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0521861535 - Mathematical Methods for Physics and Engineering, Third Edition

K. F. Riley, M. P. Hobson and S. J. Bence

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Mathematical Methods for Physics and Engineering

The third edition of this highly acclaimed undergraduate textbook is suitable for teaching all the mathematics ever likely to be needed for an undergraduate course in any of the physical sciences. As well as lucid descriptions of all the topics covered and many worked examples, it contains more than 800 exercises. A number of additional topics have been included and the text has undergone significant reorganisation in some areas. New stand-alone chapters:

- give a systematic account of the ‘special functions’ of physical science
- cover an extended range of practical applications of complex variables including WKB methods and saddle-point integration techniques
- provide an introduction to quantum operators.

Further tabulations, of relevance in statistics and numerical integration, have been added. In this edition, all 400 odd-numbered exercises are provided with complete worked solutions in a separate manual, available to both students and their teachers; these are in addition to the hints and outline answers given in the main text. The even-numbered exercises have no hints, answers or worked solutions and can be used for unaided homework; full solutions to them are available to instructors on a password-protected website.

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Contents

<i>Preface to the third edition</i>	<i>page</i> xx
<i>Preface to the second edition</i>	xxiii
<i>Preface to the first edition</i>	xxv
1 Preliminary algebra	1
1.1 Simple functions and equations	1
<i>Polynomial equations; factorisation; properties of roots</i>	
1.2 Trigonometric identities	10
<i>Single angle; compound angles; double- and half-angle identities</i>	
1.3 Coordinate geometry	15
1.4 Partial fractions	18
<i>Complications and special cases</i>	
1.5 Binomial expansion	25
1.6 Properties of binomial coefficients	27
1.7 Some particular methods of proof	30
<i>Proof by induction; proof by contradiction; necessary and sufficient conditions</i>	
1.8 Exercises	36
1.9 Hints and answers	39
2 Preliminary calculus	41
2.1 Differentiation	41
<i>Differentiation from first principles; products; the chain rule; quotients; implicit differentiation; logarithmic differentiation; Leibnitz' theorem; special points of a function; curvature; theorems of differentiation</i>	

CONTENTS

2.2	Integration	59
	<i>Integration from first principles; the inverse of differentiation; by inspection; sinusoidal functions; logarithmic integration; using partial fractions; substitution method; integration by parts; reduction formulae; infinite and improper integrals; plane polar coordinates; integral inequalities; applications of integration</i>	
2.3	Exercises	76
2.4	Hints and answers	81
3	Complex numbers and hyperbolic functions	83
3.1	The need for complex numbers	83
3.2	Manipulation of complex numbers	85
	<i>Addition and subtraction; modulus and argument; multiplication; complex conjugate; division</i>	
3.3	Polar representation of complex numbers	92
	<i>Multiplication and division in polar form</i>	
3.4	de Moivre’s theorem	95
	<i>trigonometric identities; finding the nth roots of unity; solving polynomial equations</i>	
3.5	Complex logarithms and complex powers	99
3.6	Applications to differentiation and integration	101
3.7	Hyperbolic functions	102
	<i>Definitions; hyperbolic–trigonometric analogies; identities of hyperbolic functions; solving hyperbolic equations; inverses of hyperbolic functions; calculus of hyperbolic functions</i>	
3.8	Exercises	109
3.9	Hints and answers	113
4	Series and limits	115
4.1	Series	115
4.2	Summation of series	116
	<i>Arithmetic series; geometric series; arithmetico-geometric series; the difference method; series involving natural numbers; transformation of series</i>	
4.3	Convergence of infinite series	124
	<i>Absolute and conditional convergence; series containing only real positive terms; alternating series test</i>	
4.4	Operations with series	131
4.5	Power series	131
	<i>Convergence of power series; operations with power series</i>	
4.6	Taylor series	136
	<i>Taylor’s theorem; approximation errors; standard Maclaurin series</i>	
4.7	Evaluation of limits	141
4.8	Exercises	144
4.9	Hints and answers	149

CONTENTS

5	Partial differentiation	151
5.1	Definition of the partial derivative	151
5.2	The total differential and total derivative	153
5.3	Exact and inexact differentials	155
5.4	Useful theorems of partial differentiation	157
5.5	The chain rule	157
5.6	Change of variables	158
5.7	Taylor’s theorem for many-variable functions	160
5.8	Stationary values of many-variable functions	162
5.9	Stationary values under constraints	167
5.10	Envelopes	173
5.11	Thermodynamic relations	176
5.12	Differentiation of integrals	178
5.13	Exercises	179
5.14	Hints and answers	185
6	Multiple integrals	187
6.1	Double integrals	187
6.2	Triple integrals	190
6.3	Applications of multiple integrals	191
	<i>Areas and volumes; masses, centres of mass and centroids; Pappus’ theorems; moments of inertia; mean values of functions</i>	
6.4	Change of variables in multiple integrals	199
	<i>Change of variables in double integrals; evaluation of the integral $I = \int_{-\infty}^{\infty} e^{-x^2} dx$; change of variables in triple integrals; general properties of Jacobians</i>	
6.5	Exercises	207
6.6	Hints and answers	211
7	Vector algebra	212
7.1	Scalars and vectors	212
7.2	Addition and subtraction of vectors	213
7.3	Multiplication by a scalar	214
7.4	Basis vectors and components	217
7.5	Magnitude of a vector	218
7.6	Multiplication of vectors	219
	<i>Scalar product; vector product; scalar triple product; vector triple product</i>	

