

Index

- 1-cocycle, 25, 57
- 2-cocycle, 58
- absolute Galois group, 81
 - of finite field, 83
 - of Laurent series field, 83
- acyclic complex, 51
- affine closed set, 298
- affine variety, 298
- Albert form, 13
- Albert's cyclicity problem, 209
- Albert's theorem
 - on algebras in characteristic p , 264
 - on algebras of degree four, 15
 - on biquaternion algebras, 13
- Albert–Brauer–Hasse–Noether theorem, 159
- Amitsur's conjecture, 126
- Amitsur's counterexample, 23
- Amitsur's theorem, 125
- Amitsur–Rowen–Tignol counterexample, 16
- ample divisor, 307
- Artin–Schreier theorem, 113, 180
- Artin–Schreier theory, 94
- Ax's counterexample, 142
- Bass–Tate lemma, 194
- biquaternion algebra, 12
- birational invariant, 160
- Bloch–Gabber–Kato theorem, 280
- Bloch–Kato conjecture, 108, 213
- blowup, 302
- Bockstein homomorphism, 214
- Bogomolov's theorem, 164
- Brauer equivalence, 31
- Brauer group, 32
 - m -torsion part of, 99
 - and differential forms, 268
 - and purely inseparable base extension, 180
 - as first cohomology group, 32
 - as second cohomology group, 99
 - inverse of an element in, 31
 - is a torsion group, 99
 - of \mathbf{R} , 113
 - of $\mathbf{F}(t)$, 150
- of $\mathbf{F}(C)$, 157, 159
- of $k_s((t))$, 149
- of a number field, 159
- of cyclic extension, 100, 111
- relative, 32
- unramified, 161
- C_1 -field, 140
- C_r -field, 141
- C'_r -field, 141
- Cartier isomorphism, 278
- Cartier operator, 278, 297
 - inverse, 267
 - over rings, 278
- Castelnuovo's theorem, 160
- central algebra over a field, 4
- central simple algebra, 17
 - as twisted form of matrix algebra, 20
 - base change of, 20
 - dimension of, 21
 - of prime degree, 209
 - over $\mathbf{F}((t))$, 151
 - over $\mathbf{F}(C)$, 144
 - over a Laurent series field, 149
 - over a number field, 160
 - over an algebraically closed field, 20
 - product of, 30
- Châtelet's theorem, 115, 125
- chain lemma, 216
 - for quaternion algebras, 16, 217
 - over number fields, 219
- Chevalley's theorem, 142
- Chow group, 228
- closed point, 299
- co-induced module, 60, 61
 - for cohomology of profinite groups, 88
- coboundary, 57
- coboundary map, 52
 - in group cohomology, 58
- cochain, 57
 - continuous, 112
 - inhomogeneous, 57
 - normalized, 58

- cocycle, 57
- codimension, 303
- Cohen structure theorem, 309
- cohomological dimension
 - of \hat{Z} , 136
 - of a field, 138
 - of a finite field, 140
- cohomology group, 55
 - of profinite group, 86
- completion of ring, 308
- complex of modules, 51
- conic
 - as Severi–Brauer variety, 120, 129
 - associated with quaternion algebra, 7
- conjugation action on cohomology, 64
- connecting homomorphism, 52
- connection, 270
- coordinate ring, 298
- coresidue map, 179
- corestriction map, 62
 - and coresidue map, 179
 - and Galois symbol, 211
 - for cohomology of profinite groups, 88
 - for purely inseparable extensions, 176
- crossed product, 23
- cup-product, 72
 - and conjugation, 76
 - and inflation maps, 76
 - and long exact sequences, 73, 75
 - and restriction maps, 76
 - associativity of, 73
 - for cohomology of profinite groups, 90
 - functoriality of, 72
 - graded-commutativity of, 73
- cyclic algebra, 33
 - Brauer class of, 109
 - in characteristic p , 36, 260
 - presentation of, 33, 36
 - splitting of, 111, 112
- cyclic group, cohomology of, 59, 60, 77
- d -uple embedding, 118, 301
- de Rham complex, 322
 - decomposition of, 276
- degree
 - of central simple algebra, 21
 - of divisor class on Severi–Brauer variety, 126
 - of divisor on curve, 151, 307
 - of divisor on curve over a finite field, 157
- derivation, 314
- descent for inseparable extensions of height one, 274
- differential forms, 318
 - higher, 321
 - logarithmic, 271
- differential symbol, 280
 - and norm map, 281
 - injectivity of, 288
 - surjectivity of, 282
- dimension of variety, 302
- dimension shifting, 68
- direct limit, 86
 - of cohomology groups, 91
 - of exact sequences, 91
- direct system, 86
- discrete valuation, 310
 - ring, 311
- division algebra, 1
- divisor, 305
 - exceptional, 302
 - of rational function, 306
 - positive, 307
 - support of, 305
- dlog map, 266
- double complex, 69
- excellent
 - local ring, 308
 - scheme, 224
- Faddeev’s exact sequence, 155
 - for Brauer groups, 156
 - with finite coefficients, 176
- Fischer’s theorem, 163
- flat connection, 271
- free module, 53
- Frobenius’s theorem, 113
- function field, 300
 - of conic, 9
- G -module, 50
 - continuous, 85
- Galois cohomology, 86
 - is torsion, 87
 - of \mathbb{Q} -vector space is trivial, 87
- Galois descent, 25
 - for central simple algebras, 30
 - for quadratic forms, 26
 - for Severi–Brauer varieties, 117
- Galois symbol, 108
 - and specialization map, 211
 - and tame symbol, 210
 - and transcendental extensions, 212
 - over number fields, 216
 - surjectivity of, 212

