Part I

Setting the scene

1

An introduction to the solar tachocline

Douglas Gough

1.1 Preamble

The task that I have been assigned is to set the scene for the discussions that follow: to present my view of the principal issues that had confronted us before the meeting when trying to understand the dynamics of the solar tachocline. Most of what I write here is enlarged upon, and in some cases superseded by, the chapters that follow, in which references to most of the original publications can also be found. Nevertheless, I trust that it can serve as a useful elementary introduction to the subject, setting it into its wider astronomical context.

The tachocline is interesting to astrophysicists for a variety of reasons, the most important being (i) that it couples the radiative interior of the Sun, where nearly 90% of the angular momentum resides, to the convection zone, which is being spun down by the solar wind, (ii) that it controls conditions at the lower boundary of the convection zone, and is therefore an integral component of the overall rotational dynamics of the convection, and (iii), perhaps most relevant to the interests of the greater proportion of the participants of the workshop, it is now generally recognized as being the seat of the solar dynamo. It plays some role in shaping the evolution of the Sun, and it must be taken into account when interpreting the helioseismological diagnostics of the solar structure.

It is therefore perhaps useful first for me to make a few remarks about some properties of the Sun that are pertinent to the dynamics of the tachocline, and to agree on a practical definition, or at least a description, of what we even mean by the tachocline. I shall then discuss the two existing dynamical descriptions of the tachocline, and the extent to which I consider them to represent, or not, the response of the Sun to the predominant balance of forces in what in reality are circumstances much more complicated than those to which the idealized theories really apply. This raises many theoretical issues, some of which are supported either directly or indirectly by experiment (either physical or numerical) or by

The Solar Tachocline, D. W. Hughes, R. Rosner and N. O. Weiss.
Published by Cambridge University Press. © Cambridge University Press 2007.

astronomical observation; more might also be so supported in the not-too-distant future, others not.

My discussion is not a balanced account of the various (disparate) views of the community; it is peppered with my own opinions, with many of which the reader may take issue. In so doing, I hope that it will stimulate productive thought that will advance our understanding of this topical subject.

1.2 Some basic properties of the Sun

The Sun is a star on the Main Sequence: that is to say, it is in a state of hydrostatic and thermal balance in which energy that is being produced in a hot central core from thermonuclear transmutation of hydrogen into helium is being transported down a temperature gradient through the surrounding envelope, and is finally radiated into space from the photosphere, the visible surface of the Sun. This is generally regarded as a state of one of the simplest phases of the evolution of a star. It is certainly the most well studied. It is also the longest phase, at least before the star finally condenses into a degenerate configuration such as a white dwarf or a neutron star, and so most ordinary (non-degenerate) stars are currently in their Main-Sequence phase. The duration of this phase, which ends when the supply of hydrogen fuel is exhausted from the centre of the core, depends on the mass of the star; for the Sun it is about 1.0×10^{10} years. The solar age is about 4.6×10^{9} years, so the Sun is nearly half-way through its Main-Sequence life.

The Sun's core (the region in which, say, 95% of the thermonuclear energy is generated) extends to about 20% of the radius $R_{\odot}$ of the photosphere, and contains about 35% of the total mass of the star. The surrounding envelope is divided into two principal regions: a quiescent radiative region in which radiant heat diffuses down the temperature gradient, extending to a radius, r_{c}, of about $0.71R_{\odot}$, and an overlying turbulent convective region which extends to the photosphere. These two regions merge in – some would say are separated by – a thin boundary layer, which is the tachocline, on which I shall enlarge very shortly. But before doing so I should perhaps attempt to avert misunderstanding by drawing attention to a potential source of confusion brought about by some newcomers to the field who call the radiative envelope the core. I hope that any inadvertent occurrence of that misnomer in this book will be subliminally ignored. I shall sometimes use the term 'radiative interior' to denote the entire radiative region beneath the convection zone, encompassing both the radiative envelope and the core.