CONTENTS

7.7	Equations of lines, planes and spheres	226
7.8	Using vectors to find distances <i>Point to line; point to plane; line to line; line to plane</i>	229
7.9	Reciprocal vectors	233
7.10	Exercises	234
7.11	Hints and answers	240
8	Matrices and vector spaces	241
8.1	Vector spaces <i>Basis vectors; inner product; some useful inequalities</i>	242
8.2	Linear operators	247
8.3	Matrices	249
8.4	Basic matrix algebra <i>Matrix addition; multiplication by a scalar; matrix multiplication</i>	250
8.5	Functions of matrices	255
8.6	The transpose of a matrix	255
8.7	The complex and Hermitian conjugates of a matrix	256
8.8	The trace of a matrix	258
8.9	The determinant of a matrix <i>Properties of determinants</i>	259
8.10	The inverse of a matrix	263
8.11	The rank of a matrix	267
8.12	Special types of square matrix <i>Diagonal; triangular; symmetric and antisymmetric; orthogonal; Hermitian and anti-Hermitian; unitary; normal</i>	268
8.13	Eigenvectors and eigenvalues <i>Of a normal matrix; of Hermitian and anti-Hermitian matrices; of a unitary matrix; of a general square matrix</i>	272
8.14	Determination of eigenvalues and eigenvectors <i>Degenerate eigenvalues</i>	280
8.15	Change of basis and similarity transformations	282
8.16	Diagonalisation of matrices	285
8.17	Quadratic and Hermitian forms <i>Stationary properties of the eigenvectors; quadratic surfaces</i>	288
8.18	Simultaneous linear equations <i>Range; null space; N simultaneous linear equations in N unknowns; singular value decomposition</i>	292
8.19	Exercises	307
8.20	Hints and answers	314
9	Normal modes	316
9.1	Typical oscillatory systems	317
9.2	Symmetry and normal modes	322

CONTENTS

9.3	Rayleigh–Ritz method	327
9.4	Exercises	329
9.5	Hints and answers	332
10	Vector calculus	334
10.1	Differentiation of vectors	334
	<i>Composite vector expressions; differential of a vector</i>	
10.2	Integration of vectors	339
10.3	Space curves	340
10.4	Vector functions of several arguments	344
10.5	Surfaces	345
10.6	Scalar and vector fields	347
10.7	Vector operators	347
	<i>Gradient of a scalar field; divergence of a vector field; curl of a vector field</i>	
10.8	Vector operator formulae	354
	<i>Vector operators acting on sums and products; combinations of grad, div and curl</i>	
10.9	Cylindrical and spherical polar coordinates	357
10.10	General curvilinear coordinates	364
10.11	Exercises	369
10.12	Hints and answers	375
11	Line, surface and volume integrals	377
11.1	Line integrals	377
	<i>Evaluating line integrals; physical examples; line integrals with respect to a scalar</i>	
11.2	Connectivity of regions	383
11.3	Green’s theorem in a plane	384
11.4	Conservative fields and potentials	387
11.5	Surface integrals	389
	<i>Evaluating surface integrals; vector areas of surfaces; physical examples</i>	
11.6	Volume integrals	396
	<i>Volumes of three-dimensional regions</i>	
11.7	Integral forms for grad, div and curl	398
11.8	Divergence theorem and related theorems	401
	<i>Green’s theorems; other related integral theorems; physical applications</i>	
11.9	Stokes’ theorem and related theorems	406
	<i>Related integral theorems; physical applications</i>	
11.10	Exercises	409
11.11	Hints and answers	414
12	Fourier series	415
12.1	The Dirichlet conditions	415

