>18.14</td>
<td>Hints and answers</td>
<td>646</td>
</tr>
<tr>
<td>19</td>
<td>Quantum operators</td>
<td>648</td>
</tr>
<tr>
<td>19.1</td>
<td>Operator formalism</td>
<td>648</td>
</tr>
<tr>
<td>19.2</td>
<td>Physical examples of operators</td>
<td>656</td>
</tr>
<tr>
<td>19.3</td>
<td>Exercises</td>
<td>671</td>
</tr>
<tr>
<td>19.4</td>
<td>Hints and answers</td>
<td>674</td>
</tr>
<tr>
<td>20</td>
<td>Partial differential equations: general and particular solutions</td>
<td>675</td>
</tr>
<tr>
<td>20.1</td>
<td>Important partial differential equations</td>
<td>676</td>
</tr>
<tr>
<td>20.2</td>
<td>General form of solution</td>
<td>680</td>
</tr>
<tr>
<td>20.3</td>
<td>General and particular solutions</td>
<td>681</td>
</tr>
<tr>
<td>20.4</td>
<td>The wave equation</td>
<td>693</td>
</tr>
<tr>
<td>20.5</td>
<td>The diffusion equation</td>
<td>695</td>
</tr>
<tr>
<td>20.6</td>
<td>Characteristics and the existence of solutions</td>
<td>699</td>
</tr>
<tr>
<td>20.7</td>
<td>Uniqueness of solutions</td>
<td>705</td>
</tr>
<tr>
<td>20.8</td>
<td>Exercises</td>
<td>707</td>
</tr>
<tr>
<td>20.9</td>
<td>Hints and answers</td>
<td>711</td>
</tr>
<tr>
<td>21</td>
<td>Partial differential equations: separation of variables and other methods</td>
<td>713</td>
</tr>
<tr>
<td>21.1</td>
<td>Separation of variables: the general method</td>
<td>713</td>
</tr>
<tr>
<td>21.2</td>
<td>Superposition of separated solutions</td>
<td>717</td>
</tr>
<tr>
<td>21.3</td>
<td>Separation of variables in polar coordinates</td>
<td>725</td>
</tr>
<tr>
<td>21.4</td>
<td>Integral transform methods</td>
<td>747</td>
</tr>
<tr>
<td>CONTENTS</td>
<td>PAGE</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>21.5 Inhomogeneous problems – Green’s functions</td>
<td>751</td>
<td></td>
</tr>
<tr>
<td>Similarities to Green’s functions for ordinary differential equations;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>general boundary-value problems; Dirichlet problems; Neumann problems</td>
<td></td>
<td></td>
</tr>
<tr>
<td>21.6 Exercises</td>
<td>767</td>
<td></td>
</tr>
<tr>
<td>21.7 Hints and answers</td>
<td>773</td>
<td></td>
</tr>
<tr>
<td>22 Calculus of variations</td>
<td>775</td>
<td></td>
</tr>
<tr>
<td>22.1 The Euler–Lagrange equation</td>
<td>776</td>
<td></td>
</tr>
<tr>
<td>22.2 Special cases</td>
<td>777</td>
<td></td>
</tr>
<tr>
<td>$F$ does not contain $y$ explicitly; $F$ does not contain $x$ explicitly</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.3 Some extensions</td>
<td>781</td>
<td></td>
</tr>
<tr>
<td>Several dependent variables; several independent variables; higher-order</td>
<td></td>
<td></td>
</tr>
<tr>
<td>derivatives; variable end-points</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.4 Constrained variation</td>
<td>785</td>
<td></td>
</tr>
<tr>
<td>22.5 Physical variational principles</td>
<td>787</td>
<td></td>
</tr>
<tr>
<td>Fermat’s principle in optics; Hamilton’s principle in mechanics</td>
<td></td>
<td></td>
</tr>
<tr>
<td>22.6 General eigenvalue problems</td>
<td>790</td>
<td></td>
</tr>
<tr>
<td>22.7 Estimation of eigenvalues and eigenfunctions</td>
<td>792</td>
<td></td>
</tr>
<tr>
<td>22.8 Adjustment of parameters</td>
<td>795</td>
<td></td>
</tr>
<tr>
<td>22.9 Exercises</td>
<td>797</td>
<td></td>
</tr>
<tr>
<td>22.10 Hints and answers</td>
<td>801</td>
<td></td>
</tr>
<tr>
<td>23 Integral equations</td>
<td>803</td>
<td></td>
</tr>
<tr>
<td>23.1 Obtaining an integral equation from a differential equation</td>
<td>803</td>
<td></td>
</tr>
<tr>
<td>23.2 Types of integral equation</td>
<td>804</td>
<td></td>
</tr>
<tr>
<td>23.3 Operator notation and the existence of solutions</td>
<td>805</td>
<td></td>
</tr>
<tr>
<td>23.4 Closed-form solutions</td>
<td>806</td>
<td></td>
</tr>
<tr>