At this point it is perhaps useful to mention a few more timescales. The thermal diffusion time τ_{th} through the radiative envelope, namely $r_{\mathrm{c}}^2/\pi^2\overline{\kappa}$, where $\overline{\kappa} = 7.3 \times 10^5\ \mathrm{cm}^2\ \mathrm{s}^{-1}$ is an appropriate average (the square of the harmonic mean, over radius r, of the square root) of the thermal diffusivity κ, is 1.2×10^7 years; the viscous timescale, in the absence of boundary layers, is about 10^{12} years, substantially greater than the age of the Universe; and the thermal relaxation time of the convection zone, namely the characteristic time over which the zone would return to thermal equilibrium after a putative global perturbation to conditions at its base, is 10^5 years. The timescale for Ohmic decay of a large-scale magnetic field pervading the radiative interior is of order 10^{11} years. All four timescales are much greater than the characteristic times over which most of the dynamical processes discussed in this volume operate, and so for many purposes the phenomena directly associated with them can be ignored. On the other hand, the characteristic global dynamical timescale, which is given by the acoustic travel time from the centre to the surface of the Sun, is about 1 h, which is much shorter; therefore the radiative envelope and the core remain quite precisely in hydrostatic equilibrium on the timescales of major interest here.

The (sidereal) rotation period of the essentially uniformly rotating radiative interior of the Sun is about 27 days, corresponding to the angular velocity $\Omega_0 = 2.7 \times 10^{-6}\,\mathrm{s}^{-1}$. This is comparable with the characteristic turnover time of the large convective eddies in the lower reaches of the convection zone. The angular velocity in the convection zone varies only weakly with depth, and declines gradually from $2.9 \times 10^{-6}\,\mathrm{s}^{-1}$ at the equator to about $2.0 \times 10^{-6}\,\mathrm{s}^{-1}$ at the poles.

It should be realized that the convection zone contains only about 2% of the mass of the star. Therefore any reasonable redistribution of matter in the convection zone brought about by convection dynamics has very little impact on the overall weight of the envelope pressing down on the core. Therefore the core evolves inexorably on its own timescale of 10^{10} years, untrammelled by the machinations of the dynamically active outer envelope. I should mention also the Eddington–Sweet timescale, τ_{ES}, the circulation time of large-scale meridional flow through the radiative envelope that is associated with the baroclinicity induced by rotation, and which is enabled by thermal diffusion; it is the thermal diffusion time divided by the square of the rotational Froude number $Fr = 2\Omega_0/\overline{N}$ (where $\overline{N}$ is the value of the buoyancy frequency characteristic of the body of the radiative envelope, about $2.5 \times 10^{-3}\,\mathrm{s}^{-1}$), namely $\tau_{\mathrm{ES}} = Fr^{-2}\tau_{\mathrm{th}} \simeq 2.5 \times 10^{12}$ years. There is also a contribution to the circulation from spin-down resulting from the extraction of angular momentum from the convection zone by the solar wind via the external magnetic field, which is currently slowing down the Sun on a timescale of $\tau_{\mathrm{sd}} \simeq 10^{10}$ years, a time comparable with, although a little longer than, the solar age.

There are other timescales more pertinent to the subject-matter of this volume that will emerge in due course. Let me mention here just two. The first is the global Alfvénic time τ_{A}. This is the time it takes for an Alfvén wave to traverse the quiescent radiative interior of the Sun, and is characteristic of the period of a magnetically restored global torsional oscillation. It depends, of course, on the (harmonic) mean intensity of the large-scale magnetic field, which is unknown. Presumptions in the literature of its characteristic value vary widely, from zero, a most unlikely value, to the order of megagauss. Suffice it to say that if one adopts an intermediate value, say about 2 kG, a value characteristic of, though somewhat lower than, the fields observed in sunspot umbrae, then τ_A is about 22 years, a timescale which is central to issues discussed in this volume. It is therefore evident that no theory of the solar cycle can be considered complete unless it addresses the dynamical role of the radiative envelope. The second timescale is the internal thermal or rotational equilibration time of the convection zone: the time it takes for the convection zone to propagate (rather than merely to diffuse) a thermal or angular-momentum perturbation towards an internal equilibrium without the entire convection zone necessarily getting back into balance with its surroundings. That time is about one year, again not very different from the timescale of the solar cycle.