CONTENTS

12.2	The Fourier coefficients	417
12.3	Symmetry considerations	419
12.4	Discontinuous functions	420
12.5	Non-periodic functions	422
12.6	Integration and differentiation	424
12.7	Complex Fourier series	424
12.8	Parseval's theorem	426
12.9	Exercises	427
12.10	Hints and answers	431
13	Integral transforms	433
13.1	Fourier transforms <i>The uncertainty principle; Fraunhofer diffraction; the Dirac δ-function; relation of the δ-function to Fourier transforms; properties of Fourier transforms; odd and even functions; convolution and deconvolution; correlation functions and energy spectra; Parseval's theorem; Fourier transforms in higher dimensions</i>	433
13.2	Laplace transforms <i>Laplace transforms of derivatives and integrals; other properties of Laplace transforms</i>	453
13.3	Concluding remarks	459
13.4	Exercises	460
13.5	Hints and answers	466
14	First-order ordinary differential equations	468
14.1	General form of solution	469
14.2	First-degree first-order equations <i>Separable-variable equations; exact equations; inexact equations, integrating factors; linear equations; homogeneous equations; isobaric equations; Bernoulli's equation; miscellaneous equations</i>	470
14.3	Higher-degree first-order equations <i>Equations soluble for p; for x; for y; Clairaut's equation</i>	480
14.4	Exercises	484
14.5	Hints and answers	488
15	Higher-order ordinary differential equations	490
15.1	Linear equations with constant coefficients <i>Finding the complementary function $y_c(x)$; finding the particular integral $y_p(x)$; constructing the general solution $y_c(x) + y_p(x)$; linear recurrence relations; Laplace transform method</i>	492
15.2	Linear equations with variable coefficients <i>The Legendre and Euler linear equations; exact equations; partially known complementary function; variation of parameters; Green's functions; canonical form for second-order equations</i>	503

CONTENTS

15.3	General ordinary differential equations	518
	<i>Dependent variable absent; independent variable absent; non-linear exact equations; isobaric or homogeneous equations; equations homogeneous in x or y alone; equations having $y = Ae^x$ as a solution</i>	
15.4	Exercises	523
15.5	Hints and answers	529
16	Series solutions of ordinary differential equations	531
16.1	Second-order linear ordinary differential equations	531
	<i>Ordinary and singular points</i>	
16.2	Series solutions about an ordinary point	535
16.3	Series solutions about a regular singular point	538
	<i>Distinct roots not differing by an integer; repeated root of the indicial equation; distinct roots differing by an integer</i>	
16.4	Obtaining a second solution	544
	<i>The Wronskian method; the derivative method; series form of the second solution</i>	
16.5	Polynomial solutions	548
16.6	Exercises	550
16.7	Hints and answers	553
17	Eigenfunction methods for differential equations	554
17.1	Sets of functions	556
	<i>Some useful inequalities</i>	
17.2	Adjoint, self-adjoint and Hermitian operators	559
17.3	Properties of Hermitian operators	561
	<i>Reality of the eigenvalues; orthogonality of the eigenfunctions; construction of real eigenfunctions</i>	
17.4	Sturm–Liouville equations	564
	<i>Valid boundary conditions; putting an equation into Sturm–Liouville form</i>	
17.5	Superposition of eigenfunctions: Green’s functions	569
17.6	A useful generalisation	572
17.7	Exercises	573
17.8	Hints and answers	576
18	Special functions	577
18.1	Legendre functions	577
	<i>General solution for integer ℓ; properties of Legendre polynomials</i>	
18.2	Associated Legendre functions	587
18.3	Spherical harmonics	593
18.4	Chebyshev functions	595
18.5	Bessel functions	602
	<i>General solution for non-integer ν; general solution for integer ν; properties of Bessel functions</i>	

CONTENTS

18.6	Spherical Bessel functions	614
18.7	Laguerre functions	616
18.8	Associated Laguerre functions	621
18.9	Hermite functions	624
18.10	Hypergeometric functions	628
18.11	Confluent hypergeometric functions	633
18.12	The gamma function and related functions	635
18.13	Exercises	640
18.14	Hints and answers	646
19	Quantum operators	648
19.1	Operator formalism <i>Commutators</i>	648
19.2	Physical examples of operators <i>Uncertainty principle; angular momentum; creation and annihilation operators</i>	656
19.3	Exercises	671
19.4	Hints and answers	674
20	Partial differential equations: general and particular solutions	675
20.1	Important partial differential equations <i>The wave equation; the diffusion equation; Laplace's equation; Poisson's equation; Schrödinger's equation</i>	676
20.2	General form of solution	680
20.3	General and particular solutions <i>First-order equations; inhomogeneous equations and problems; second-order equations</i>	681
20.4	The wave equation	693
20.5	The diffusion equation	695
20.6	Characteristics and the existence of solutions <i>First-order equations; second-order equations</i>	699
20.7	Uniqueness of solutions	705
20.8	Exercises	707
20.9	Hints and answers	711
21	Partial differential equations: separation of variables and other methods	713
21.1	Separation of variables: the general method	713
21.2	Superposition of separated solutions	717
21.3	Separation of variables in polar coordinates <i>Laplace's equation in polar coordinates; spherical harmonics; other equations in polar coordinates; solution by expansion; separation of variables for inhomogeneous equations</i>	725
21.4	Integral transform methods	747