<td>Separable kernels; integral transform methods; differentiation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>23.5 Neumann series</td>
<td>813</td>
<td></td>
</tr>
<tr>
<td>23.6 Fredholm theory</td>
<td>815</td>
<td></td>
</tr>
<tr>
<td>23.7 Schmidt–Hilbert theory</td>
<td>816</td>
<td></td>
</tr>
<tr>
<td>23.8 Exercises</td>
<td>819</td>
<td></td>
</tr>
<tr>
<td>23.9 Hints and answers</td>
<td>823</td>
<td></td>
</tr>
<tr>
<td>24 Complex variables</td>
<td>824</td>
<td></td>
</tr>
<tr>
<td>24.1 Functions of a complex variable</td>
<td>825</td>
<td></td>
</tr>
<tr>
<td>24.2 The Cauchy–Riemann relations</td>
<td>827</td>
<td></td>
</tr>
<tr>
<td>24.3 Power series in a complex variable</td>
<td>830</td>
<td></td>
</tr>
<tr>
<td>24.4 Some elementary functions</td>
<td>832</td>
<td></td>
</tr>
<tr>
<td>24.5 Multivalued functions and branch cuts</td>
<td>835</td>
<td></td>
</tr>
<tr>
<td>24.6 Singularities and zeros of complex functions</td>
<td>837</td>
<td></td>
</tr>
<tr>
<td>24.7 Conformal transformations</td>
<td>839</td>
<td></td>
</tr>
<tr>
<td>24.8 Complex integrals</td>
<td>845</td>
<td></td>
</tr>
</tbody>
</table>
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.9 Cauchy’s theorem</td>
<td>849</td>
</tr>
<tr>
<td>24.10 Cauchy’s integral formula</td>
<td>851</td>
</tr>
<tr>
<td>24.11 Taylor and Laurent series</td>
<td>853</td>
</tr>
<tr>
<td>24.12 Residue theorem</td>
<td>858</td>
</tr>
<tr>
<td>24.13 Definite integrals using contour integration</td>
<td>861</td>
</tr>
<tr>
<td>24.14 Exercises</td>
<td>867</td>
</tr>
<tr>
<td>24.15 Hints and answers</td>
<td>870</td>
</tr>
<tr>
<td><strong>25 Applications of complex variables</strong></td>
<td><strong>871</strong></td>
</tr>
<tr>
<td>25.1 Complex potentials</td>
<td>871</td>
</tr>
<tr>
<td>25.2 Applications of conformal transformations</td>
<td>876</td>
</tr>
<tr>
<td>25.3 Location of zeros</td>
<td>879</td>
</tr>
<tr>
<td>25.4 Summation of series</td>
<td>882</td>
</tr>
<tr>
<td>25.5 Inverse Laplace transform</td>
<td>884</td>
</tr>
<tr>
<td>25.6 Stokes’ equation and Airy integrals</td>
<td>888</td>
</tr>
<tr>
<td>25.7 WKB methods</td>
<td>895</td>
</tr>
<tr>
<td>25.8 Approximations to integrals</td>
<td>905</td>
</tr>
<tr>
<td><strong>Level lines and saddle points; steepest descents; stationary phase</strong></td>
<td><strong>920</strong></td>
</tr>
<tr>
<td>25.9 Exercises</td>
<td>920</td>
</tr>
<tr>
<td>25.10 Hints and answers</td>
<td>925</td>
</tr>
<tr>
<td><strong>26 Tensors</strong></td>
<td><strong>927</strong></td>
</tr>
<tr>
<td>26.1 Some notation</td>
<td>928</td>
</tr>
<tr>
<td>26.2 Change of basis</td>
<td>929</td>
</tr>
<tr>
<td>26.3 Cartesian tensors</td>
<td>930</td>
</tr>
<tr>
<td>26.4 First- and zero-order Cartesian tensors</td>
<td>932</td>
</tr>
<tr>
<td>26.5 Second- and higher-order Cartesian tensors</td>
<td>935</td>
</tr>
<tr>
<td>26.6 The algebra of tensors</td>
<td>938</td>
</tr>
<tr>
<td>26.7 The quotient law</td>
<td>939</td>
</tr>
<tr>
<td>26.8 The tensors $\delta_{ij}$ and $\epsilon_{ijk}$</td>
<td>941</td>
</tr>
<tr>
<td>26.9 Isotropic tensors</td>
<td>944</td>
</tr>
<tr>
<td>26.10 Improper rotations and pseudotensors</td>
<td>946</td>
</tr>
<tr>
<td>26.11 Dual tensors</td>
<td>949</td>
</tr>
<tr>
<td>26.12 Physical applications of tensors</td>
<td>950</td>
</tr>
<tr>
<td>26.13 Integral theorems for tensors</td>
<td>954</td>
</tr>
<tr>
<td>26.14 Non-Cartesian coordinates</td>
<td>955</td>
</tr>
<tr>
<td>26.15 The metric tensor</td>
<td>957</td>
</tr>
<tr>
<td>26.16 General coordinate transformations and tensors</td>
<td>960</td>
</tr>
<tr>
<td>26.17 Relative tensors</td>
<td>963</td>
</tr>
<tr>
<td>26.18 Derivatives of basis vectors and Christoffel symbols</td>
<td>965</td>
</tr>
<tr>
<td>26.19 Covariant differentiation</td>
<td>968</td>
</tr>
<tr>
<td>26.20 Vector operators in tensor form</td>
<td>971</td>
</tr>
</tbody>
</table>
## CONTENTS