To conclude, I record in Table 1.1 characteristic values of various physical variables in the tachocline. They are evaluated at a radius $r = 0.70R_\odot$. The density, ρ, pressure, p, sound speed, c, and acceleration due to gravity, g, were obtained seismologically; the remaining, non-seismic, variables were inferred from a solar model (Model S) of Jørgen Christensen-Dalsgaard by adjusting it appropriately to be consistent with the seismic variables, and taking the relative hydrogen abundance by mass to be $X = 0.737$, as in the model. Under these conditions one can compute diffusion coefficients. There is considerable diversity amongst the values that one finds quoted in the literature; here I adopt what I consider to be the most reliable estimates: I evaluate the magnetic diffusivity η and the ion contribution ν_i to the kinematic viscosity from the formulae of Lyman Spitzer, using Georges Michaud and Charles Proffitt's more recent estimate of the Coulomb logarithm ($\ln \Lambda = 2.5$); I evaluate the photon-transport contribution ν_r to the viscosity from the formula of L. H. Thomas; I evaluate the helium–hydrogen diffusion coefficients from Michaud and Proffitt's extraction from the work of Paquette, Pelletier, Fontaine and Michaud. About 10% of the total kinematic viscosity $\nu = \nu_i + \nu_r$ and all of the thermal diffusivity κ come from photon transport; the magnetic diffusivity comes entirely from particle transport. From these coefficients one deduces a Prandtl number $\nu/\kappa \simeq 1.9 \times 10^{-6}$ and a magnetic Prandtl number $\nu/\eta \simeq 6.6 \times 10^{-2}$ characteristic of the tachocline. One can extrapolate the diffusion coefficients downwards through the radiative envelope using the approximate scaling laws $\eta \propto T^{-3/2} \ln \Lambda$, $\nu_i \propto T^{5/2}/\rho \ln \Lambda$, $\nu_r \propto T^4/\rho^2\hat{\kappa}$, $\kappa \propto T^3/\rho^2\hat{\kappa}$, $\chi \propto T^{5/2}/\rho \ln \Lambda$, with $\Lambda \propto \rho^{-1/2}T^{3/2}$, and k_p and k_T constant; $\hat{\kappa}$ is the opacity, which in the radiative interior satisfies roughly (to within 15%) the empirical law $\hat{\kappa} \propto \rho^{0.4}T^{-3}$. The Soret coefficients depend on chemical composition, and are each approximately proportional to $(1 - X^2)(3 + 5X)/(3 + X)$; they increase inwards through the core by about 50%. Beneath the tachocline the buoyancy frequency N increases gradually

Table 1.1. *Properties of the tachocline at $r = 0.70R_\odot$*

density	ρ	0.21	$\mathrm{g\,cm^{-3}}$
pressure	p	6.7×10^{13}	$\mathrm{g\,cm^{-2}s^{-2}}$
temperature	T	2.3×10^{6}	K
sound speed	c	2.3×10^{7}	$\mathrm{cm\,s^{-1}}$
opacity	$\hat{\kappa}$	19	$\mathrm{g^{-1}\,cm^{2}}$
gravitational acceleration	g	5.4×10^{4}	$\mathrm{cm\,s^{-2}}$
density scale height	H_ρ	$0.12R_\odot$	
pressure scale height	H_p	$0.08R_\odot$	
adiabatic exponent	γ_1	1.665	
buoyancy frequency	N	8×10^{-4}	$\mathrm{s^{-1}}$
magnetic diffusivity	η	4.1×10^{2}	$\mathrm{cm^{2}\,s^{-1}}$
kinematic viscosity	ν	2.7×10^{1}	$\mathrm{cm^{2}\,s^{-1}}$
thermal diffusivity	κ	1.4×10^{7}	$\mathrm{cm^{2}\,s^{-1}}$
helium diffusion coefficient	χ	8.7	$\mathrm{cm^{2}\,s^{-1}}$
pressure Soret factor	k_p	2.9	
temperature Soret factor	k_T	2.6	