CONTENTS

21.5	Inhomogeneous problems – Green’s functions	751
	<i>Similarities to Green’s functions for ordinary differential equations; general boundary-value problems; Dirichlet problems; Neumann problems</i>	
21.6	Exercises	767
21.7	Hints and answers	773
22	Calculus of variations	775
22.1	The Euler–Lagrange equation	776
22.2	Special cases	777
	<i>F does not contain y explicitly; F does not contain x explicitly</i>	
22.3	Some extensions	781
	<i>Several dependent variables; several independent variables; higher-order derivatives; variable end-points</i>	
22.4	Constrained variation	785
22.5	Physical variational principles	787
	<i>Fermat’s principle in optics; Hamilton’s principle in mechanics</i>	
22.6	General eigenvalue problems	790
22.7	Estimation of eigenvalues and eigenfunctions	792
22.8	Adjustment of parameters	795
22.9	Exercises	797
22.10	Hints and answers	801
23	Integral equations	803
23.1	Obtaining an integral equation from a differential equation	803
23.2	Types of integral equation	804
23.3	Operator notation and the existence of solutions	805
23.4	Closed-form solutions	806
	<i>Separable kernels; integral transform methods; differentiation</i>	
23.5	Neumann series	813
23.6	Fredholm theory	815
23.7	Schmidt–Hilbert theory	816
23.8	Exercises	819
23.9	Hints and answers	823
24	Complex variables	824
24.1	Functions of a complex variable	825
24.2	The Cauchy–Riemann relations	827
24.3	Power series in a complex variable	830
24.4	Some elementary functions	832
24.5	Multivalued functions and branch cuts	835
24.6	Singularities and zeros of complex functions	837
24.7	Conformal transformations	839
24.8	Complex integrals	845

CONTENTS

24.9	Cauchy’s theorem	849
24.10	Cauchy’s integral formula	851
24.11	Taylor and Laurent series	853
24.12	Residue theorem	858
24.13	Definite integrals using contour integration	861
24.14	Exercises	867
24.15	Hints and answers	870
25	Applications of complex variables	871
25.1	Complex potentials	871
25.2	Applications of conformal transformations	876
25.3	Location of zeros	879
25.4	Summation of series	882
25.5	Inverse Laplace transform	884
25.6	Stokes’ equation and Airy integrals	888
25.7	WKB methods	895
25.8	Approximations to integrals <i>Level lines and saddle points; steepest descents; stationary phase</i>	905
25.9	Exercises	920
25.10	Hints and answers	925
26	Tensors	927
26.1	Some notation	928
26.2	Change of basis	929
26.3	Cartesian tensors	930
26.4	First- and zero-order Cartesian tensors	932
26.5	Second- and higher-order Cartesian tensors	935
26.6	The algebra of tensors	938
26.7	The quotient law	939
26.8	The tensors δ_{ij} and ϵ_{ijk}	941
26.9	Isotropic tensors	944
26.10	Improper rotations and pseudotensors	946
26.11	Dual tensors	949
26.12	Physical applications of tensors	950
26.13	Integral theorems for tensors	954
26.14	Non-Cartesian coordinates	955
26.15	The metric tensor	957
26.16	General coordinate transformations and tensors	960
26.17	Relative tensors	963
26.18	Derivatives of basis vectors and Christoffel symbols	965
26.19	Covariant differentiation	968
26.20	Vector operators in tensor form	971

CONTENTS

26.21	Absolute derivatives along curves	975
26.22	Geodesics	976
26.23	Exercises	977
26.24	Hints and answers	982
27	Numerical methods	984
27.1	Algebraic and transcendental equations <i>Rearrangement of the equation; linear interpolation; binary chopping; Newton–Raphson method</i>	985
27.2	Convergence of iteration schemes	992
27.3	Simultaneous linear equations <i>Gaussian elimination; Gauss–Seidel iteration; tridiagonal matrices</i>	994
27.4	Numerical integration <i>Trapezium rule; Simpson’s rule; Gaussian integration; Monte Carlo methods</i>	1000
27.5	Finite differences	1019
27.6	Differential equations <i>Difference equations; Taylor series solutions; prediction and correction; Runge–Kutta methods; isoclines</i>	1020
27.7	Higher-order equations	1028
27.8	Partial differential equations	1030
27.9	Exercises	1033
27.10	Hints and answers	1039
28	Group theory	1041
28.1	Groups <i>Definition of a group; examples of groups</i>	1041
28.2	Finite groups	1049
28.3	Non-Abelian groups	1052
28.4	Permutation groups	1056
28.5	Mappings between groups	1059
28.6	Subgroups	1061
28.7	Subdividing a group <i>Equivalence relations and classes; congruence and cosets; conjugates and classes</i>	1063
28.8	Exercises	1070
28.9	Hints and answers	1074
29	Representation theory	1076
29.1	Dipole moments of molecules	1077
29.2	Choosing an appropriate formalism	1078
29.3	Equivalent representations	1084
29.4	Reducibility of a representation	1086
29.5	The orthogonality theorem for irreducible representations	1090