Cambridge University Press

978-0-521-86103-8 - Central Simple Algebras and Galois Cohomology

Philippe Gille and Tamás Szamuely

Index

[More information](#)*Index*

341

- Galois theory for infinite extensions, 84
 generic point, 299
 of conic, 9
 Gersten complex in Milnor K-theory, 224
 localization property of, 228
 Mayer-Vietoris property of, 228
 Gersten conjecture, 231
 graded-commutative, 69
 Greenberg's approximation theorem,
 144
 group extension, 58
 and alternating forms, 167
 cohomology class of, 59
 pullback of, 64
 pushforward of, 59
- $H^1(G, A)$ for non-commutative A , 25
 Hasse invariant, 150
 height of prime ideal, 303
 Hensel's lemma, 309
 refined, 310
 Hilbert's Theorem 90, 26
 for K_2 , 239, 257
 for Galois cohomology, 93
 for the additive group, 95
 generalization of, 41
 original form of, 26, 60
 Hochschild's formula, 315
 Hochschild's theorem, 260
 homogeneous coordinate ring, 299
- index
 of central simple algebra, 100
 and base extension, 104
 and degrees of splitting fields, 103
 of Brauer class, 100
 of Severi–Brauer variety, 125
 inertia group, 313
 inflation map, 63
 for cohomology of profinite groups,
 88
 inflation-restriction sequence, 65, 67
 for cohomology of profinite groups, 93
 inverse Cartier operator, 267
 inverse limit, 81
 inverse system, 81
 irreducible component, 298
 Izhboldin's theorem, 281
- Jacobson correspondence, 275
 Jacobson polynomial, 315
 Jacobson's formula, 316
 Jacobson–Cartier theorem, 267, 271, 278, 297
- K_1 of a ring, 42
 Künneth formula, 276
 Kang's construction, 118
 K^M -homology, 228
 and Zariski cohomology, 232
 homotopy invariance of, 229
 of projective space, 231
 Krull dimension
 of ring, 303
 of variety, 303
 Kummer theory, 93
- Lüroth's theorem, 160
 Lang's theorem
 on abelian varieties over finite fields,
 157
 on Laurent series fields, 144
 Lichtenbaum's theorem, 129
 Lie algebra of derivations, 317
 linear system, 307
 linearly equivalent divisors, 306
 local parameter, 311
 local ring, 300
 of subvariety, 300
 long exact sequence, 51
 for cohomology of profinite groups, 90
 for noncommutative H^1 , 40, 95
- matrix algebra
 automorphism group of, 29
 is simple, 18
 left ideals in, 18
 maximal prime to p extension, 195
 Merkurjev's theorem, 16
 Merkurjev–Suslin theorem, 37, 108,
 109, 255
 Milnor K-theory, 108
 p -torsion in, 245
 functoriality of, 183
 graded-commutativity of, 184
 of a field of cohomological dimension one,
 221
 of a finite field, 184
 of an algebraically closed field, 220
 products in, 183
 torsion in, 244
 Milnor's exact sequence, 190
 and Faddeev's exact sequence, 211
 morphism of varieties, 301
- No-name lemma, 181
 Noether's problem, 171
 norm group, 134