The density and pressure scale heights are defined as $H_\rho = (-\mathrm{d}\ln\rho/\mathrm{d}r)^{-1}$ and $H_p = (-\mathrm{d}\ln p/\mathrm{d}r)^{-1} = c^2/\gamma_1 g$, where $\gamma_1 = (\partial\ln p/\partial\ln\rho)_s$ is the first adiabatic exponent, the partial thermodynamic derivative being taken at constant specific entropy s. The buoyancy frequency is defined by $N^2 = g(H_\rho^{-1} - gc^{-2})$; it rises with depth from essentially zero at the base of the convection zone to about 1×10^{-3} at the top of the tachopause. The (upwards) helium diffusion velocity v_{He} through hydrogen is given by $v_{\mathrm{He}} = -\chi(\mathrm{d}\ln Y/\mathrm{d}r - k_p\mathrm{d}\ln p/\mathrm{d}r - k_T\mathrm{d}\ln T/\mathrm{d}r)$, where Y is the helium abundance, whose value in the tachocline is 0.245. The diffusion velocity is defined such that $v_{\mathrm{He}}Y$ is the mass flux of helium through hydrogen. The total radius, mass and luminosity of the Sun are $R_\odot = 6.96 \times 10^{10}$ cm, $M_\odot = 1.99 \times 10^{33}$ g, and $L_\odot = 3.84 \times 10^{33}$ erg s^{-1}.

with depth to a maximum of about $2.9 \times 10^{-3}\,\mathrm{s^{-1}}$ at $r \simeq 0.1R_\odot$, and then declines to zero at $r = 0$.

1.3 Solar spin-down

Perhaps the most serious problem to be faced by any protostar is how to dispose of almost all of its angular momentum and magnetic field as it collapses to its eventual hydrostatic state. A globular region of mass $1M_\odot$ of dense interstellar gas, say with a density of 100 hydrogen atoms per cubic centimetre, rotating with an angular velocity equal to half the mean vorticity of the galactic rotation (namely, 1×10^{-8} y^{-1}, Oort's second constant) and pervaded by a not atypical 3 μG magnetic field, would, if it could conserve angular momentum and magnetic flux as it contracted to solar dimensions, end up rotating some 3×10^5 times faster than the Sun, and would possess a global magnetic field in the radiative interior of about 1TG, exceeding the solar value by perhaps a similar factor; the disrupting Lorentz force would be comparable with the binding gravitational force, and the centrifugal force would exceed the gravitational force by a factor in excess of 10^4. There has been a fair amount of intellectual effort expended on the angular momentum issue, on which I shall not dwell here, except to note that when the Sun first attained hydrostatic equilibrium, having lost almost all of its angular momentum, it was still rotating considerably faster than it is today. From that time on the solar-wind torque, acting on a timescale comparable with, although at present apparently somewhat longer than, the Main-Sequence age, has slowed down the convection zone, which, as I pointed out earlier, equilibrates on the very short timescale of about a year. A consequent issue that has exercised the minds of stellar physicists quite considerably is the extent to which the spin-down of the convection zone has been transmitted to the radiative interior. In the early days opinions varied widely, but now the matter has essentially been settled by helioseismology. I raise it here because it was in that context that the first idea of a tachocline emerged.