26.21 Absolute derivatives along curves 975
26.22 Geodesics 976
26.23 Exercises 977
26.24 Hints and answers 982

27 Numerical methods 984
27.1 Algebraic and transcendental equations 985
   Rearrangement of the equation; linear interpolation; binary chopping;
   Newton–Raphson method
27.2 Convergence of iteration schemes 992
27.3 Simultaneous linear equations 994
   Gaussian elimination; Gauss–Seidel iteration; tridiagonal matrices
27.4 Numerical integration 1000
   Trapezium rule; Simpson’s rule; Gaussian integration; Monte Carlo methods
27.5 Finite differences 1019
27.6 Differential equations 1020
   Difference equations; Taylor series solutions; prediction and correction;
   Runge–Kutta methods; isoclines
27.7 Higher-order equations 1028
27.8 Partial differential equations 1030
27.9 Exercises 1033
27.10 Hints and answers 1039

28 Group theory 1041
28.1 Groups 1041
   Definition of a group; examples of groups
28.2 Finite groups 1049
28.3 Non-Abelian groups 1052
28.4 Permutation groups 1056
28.5 Mappings between groups 1059
28.6 Subgroups 1061
28.7 Subdividing a group 1063
   Equivalence relations and classes; congruence and cosets; conjugates and
   classes
28.8 Exercises 1070
28.9 Hints and answers 1074

29 Representation theory 1076
29.1 Dipole moments of molecules 1077
29.2 Choosing an appropriate formalism 1078
29.3 Equivalent representations 1084
29.4 Reducibility of a representation 1086
29.5 The orthogonality theorem for irreducible representations 1090
## CONTENTS

29.6 Characters 1092  
Orthogonality property of characters

29.7 Counting irreps using characters 1095  
Summation rules for irreps

29.8 Construction of a character table 1100

29.9 Group nomenclature 1102

29.10 Product representations 1103

29.11 Physical applications of group theory 1105  
Bonding in molecules; matrix elements in quantum mechanics; degeneracy of normal modes; breaking of degeneracies