CONTENTS

29.6	Characters	1092
	<i>Orthogonality property of characters</i>	
29.7	Counting irreps using characters	1095
	<i>Summation rules for irreps</i>	
29.8	Construction of a character table	1100
29.9	Group nomenclature	1102
29.10	Product representations	1103
29.11	Physical applications of group theory	1105
	<i>Bonding in molecules; matrix elements in quantum mechanics; degeneracy of normal modes; breaking of degeneracies</i>	
29.12	Exercises	1113
29.13	Hints and answers	1117
30	Probability	1119
30.1	Venn diagrams	1119
30.2	Probability	1124
	<i>Axioms and theorems; conditional probability; Bayes' theorem</i>	
30.3	Permutations and combinations	1133
30.4	Random variables and distributions	1139
	<i>Discrete random variables; continuous random variables</i>	
30.5	Properties of distributions	1143
	<i>Mean; mode and median; variance and standard deviation; moments; central moments</i>	
30.6	Functions of random variables	1150
30.7	Generating functions	1157
	<i>Probability generating functions; moment generating functions; characteristic functions; cumulant generating functions</i>	
30.8	Important discrete distributions	1168
	<i>Binomial; geometric; negative binomial; hypergeometric; Poisson</i>	
30.9	Important continuous distributions	1179
	<i>Gaussian; log-normal; exponential; gamma; chi-squared; Cauchy; Breit–Wigner; uniform</i>	
30.10	The central limit theorem	1195
30.11	Joint distributions	1196
	<i>Discrete bivariate; continuous bivariate; marginal and conditional distributions</i>	
30.12	Properties of joint distributions	1199
	<i>Means; variances; covariance and correlation</i>	
30.13	Generating functions for joint distributions	1205
30.14	Transformation of variables in joint distributions	1206
30.15	Important joint distributions	1207
	<i>Multinomial; multivariate Gaussian</i>	
30.16	Exercises	1211
30.17	Hints and answers	1219

CONTENTS

31	Statistics	1221
31.1	Experiments, samples and populations	1221
31.2	Sample statistics	1222
	<i>Averages; variance and standard deviation; moments; covariance and correlation</i>	
31.3	Estimators and sampling distributions	1229
	<i>Consistency, bias and efficiency; Fisher's inequality; standard errors; confidence limits</i>	
31.4	Some basic estimators	1243
	<i>Mean; variance; standard deviation; moments; covariance and correlation</i>	
31.5	Maximum-likelihood method	1255
	<i>ML estimator; transformation invariance and bias; efficiency; errors and confidence limits; Bayesian interpretation; large-N behaviour; extended ML method</i>	
31.6	The method of least squares	1271
	<i>Linear least squares; non-linear least squares</i>	
31.7	Hypothesis testing	1277
	<i>Simple and composite hypotheses; statistical tests; Neyman–Pearson; generalised likelihood-ratio; Student's <i>t</i>; Fisher's <i>F</i>; goodness of fit</i>	
31.8	Exercises	1298
31.9	Hints and answers	1303
	<i>Index</i>	1305

CONTENTS

I am the very Model for a Student Mathematical

I am the very model for a student mathematical;
I've information rational, and logical and practical.
I know the laws of algebra, and find them quite symmetrical,
And even know the meaning of 'a variate antithetical'.

I'm extremely well acquainted, with all things mathematical.
I understand equations, both the simple and quadratical.
About binomial theorems I'm teeming with a lot o'news,
With many cheerful facts about the square of the hypotenuse.

I'm very good at integral and differential calculus,
And solving paradoxes that so often seem to rankle us.
In short in matters rational, and logical and practical,
I am the very model for a student mathematical.

I know the singularities of equations differential,
And some of these are regular, but the rest are quite essential.
I quote the results of giants; with Euler, Newton, Gauss, Laplace,
And can calculate an orbit, given a centre, force and mass.

I can reconstruct equations, both canonical and formal,
And write all kinds of matrices, orthogonal, real and normal.
I show how to tackle problems that one has never met before,
By analogy or example, or with some clever metaphor.

I seldom use equivalence to help decide upon a class,
But often find an integral, using a contour o'er a pass.
In short in matters rational, and logical and practical,
I am the very model for a student mathematical.

When you have learnt just what is meant by 'Jacobian' and 'Abelian';
When you at sight can estimate, for the modal, mean and median;
When describing normal subgroups is much more than recitation;
When you understand precisely what is 'quantum excitation';

When you know enough statistics that you can recognise RV;
When you have learnt all advances that have been made in SVD;
And when you can spot the transform that solves some tricky PDE,
You will feel no better student has ever sat for a degree.

Your accumulated knowledge, whilst extensive and exemplary,
Will have only been brought down to the beginning of last century,
But still in matters rational, and logical and practical,
You'll be the very model of a student mathematical.