There was considerable debate in the 1960s and 1970s on the rotation of the solar interior, brought about partly by Bob Dicke's claim that the surface of the Sun is some six times more oblate (implying a 2.5-fold increase in ellipticity) than one would have expected had the Sun been rotating more-or-less uniformly throughout. The import of the claim was that the external gravitational equipotentials deviate from sphericity by more than 100 times expectation, and thereby destroy the precise agreement between observations of the rate of precession of the perihelion of the orbit of the planet Mercury and the prediction by General Relativity. (The reason why the Sun's surface and the gravitational equipotentials do not deform proportionately is that in near-uniform rotation only 4% of the deviation from sphericity of the surface layers arises from the asphericity of the gravitational equipotentials, the rest being the direct centrifugal distortion arising from the observed rotation of those layers.) Dicke argued that because the timescale for viscous diffusion of angular momentum through the quiescent radiative interior is substantially greater than the age of the Sun – indeed, as I mentioned earlier, it is greater than the age of the Universe – the core is still spinning with angular velocity comparable with the high value it had when the Sun arrived on the Main Sequence, and is therefore still substantially flattened by the centrifugal force, inducing oblateness in the gravitational field. Against that view, Ed Spiegel was the standard-bearer for realism, pointing out that, as for any issue with non-uniformly rotating fluids, spin-down must be considered in a dynamically consistent way, and that statements based on diffusion alone are highly misleading. (I should take this opportunity to remind the reader that that is so also for issues concerning the tachocline today.) After some preliminary toy-model studies with Derek Moore and Francis Bretherton, Spiegel addressed the influence of the strongly stable stratification on the baroclinic angular-momentum-transporting meridional flow induced by spin-down, and conceived of a growing sequence of thin Holton shear layers which transported angular momentum principally by advection, and which advanced from the base of the convection zone to the core, spinning down the entire Sun. Although the advance occurred on a timescale much less than the age of the Sun, it was nonetheless quite slow by dynamical standards, so perhaps Spiegel's original appellation 'tachycline' was not entirely appropriate.

It appears today that the existence of the tachocline is not directly dependent on spin-down, and is instead driven principally by the stresses maintaining the latitudinal differential rotation of the convection zone against a rigidly rotating envelope below. This conclusion derives from the helioseismological measurements of the variation of angular velocity in the solar interior (in addition to observational estimates of the spin-down rate), which it is appropriate now to describe.

1.4 The rotation of the Sun today

I should point out immediately, to avoid possible misunderstanding, that by angular velocity I mean the azimuthally averaged value, any deviations from that value being regarded as zonal flow. I must also point out that, except when I state explicitly to the contrary, all descriptions of the angular velocity Ω actually refer to an average of values at equal latitudes in the northern and southern hemispheres, which is what global helioseismology tells us. Observers almost invariably talk about rotation rate, whose value they quote typically in nanohertz. For brevity, values of Ω are sometimes quoted in nanohertz, but what is meant, of course, is values of $\Omega/2\pi$.

It has long been known from direct visual observation of tracers that the surface layers of the Sun rotate differentially, with angular velocity which is quite accurately described by the three-term expression

$$\Omega_s(\theta) = \Omega_e(1 - \alpha_2\mu^2 - \alpha_4\mu^4), \qquad (1.1)$$

where $\mu = \cos\theta$, θ being colatitude, and Ω_e, α_2 and α_4 are constants. Helioseismological inferences from the rotational degeneracy splitting of acoustic modes obtained originally and separately by Tim Brown, Ken Libbrecht and Jesper Schou, and their colleagues, have demonstrated that this kind of latitudinal variation persists almost unaltered down to the base of the convection zone, although, as has been emphasized recently by Peter Gilman and Rachel Howe, through the main body of the convection zone, at latitudes below about 70°, the angular-velocity contours depicted by Schou and his collaborators are more closely described as being inclined at a constant angle, about 30°, to the axis of rotation, except, of course, very near to the equator. At the base of the convection zone there is an abrupt transition to almost uniform angular velocity throughout the radiative envelope, with a value Ω_0 intermediate between the extremes of Ω_s. The angular velocity in the core is uncertain, although there is some evidence that its average is somewhat less than that in the surrounding envelope, a property which was not expected by anyone who had contemplated solar spin-down before helioseismology. The values of the constants defining Ω_s in the subsurface layers, at radius $r = 0.995R_\odot$, obtained seismologically by Schou and his colleagues, are $\Omega_e/2\pi \simeq 455\,\text{nHz}$, $\alpha_2 \simeq 0.12$, $\alpha_4 \simeq 0.17$; at $r = 0.75R_\odot$ the latitudinal variation of the angular velocity can be represented by the same expression with $\Omega_e/2\pi \simeq 463\,\text{nHz}$, $\alpha_2 \simeq 0.17$ and $\alpha_4 \simeq 0.08$ (implying a specific angular momentum, integrated over the sphere, the same as at $r = 0.995R_\odot$). Similar values are given by other investigators. The radiative interior rotates at a rate $\Omega_0/2\pi \simeq 430\,\text{nHz}$. The transition shear layer at the base of the convection zone – the object of study in this volume – is too thin to be resolved by current seismic data, but fits made by Sasha Kosovichev, Paul Charbonneau and others of simple functional forms have suggested values of its thickness Δ ranging from about $0.02R_\odot$ to $0.05R_\odot$. Most of the estimates rest on the assumption that the base of the convection zone is spherical. Any deviation from sphericity, for which there is some weak seismological evidence, smears a spherical view. Therefore the estimates of the thickness of the shear layer should probably be considered to be upper bounds.