- norm map
 - on K_1 , 192
 - and Galois symbol, 211
 - for Milnor K-theory, 192, 196, 221
 - for Milnor K-theory and base extension, 197
 - on the cohomology of a subgroup, 79
- opposite algebra, 32
- p -basis, 320
- p -cohomological dimension, 136
 - and the Brauer group, 138
 - of a field, 138
 - of a pro- p group, 137
 - of fields of characteristic p , 139
 - strict, 136
- p -Lie algebra, 317
- p -connection, 271
- period
 - of central simple algebra, 104
 - of Severi–Brauer variety, 118
- period-index questions, 105
- Picard group, 306
 - of affine space, 306
 - of projective space, 306
- point
 - rational, on conic, 7
- pointed set, 25
- primary decomposition of central simple algebra, 105
- pro- p -Sylow subgroup, 89
- profinite completion, 82
- profinite group, 81
 - cohomology of, 86
 - has torsion cohomology groups, 87
 - topology of, 83
- projection formula
 - in group cohomology, 76
 - in Milnor K-theory, 193
- projective
 - module, 53
 - resolution, 53
- projective closed set, 299
- projective variety, 299
- pure quaternion, 3
- purely inseparable extension of height one, 263
- quaternion, 1
 - conjugate of, 2
- quaternion algebra, 2
 - has period two, 13
 - over finite field, 8
 - split, 3
- quaternion norm, 2
 - as reduced norm, 6
- ramification
 - tame, 313
 - wild, 313
- ramification index, 312
- rational function, 300
- rational map, 301
- rational point, 299
- rational variety, 160
- reduced norm, 37
 - and norm group of Severi–Brauer variety, 134
 - cokernel of, 41
 - for K_1 , 46
 - on division algebra, 38
- reduced trace, 38
- residue field of a point, 300
- residue map
 - and cup-products, 173
 - for Milnor K-theory, 185
 - for the Brauer group, 147
 - for the cohomology of function fields, 153
 - with finite coefficients, 174
- Residue Theorem, 154, 181
- restriction map, 62
 - for cohomology of profinite groups, 87
- Rieffel's lemma, 19
- Rosset–Tate reciprocity law, 207
- Rosset–Tate symbol, 206
- Saltman's example, 170
- Saltman's theorem, 131
- scheme-theoretic point, 299
- Schur's lemma, 19
- separably generated field, 303
- Severi–Brauer variety, 115
 - Brauer class of, 124
 - minimal, 124
- Shafarevich's example, 167
- Shapiro's lemma, 61
 - for cohomology of profinite groups, 88
- simple algebra, 17
- singular
 - locus, 300
 - point, 300
- Skolem–Noether theorem, 40
- smooth
 - point, 300
 - variety, 300
- snake lemma, 52

Index

343

- specialization map
 - and Galois symbol, 211
 - and norm map, 204
 - for Galois cohomology, 175
 - for Milnor K-theory, 185
- Speiser's lemma, 27
- splitting field, 21
 - Galois, 22, 117
 - of Severi–Brauer variety, 115
 - separable, 22, 101, 117
 - solvable, 37
- stably birational, 126
- standard resolution, 56
- Steinberg relation
 - for cyclic algebras, 49
 - for Galois symbol, 106
 - for Milnor K-theory, 183
 - for quaternion algebras, 8
 - for the Galois symbol, 182
- symbol
 - in Milnor K-theory, 108
 - of length one, 216
 - splitting of, 112
- tame symbol, 185, 187
 - and base extension, 187
 - and Galois symbol, 210
 - and norm map, 204
 - and specialization map, 187
 - kernel and cokernel of, 187
- Tate's theorem on the Galois symbol, 216
- Teichmüller's theorem, 263, 269
- tensor product of complexes, 69
- trace map for differential forms, 281
- transcendence basis, 302
- transcendence degree, 302
- transgression map, 67
- Tsen's theorem, 143
- twisted form, 25
- twisted-linear subvariety, 115
 - Brauer class of, 121, 122
- twisting by a cocycle, 26
- unirational variety, 160
- unramified Brauer group, 161
- unramified extension, 312
- Voevodsky's theorem, 108
- Wang's theorem, 47
- Wedderburn's theorem
 - on algebras of degree 3, 222
 - on finite fields, 143
 - on simple algebras over a field, 18
- Weil reciprocity law, 192, 205
- Whitehead's Lemma, 42
- Witt's theorem
 - on quaternion algebras, 9, 129
 - on the Brauer group of a Laurent series field, 149
- Zariski topology, 299