Central Simple Algebras and Galois Cohomology

This book is the first comprehensive, modern introduction to the theory of central simple algebras over arbitrary fields. Starting from the basics, it reaches such advanced results as the Merkurjev–Suslin theorem. This theorem is both the culmination of work initiated by Brauer, Noether, Hasse and Albert in the 1930s and the starting point of motivic cohomology theory, a domain at the forefront of current research in algebraic geometry and K-theory and the setting of recent spectacular work by Voevodsky, Suslin, Rost and others.

Assuming only a solid background in algebra, but no homological algebra, the book covers the basic theory of central simple algebras, methods of Galois descent and Galois cohomology, Severi–Brauer varieties, residue maps and, finally, Milnor K-theory and K-cohomology. A number of noteworthy additional topics are also covered. The last chapter rounds off the theory by presenting the results in positive characteristic, including the theorem of Bloch–Gabber–Kato. The book is suitable as a textbook for graduate students and as a reference for researchers working in algebra, algebraic geometry or K-theory.

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Cambridge University Press
978-0-521-86103-8 - Central Simple Algebras and Galois Cohomology
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CAMBRIDGE
UNIVERSITY PRESS

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UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

Published in the United States of America by Cambridge University Press, New York

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It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9780521861038

First published 2006

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-86103-8 Hardback

Cambridge University Press
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Preface

This book provides a comprehensive and up-to-date introduction to the theory of central simple algebras over arbitrary fields, emphasizing methods of Galois cohomology and (mostly elementary) algebraic geometry. The central result is the Merkurjev–Suslin theorem. As we see it today, this fundamental theorem is at the same time the culmination of the theory of Brauer groups of fields initiated by Brauer, Noether, Hasse and Albert in the 1930s, and a starting point of motivic cohomology theory, a domain which is at the forefront of current research in algebraic geometry and K-theory – suffice it here to mention the recent spectacular results of Voevodsky, Suslin, Rost and others. As a gentle ascent towards the Merkurjev–Suslin theorem, we cover the basic theory of central simple algebras, methods of Galois descent and Galois cohomology, Severi–Brauer varieties, residue maps and finally, Milnor K-theory and K-cohomology. These chapters also contain a number of noteworthy additional topics. The last chapter of the book rounds off the theory by presenting the results in positive characteristic. For an overview of the contents of each chapter we refer to their introductory sections.

Prerequisites The book should be accessible to a graduate student or a non-specialist reader with a solid training in algebra including Galois theory and basic commutative algebra, but no homological algebra. Some familiarity with algebraic geometry is also helpful. Most of the text can be read with a basic knowledge corresponding to, say, the first volume of Shafarevich's text. To help the novice, we summarize in an appendix the results from algebraic geometry we need. The first three sections of Chapter 8 require some familiarity with schemes, and in the proof of one technical statement we are forced to use techniques from Quillen K-theory. However, these may be skipped in a first reading by those willing to accept some 'black boxes'.

Acknowledgments

Our first words of thanks go to Jean-Louis Colliot-Thélène and Jean-Pierre Serre, from whom we learned much of what we know about the subject and who, to our great joy, have also been the most assiduous readers of the manuscript, and suggested many improvements. Numerous other colleagues helped us with their advice during the preparation of the text, or spotted inaccuracies in previous versions. Thanks are due to Spencer Bloch, Jean-Benoît Bost, Irene Bouw, Gábor Braun, Ferenc Bródy, Jérôme Burési, Baptiste Calmès, Vladimir Chernousov, Mathieu Florence, Ofer Gabber, Skip Garibaldi, Luc Illusie, Bruno Kahn, Max-Albert Knus, David Leep, David Madore, Alexander Merkurjev, Ján Mináč, Arturo Pianzola, Peter Roquette, Joël Riou, Christophe Soulé, Jean-Pierre Tignol, Burt Totaro and Stefan Wewers.

Parts of the book formed the basis of a graduate course by the first author at Université de Paris-Sud and of a lecture series by the two authors at the Alfréd Rényi Institute. We thank both audiences for their pertinent questions and comments, and in particular Endre Szabó who shared his geometric insight with us. Most of the book was written while the first author visited the Rényi Institute in Budapest with a Marie Curie Intra-European Fellowship. The support of the Commission and the hospitality of the Institute are gratefully acknowledged. Last but not least, we are indebted to Diana Gillooly for assuring us a smooth and competent publishing procedure.