29.12 Exercises 1113

29.13 Hints and answers 1117

30 Probability 1119

30.1 Venn diagrams 1119

30.2 Probability 1124  
Axioms and theorems; conditional probability; Bayes’ theorem

30.3 Permutations and combinations 1133

30.4 Random variables and distributions 1139  
Discrete random variables; continuous random variables

30.5 Properties of distributions 1143  
Mean; mode and median; variance and standard deviation; moments; central moments

30.6 Functions of random variables 1150

30.7 Generating functions 1157  
Probability generating functions; moment generating functions; characteristic functions; cumulant generating functions

30.8 Important discrete distributions 1168  
Binomial; geometric; negative binomial; hypergeometric; Poisson

30.9 Important continuous distributions 1179  
Gaussian; log-normal; exponential; gamma; chi-squared; Cauchy; Breit–Wigner; uniform

30.10 The central limit theorem 1195

30.11 Joint distributions 1196  
Discrete bivariate; continuous bivariate; marginal and conditional distributions

30.12 Properties of joint distributions 1199  
Means; variances; covariance and correlation

30.13 Generating functions for joint distributions 1205

30.14 Transformation of variables in joint distributions 1206

30.15 Important joint distributions 1207  
Multinomial; multivariate Gaussian

30.16 Exercises 1211

30.17 Hints and answers 1219
CONTENTS

31 Statistics 1221
31.1 Experiments, samples and populations 1221
31.2 Sample statistics 1222
Averages; variance and standard deviation; moments; covariance and correlation
31.3 Estimators and sampling distributions 1229
Consistency, bias and efficiency; Fisher’s inequality; standard errors; confidence limits
31.4 Some basic estimators 1243
Mean; variance; standard deviation; moments; covariance and correlation
31.5 Maximum-likelihood method 1255
ML estimator; transformation invariance and bias; efficiency; errors and confidence limits; Bayesian interpretation; large-N behaviour; extended ML method
31.6 The method of least squares 1271
Linear least squares; non-linear least squares
31.7 Hypothesis testing 1277
Simple and composite hypotheses; statistical tests; Neyman–Pearson; generalised likelihood-ratio; Student’s t; Fisher’s F; goodness of fit
31.8 Exercises 1298
31.9 Hints and answers 1303
Index 1305
I am the very Model for a Student Mathematical

I am the very model for a student mathematical;
I've information rational, and logical and practical.
I know the laws of algebra, and find them quite symmetrical,
And even know the meaning of 'a variate antithetical'.

I'm extremely well acquainted, with all things mathematical.
I understand equations, both the simple and quadratical.
About binomial theorems I'm teeming with a lot o'news,
With many cheerful facts about the square of the hypotenuse.

I'm very good at integral and differential calculus,
And solving paradoxes that so often seem to rankle us.
In short in matters rational, and logical and practical,
I am the very model for a student mathematical.

I know the singularities of equations differential,
And some of these are regular, but the rest are quite essential.
I quote the results of giants; with Euler, Newton, Gauss, Laplace,
And can calculate an orbit, given a centre, force and mass.

I can reconstruct equations, both canonical and formal,
And write all kinds of matrices, orthogonal, real and normal.
I show how to tackle problems that one has never met before,
By analogy or example, or with some clever metaphor.

I seldom use equivalence to help decide upon a class,
But often find an integral, using a contour o'er a pass.
In short in matters rational, and logical and practical,
I am the very model for a student mathematical.

When you have learnt just what is meant by 'Jacobian' and 'Abelian';
When you at sight can estimate, for the modal, mean and median;
When describing normal subgroups is much more than recitation;
When you understand precisely what is 'quantum excitation';

When you know enough statistics that you can recognise RV;
When you have learnt all advances that have been made in SVD;
And when you can spot the transform that solves some tricky PDE,
You will feel no better student has ever sat for a degree.

Your accumulated knowledge, whilst extensive and exemplary,
Will have only been brought down to the beginning of last century,
But still in matters rational, and logical and practical,
You'll be the very model of a student mathematical.

KFR, with apologies to W.S. Gilbert

xix