

THE SOLAR TACHOCLINE

Helioseismology has enabled us to probe the internal structure and dynamics of the Sun, including how its rotation varies in the solar interior. The unexpected discovery of an abrupt transition – the tachocline – between the differentially rotating convection zone and the uniformly rotating radiative interior has generated considerable interest and raised many fundamental issues. This volume contains invited reviews from distinguished speakers at the first meeting devoted to the tachocline, held at the Isaac Newton Institute. It provides the only comprehensive account of the current understanding of the properties and dynamics of the tachocline, including both observational results and major theoretical issues, involving both hydrodynamic and magnetohydrodynamic behaviour.

The Solar Tachocline is a valuable reference for researchers and graduate students in astrophysics, heliospheric physics and geophysics, and the dynamics of fluids and plasmas.

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CAMBRIDGE UNIVERSITY PRESS Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521861014

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First published 2007

Printed in the United Kingdom at the University Press, Cambridge

A catalogue record for this publication is available from the British Library

ISBN-13 978-0-521-86101-4 hardback

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Preface

Over the past 25 years helioseismology has at last enabled us to probe the internal structure and dynamics of our local star, the Sun. Perhaps its greatest triumph has been to determine how the rotation varies in the solar interior. Although the bulk of the radiative zone, occupying the innermost 70% by radius, rotates more or less uniformly, the known variation with latitude of angular velocity at the surface persists down to the base of the outer convective envelope. Since it had previously been supposed that the Sun rotates sufficiently rapidly for the angular velocity to be constant on cylindrical surfaces in the convection zone it was a surprise to find that it is actually constant on conical surfaces. It came as an even greater surprise to discover that the transition between the differentially rotating exterior and the uniformly rotating interior is effected through an extremely thin layer – the tachocline – whose thickness is less than 4% of the solar radius.

This unexpectedly abrupt transition has forced us all to refine our ideas on the interactions between turbulent convection, rotation and magnetic fields, for it seems that these last play a key role in preventing the tachocline from spreading downwards into the radiative zone. To describe the internal structure of the tachocline requires an understanding of convective penetration, turbulent diffusion, mixing and angular momentum transport. This shear layer also hosts a great range of potential instabilities and, furthermore, is the most likely seat for the dynamo responsible for the cyclic magnetic activity that is observed at the surface of the Sun. These physical processes are all of interest in themselves and not only raise major issues in astrophysical fluid dynamics, but also relate to significant problems in the physics of the Earth's oceans and atmosphere, in planetary physics, and in plasma confinement. More importantly, an understanding of the dynamics of the tachocline is essential not just in order to match the interior to the exterior of the Sun – and hence to establish its global properties – but also to explain the structure, evolution and magnetism of all similar stars with deep convection zones.

The programme on *Magnetohydrodynamics of Stellar Interiors* held at the Issac Newton Institute for Mathematical Sciences in Cambridge, which we organized in



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2004, provided a timely opportunity for holding a workshop on *Tachocline Dynamics*. This one week meeting (8–12 November 2004) was the first ever to be entirely devoted to the subject and it brought almost all the key players together to discuss it. The workshop was informally structured, our aim being to maximize opportunities for argument and discussion. The number of invited lectures was therefore restricted, so as to leave plenty of time for structured discussions, led and organized by appropriate experts. In all, there were a dozen talks that introduced the principal topics, and eight hours of scheduled discussion, which continued informally among the 48 participants outside the lecture room. This format worked extremely well and the workshop was felt to be a great success. It certainly identified all the main issues, although no consensus on detailed models was expected or achieved.

There was a general feeling among the participants that, while tachocline dynamics remained a young and rapidly developing subject, it had now reached a stage where it was ready for description in a book. The workshop provided the ideal basis for such a volume and this is the result. Rather than produce a volume of proceedings, it was agreed that we should edit a book containing invited chapters from selected participants, each of which would be refereed by another participant. We are pleased that there is a wide range of age and experience among the authors; most of them are known for their work in Astrophysical Fluid Dynamics, but we also include chapters by experts in Geophysical Fluid Dynamics and Plasma Physics.

The chapters are grouped into different sections covering the main areas of our subject. The opening section contains two essays; the first provides a comprehensive introductory survey, while the second focuses on the history and pre-history of the relevant theoretical ideas. The next section presents the fundamental observational results, which underpin all of the theory that follows. Section III contains three chapters on purely hydrodynamic aspects of the tachocline, covering its structure, turbulence, and differential rotation. Magnetic fields enter next, in Section IV, with two chapters on the magnetic confinement of the tachocline and one on the influence of rotation on magnetohydrodynamic turbulence. This is followed by a section containing three chapters reviewing instabilities driven by rotational shear, magnetic fields and buoyancy. Section VI is concerned with the generation of large-scale magnetic fields by dynamo action, whether in the turbulent convection zone or, more likely, in the tachocline itself. The book then concludes with an overview that summarizes current controversies and points to future progress. There is inevitably some overlap between the different chapters and, given the lack of certainty about the nature of the tachocline, authors have not been restrained from expressing contradictory views. While we have tried to maintain some degree of uniformity regarding notation, we have not attempted to impose complete uniformity, nor have we prevented authors from using their preferred choices of electromagnetic and other units.



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We thank all those who have helped to make this book possible. We are grateful to the staff of the Isaac Newton Institute, particularly Tracey Andrew and Christine West, for making the workshop possible as part of the overall programme at the Institute. We are also indebted to the members of the Scientific Organising Committee, Pascale Garaud, Douglas Gough and Jean-Paul Zahn.

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