KFR, with apologies to W.S. Gilbert

Cambridge University Press

0521861535 - Mathematical Methods for Physics and Engineering, Third Edition

K. F. Riley, M. P. Hobson and S. J. Bence

Frontmatter

[More information](#)

Preface to the third edition

As is natural, in the four years since the publication of the second edition of this book we have somewhat modified our views on what should be included and how it should be presented. In this new edition, although the range of topics covered has been extended, there has been no significant shift in the general level of difficulty or in the degree of mathematical sophistication required. Further, we have aimed to preserve the same style of presentation as seems to have been well received in the first two editions. However, a significant change has been made to the format of the chapters, specifically to the way that the exercises, together with their hints and answers, have been treated; the details of the change are explained below.

The two major chapters that are new in this third edition are those dealing with ‘special functions’ and the applications of complex variables. The former presents a systematic account of those functions that appear to have arisen in a more or less haphazard way as a result of studying particular physical situations, and are deemed ‘special’ for that reason. The treatment presented here shows that, in fact, they are nearly all particular cases of the hypergeometric or confluent hypergeometric functions, and are special only in the sense that the parameters of the relevant function take simple or related values.

The second new chapter describes how the properties of complex variables can be used to tackle problems arising from the description of physical situations or from other seemingly unrelated areas of mathematics. To topics treated in earlier editions, such as the solution of Laplace’s equation in two dimensions, the summation of series, the location of zeros of polynomials and the calculation of inverse Laplace transforms, has been added new material covering Airy integrals, saddle-point methods for contour integral evaluation, and the WKB approach to asymptotic forms.

Other new material includes a stand-alone chapter on the use of coordinate-free operators to establish valuable results in the field of quantum mechanics; amongst

Cambridge University Press

0521861535 - Mathematical Methods for Physics and Engineering, Third Edition

K. F. Riley, M. P. Hobson and S. J. Bence

Frontmatter

[More information](#)

PREFACE TO THE THIRD EDITION

the physical topics covered are angular momentum and uncertainty principles. There are also significant additions to the treatment of numerical integration. In particular, Gaussian quadrature based on Legendre, Laguerre, Hermite and Chebyshev polynomials is discussed, and appropriate tables of points and weights are provided.

We now turn to the most obvious change to the format of the book, namely the way that the exercises, hints and answers are treated. The second edition of *Mathematical Methods for Physics and Engineering* carried more than twice as many exercises, based on its various chapters, as did the first. In its preface we discussed the general question of how such exercises should be treated but, in the end, decided to provide hints and outline answers to all problems, as in the first edition. This decision was an uneasy one as, on the one hand, it did not allow the exercises to be set as totally unaided homework that could be used for assessment purposes but, on the other, it did not give a full explanation of how to tackle a problem when a student needed explicit guidance or a model answer.

In order to allow both of these educationally desirable goals to be achieved, we have, in this third edition, completely changed the way in which this matter is handled. A large number of exercises have been included in the penultimate subsections of the appropriate, sometimes reorganised, chapters. Hints and outline answers are given, as previously, in the final subsections, *but only for the odd-numbered exercises*. This leaves all even-numbered exercises free to be set as unaided homework, as described below.

For the four hundred plus **odd-numbered** exercises, *complete* solutions are available, to both students and their teachers, in the form of a separate manual, *Student Solutions Manual for Mathematical Methods for Physics and Engineering* (Cambridge: Cambridge University Press, 2006); the hints and outline answers given in this main text are brief summaries of the model answers given in the manual. There, each original exercise is reproduced and followed by a fully worked solution. For those original exercises that make internal reference to this text or to other (even-numbered) exercises not included in the solutions manual, the questions have been reworded, usually by including additional information, so that the questions can stand alone.

In many cases, the solution given in the manual is even fuller than one that might be expected of a good student that has understood the material. This is because we have aimed to make the solutions instructional as well as utilitarian. To this end, we have included comments that are intended to show how the plan for the solution is formulated and have given the justifications for particular intermediate steps (something not always done, even by the best of students). We have also tried to write each individual substituted formula in the form that best indicates how it was obtained, before simplifying it at the next or a subsequent stage. Where several lines of algebraic manipulation or calculus are needed to obtain a final result, they are normally included in full; this should enable the

Cambridge University Press

0521861535 - Mathematical Methods for Physics and Engineering, Third Edition

K. F. Riley, M. P. Hobson and S. J. Bence

Frontmatter

[More information](#)

PREFACE TO THE THIRD EDITION

student to determine whether an incorrect answer is due to a misunderstanding of principles or to a technical error.

The remaining four hundred or so **even-numbered** exercises have no hints or answers, outlined or detailed, available for general access. They can therefore be used by instructors as a basis for setting unaided homework. Full solutions to these exercises, in the same general format as those appearing in the manual (though they may contain references to the main text or to other exercises), are available without charge to accredited teachers as downloadable pdf files on the password-protected website <http://www.cambridge.org/9780521679718>. Teachers wishing to have access to the website should contact solutions@cambridge.org for registration details.

