Magnetic fields permeate space and affect many major astrophysical phenomena, but they are often ignored due to their perceived complexity. This self-contained introduction to astrophysical magnetic fields provides both a comprehensive review of the current state of the subject and a critical discussion of the latest research. It presents our knowledge of magnetic fields from the Early Universe, their evolution in cosmic time through to their roles in present-day galaxies, galaxy clusters and the wider intergalactic medium, with attention given to both theory and observations. This volume also contains an extensive introduction into magnetohydrodynamics, numerous worked examples, observational and mathematical techniques and interpretations of the observations. Its review of our current knowledge, with an emphasis on results that are likely to form the basis for future progress, benefits a broad audience of advanced students and active researchers, including those from fields such as cosmology and general relativity.

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From Galaxies to the Early Universe

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Preface

This is a book on the astrophysics of magnetic fields on galactic and extragalactic scales, on their origin, structure and evolution with cosmic time. When discussing magnetic fields and their significance, we emphasize the physical foundations attempting to present a coherent, unified picture of the astronomical objects and processes. Astrophysics is an observational science. Therefore, we present the theoretical picture within a rich framework of observational information: this picture cannot exist without its frame. When discussing the observations, we try to put them, again, on a firm physical ground presenting observational results in a physical context rather than as isolated facts. The theoretical picture and its observational framework thus merge and become inseparable. We make an effort to present observations in a form understandable to theoreticians while discussing theory in a form suitable for observers. This is a difficult task, and our success can only be judged by the reader.

We believe that now is the right time to write a new book on the subject. Both observations and theory have made very substantial progress since the earlier book on a similar subject (Ruzmaikin et al., 1988b). These new results need to be summarized, generalized, unified, and put into the general physical context in a manner only a self-contained book can do. This book attempts to do just that.

With this attitude in mind, we felt it necessary to introduce the physical background of magnetohydrodynamics (MHD), elements of plasma physics, some of cosmic-ray dynamics, theory of synchrotron radiation, general relativity and cosmology. As a consequence, some parts of the book balance on a narrow borderline between a textbook and a research monograph. The result is reasonably self-contained but the length of the text has unavoidably increased. We are neither the first nor the last to admit this problem (‘I am sorry this letter is so long; I did not have time to make it shorter’ – Blaise Pascal), but we have made an effort to make it as short as practical. Many readers will find that they can skip the basics and read sections and chapters on current research, where we present the latest results and try to anticipate future developments.
The parts of the text that summarize the necessary background may be useful as an introduction to advanced textbooks, they may also help to consolidate any existing knowledge. Throughout the text, mathematical derivations and physical arguments are supplied with sufficient detail as to allow the reader to reproduce them without excessive effort. We have been systematically replacing phrases like ‘it can easily be shown’, which are numerous in the first draft, with details of the calculations. Many examples and calculations can be converted into problems and projects for an advanced undergraduate or postgraduate course on astrophysical magnetohydrodynamics.

Our exposition is unified by the common basis of the general physical theory of magnetic fields in a moving, electrically conducting medium, especially in turbulent flows. We treat magnetic fields in a broad variety of astrophysical objects within a coherent physical and mathematical framework with the dynamo theory as its cornerstone. The discussion also extends further to processes in the Early Universe. Our goal is to provide a logical and, as far as possible, reasonably complete picture of the origin, evolution, observed properties and physical effects associated with magnetic fields in a range of astrophysical objects from galactic to cosmological scales. We do not discuss the magnetic fields of stars and planets because the physical processes involved are somewhat different. However, the significance and applicability of the general dynamo paradigm developed here extend to radio galaxies, active galactic nuclei, stars, accretion discs and planets.

A reader interested in just one type of object (say, elliptical galaxies) would not need to read the whole book. We have structured the text with such a reader in mind. The book consists of a few core chapters where the basic physical ideas and mathematical techniques are introduced (at both heuristic and more formal levels), and several themes are threaded throughout the whole text which causes unavoidable repetition justified by our attempt to make some parts of the text as self-contained as practical. Various parts of the text contain models and results which were not published before as they developed in the process of writing this book.

We dedicate this book to our extended families. The intellectual accomplishments of the previous two generations were our lifelong inspiration, and the daily support, patience and encouragement of the next two generations have made our work possible. We were fortunate to have an opportunity to learn from Shashikumar Chitre, Jayant Narlikar and Yakov Borisovich Zeldovich from the early stages of our academic lives; their wisdom and depth remain our guiding light. We are grateful to John Barrow, Rainer Beck, Elly Berkhuijsen, Axel Brandenburg, Leon Mestel, Rajaram Nityananda, Jeremiah Ostriker, Martin Rees, Alexander Ruzmaikin, Dmitry Sokoloff, Ethan Vishniac, Richard Wielebinski and many others who have deeply influenced our thoughts and opinions. It is our pleasure to
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