In all new publications, errors and typographical mistakes are virtually unavoidable, and we would be grateful to any reader who brings instances to our attention. Retrospectively, we would like to record our thanks to Reinhard Gerndt, Paul Renteln and Joe Tenn for making us aware of some errors in the second edition. Finally, we are extremely grateful to Dave Green for his considerable and continuing advice concerning L^AT_EX.

Ken Riley, Michael Hobson,
Cambridge, 2006

Cambridge University Press

0521861535 - Mathematical Methods for Physics and Engineering, Third Edition

K. F. Riley, M. P. Hobson and S. J. Bence

Frontmatter

[More information](#)

Preface to the second edition

Since the publication of the first edition of this book, both through teaching the material it covers and as a result of receiving helpful comments from colleagues, we have become aware of the desirability of changes in a number of areas. The most important of these is that the mathematical preparation of current senior college and university entrants is now less thorough than it used to be. To match this, we decided to include a preliminary chapter covering areas such as polynomial equations, trigonometric identities, coordinate geometry, partial fractions, binomial expansions, necessary and sufficient condition and proof by induction and contradiction.

Whilst the general level of what is included in this second edition has not been raised, some areas have been expanded to take in topics we now feel were not adequately covered in the first. In particular, increased attention has been given to non-square sets of simultaneous linear equations and their associated matrices. We hope that this more extended treatment, together with the inclusion of singular value matrix decomposition, will make the material of more practical use to engineering students. In the same spirit, an elementary treatment of linear recurrence relations has been included. The topic of normal modes has been given a small chapter of its own, though the links to matrices on the one hand, and to representation theory on the other, have not been lost.

Elsewhere, the presentation of probability and statistics has been reorganised to give the two aspects more nearly equal weights. The early part of the probability chapter has been rewritten in order to present a more coherent development based on Boolean algebra, the fundamental axioms of probability theory and the properties of intersections and unions. Whilst this is somewhat more formal than previously, we think that it has not reduced the accessibility of these topics and hope that it has increased it. The scope of the chapter has been somewhat extended to include all physically important distributions and an introduction to cumulants.

Cambridge University Press

0521861535 - Mathematical Methods for Physics and Engineering, Third Edition

K. F. Riley, M. P. Hobson and S. J. Bence

Frontmatter

[More information](#)

PREFACE TO THE SECOND EDITION

Statistics now occupies a substantial chapter of its own, one that includes systematic discussions of estimators and their efficiency, sample distributions and t - and F -tests for comparing means and variances. Other new topics are applications of the chi-squared distribution, maximum-likelihood parameter estimation and least-squares fitting. In other chapters we have added material on the following topics: curvature, envelopes, curve-sketching, more refined numerical methods for differential equations and the elements of integration using Monte Carlo techniques.

Over the last four years we have received somewhat mixed feedback about the number of exercises at the ends of the various chapters. After consideration, we decided to increase the number substantially, partly to correspond to the additional topics covered in the text but mainly to give both students and their teachers a wider choice. There are now nearly 800 such exercises, many with several parts. An even more vexed question has been whether to provide hints and answers to all the exercises or just to ‘the odd-numbered’ ones, as is the normal practice for textbooks in the United States, thus making the remainder more suitable for setting as homework. In the end, we decided that hints and outline solutions should be provided for all the exercises, in order to facilitate independent study while leaving the details of the calculation as a task for the student.

In conclusion, we hope that this edition will be thought by its users to be ‘heading in the right direction’ and would like to place on record our thanks to all who have helped to bring about the changes and adjustments. Naturally, those colleagues who have noted errors or ambiguities in the first edition and brought them to our attention figure high on the list, as do the staff at The Cambridge University Press. In particular, we are grateful to Dave Green for continued $\text{\LaTeX}$ advice, Susan Parkinson for copy-editing the second edition with her usual keen eye for detail and flair for crafting coherent prose and Alison Woollatt for once again turning our basic $\text{\LaTeX}$ into a beautifully typeset book. Our thanks go to all of them, though of course we accept full responsibility for any remaining errors or ambiguities, of which, as with any new publication, there are bound to be some.

On a more personal note, KFR again wishes to thank his wife Penny for her unwavering support, not only in his academic and tutorial work, but also in their joint efforts to convert time at the bridge table into ‘green points’ on their record. MPH is once more indebted to his wife, Becky, and his mother, Pat, for their tireless support and encouragement above and beyond the call of duty. MPH dedicates his contribution to this book to the memory of his father, Ronald Leonard Hobson, whose gentle kindness, patient understanding and unbreakable spirit made all things seem possible.

Ken Riley, Michael Hobson
Cambridge, 2002

Cambridge University Press

0521861535 - Mathematical Methods for Physics and Engineering, Third Edition

K. F. Riley, M. P. Hobson and S. J. Bence

Frontmatter

[More information](#)

Preface to the first edition

A knowledge of mathematical methods is important for an increasing number of university and college courses, particularly in physics, engineering and chemistry, but also in more general science. Students embarking on such courses come from diverse mathematical backgrounds, and their core knowledge varies considerably. We have therefore decided to write a textbook that assumes knowledge only of material that can be expected to be familiar to all the current generation of students starting physical science courses at university. In the United Kingdom this corresponds to the standard of Mathematics A-level, whereas in the United States the material assumed is that which would normally be covered at junior college.

Starting from this level, the first six chapters cover a collection of topics with which the reader may already be familiar, but which are here extended and applied to typical problems encountered by first-year university students. They are aimed at providing a common base of general techniques used in the development of the remaining chapters. Students who have had additional preparation, such as Further Mathematics at A-level, will find much of this material straightforward.

Following these opening chapters, the remainder of the book is intended to cover at least that mathematical material which an undergraduate in the physical sciences might encounter up to the end of his or her course. The book is also appropriate for those beginning graduate study with a mathematical content, and naturally much of the material forms parts of courses for mathematics students. Furthermore, the text should provide a useful reference for research workers.

The general aim of the book is to present a topic in three stages. The first stage is a qualitative introduction, wherever possible from a physical point of view. The second is a more formal presentation, although we have deliberately avoided strictly mathematical questions such as the existence of limits, uniform convergence, the interchanging of integration and summation orders, etc. on the

Cambridge University Press

0521861535 - Mathematical Methods for Physics and Engineering, Third Edition

K. F. Riley, M. P. Hobson and S. J. Bence

Frontmatter

[More information](#)

PREFACE TO THE FIRST EDITION

grounds that ‘this is the real world; it must behave reasonably’. Finally a worked example is presented, often drawn from familiar situations in physical science and engineering. These examples have generally been fully worked, since, in the authors’ experience, partially worked examples are unpopular with students. Only in a few cases, where trivial algebraic manipulation is involved, or where repetition of the main text would result, has an example been left as an exercise for the reader. Nevertheless, a number of exercises also appear at the end of each chapter, and these should give the reader ample opportunity to test his or her understanding. Hints and answers to these exercises are also provided.

With regard to the presentation of the mathematics, it has to be accepted that many equations (especially partial differential equations) can be written more compactly by using subscripts, e.g. u_{xy} for a second partial derivative, instead of the more familiar $\partial^2 u / \partial x \partial y$, and that this certainly saves typographical space. However, for many students, the labour of mentally unpacking such equations is sufficiently great that it is not possible to think of an equation’s physical interpretation at the same time. Consequently, wherever possible we have decided to write out such expressions in their more obvious but longer form.

During the writing of this book we have received much help and encouragement from various colleagues at the Cavendish Laboratory, Clare College, Trinity Hall and Peterhouse. In particular, we would like to thank Peter Scheuer, whose comments and general enthusiasm proved invaluable in the early stages. For reading sections of the manuscript, for pointing out misprints and for numerous useful comments, we thank many of our students and colleagues at the University of Cambridge. We are especially grateful to Chris Doran, John Huber, Garth Leder, Tom Körner and, not least, Mike Stobbs, who, sadly, died before the book was completed. We also extend our thanks to the University of Cambridge and the Cavendish teaching staff, whose examination questions and lecture hand-outs have collectively provided the basis for some of the examples included. Of course, any errors and ambiguities remaining are entirely the responsibility of the authors, and we would be most grateful to have them brought to our attention.

We are indebted to Dave Green for a great deal of advice concerning typesetting in $\text{\LaTeX}$ and to Andrew Lovatt for various other computing tips. Our thanks also go to Anja Visser and Graça Rocha for enduring many hours of (sometimes heated) debate. At Cambridge University Press, we are very grateful to our editor Adam Black for his help and patience and to Alison Woollatt for her expert typesetting of such a complicated text. We also thank our copy-editor Susan Parkinson for many useful suggestions that have undoubtedly improved the style of the book.

Finally, on a personal note, KFR wishes to thank his wife Penny, not only for a long and happy marriage, but also for her support and understanding during his recent illness – and when things have not gone too well at the bridge table! MPH is indebted both to Rebecca Morris and to his parents for their tireless

Cambridge University Press

0521861535 - Mathematical Methods for Physics and Engineering, Third Edition

K. F. Riley, M. P. Hobson and S. J. Bence

Frontmatter

[More information](#)

PREFACE TO THE FIRST EDITION

support and patience, and for their unending supplies of tea. SJB is grateful to Anthony Gritten for numerous relaxing discussions about J. S. Bach, to Susannah Ticciati for her patience and understanding, and to Kate Isaak for her calming late-night e-mails from the USA.

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