To put our study into its context, I shall elaborate a little on the seismologically inferred angular velocity. These extra details do not all bear directly on the tachocline, but they must surely be accounted for in any comprehensive theory of the solar cycle. It is often said that the form $\Omega_s(\theta)$ persists unaltered through the convection zone, implying that the angular velocity contours are radial. As I have already pointed out, it has been shown, by work of nearly a decade ago, that throughout much of the convection zone the contours are more nearly uniformly inclined by about 30° from the axis of rotation; and equatorwards of latitude 20° or so there is a tendency for them to be even more nearly aligned with the axis of rotation. It should be realized, however, that in this equatorial region Ω deviates only slightly from being constant, so the slopes of the isotachs are not accurately determined. Recently, Howe and her colleagues, using more extensive, more highly resolved data, have concluded that the angle of inclination of the isotachs is closer to 25°. Near the poles the inclination of the isotachs from the axis of rotation continues to increase with latitude, and in the lower reaches of the convection zone the angular-velocity contours become more nearly horizontal: the angular velocity increases more rapidly with depth at constant latitude towards the base of the convection zone, as though there the stresses imposed by the more rapidly rotating radiative envelope extend further into the convection zone than do the corresponding stresses near the equator. In addition, there is another shear layer, entirely within the convection zone, and not far beneath the photosphere: the rotation rate rises with depth at constant latitude to a maximum at radius $r \simeq 0.94R_\odot$ some 20 nHz greater than the corresponding photospheric value, and then declines gradually to roughly its surface value. This is so equatorward of latitudes 50° or so. At higher latitudes seismological inferences are less reliable, although it appears that the general trend continues, but with an additional thinner shear layer of opposite sign immediately beneath the surface associated with which is a local minimum in rotation rate at $r \simeq 0.995R_\odot$, some 5–10 nHz slower than the corresponding surface rate. Also at high latitudes is an intriguing deviation of Ω from the parametrized function $\Omega_s(\theta)$; there is an abrupt decrease by 20 nHz or more in the rate of rotation of the surface layers in the vicinity of latitude 75°, extending to a depth of about $0.05R_\odot$. It is not possible at present to measure the rotation poleward of 80°, but I guess the regions of slow rotation extend all the way to the poles, and thereby, perhaps not accidentally, are coincident with the regions from which the fast solar wind blows.

I have already mentioned that the tachocline may not be spherical. By fitting parametrized shear-functions to the helioseismic data, Charbonneau and his collaborators inferred that the tachocline is prolate, with a likely ellipticity of about 0.25 (corresponding to a prolateness $(r_{\rm pole} - r_{\rm eq})/r_{\rm eq} \simeq 0.03$), although with considerable uncertainty. How much this result is contaminated by the greater spreading of the tachocline shear into the convection zone in polar regions is unclear. However, the result is not inconsistent with an earlier finding that the base of the convection zone, determined from the sound-speed stratification, appears to be similarly prolate with an ellipticity of about 0.20. There is no even half-convincing evidence of a static spatial variation of any other tachocline property.

Superposed on the basic pattern of angular velocity described in the previous paragraph are subphotospheric zonal bands of alternately fast and slow rotation which have been observed by Howe, Frank Hill, Rudi Komm, Christensen-Dalsgaard, Michael Thompson, Schou and others, the surface manifestation of which are the so-called torsional oscillations discovered earlier by Bob Howard and Barry LaBonte. The bands are about 15° wide and penetrate at least $0.15R_\odot$ into the convection zone; they are more-or-less symmetrically placed about the equator, and, according to Sergei Vorontsov and his colleagues, they migrate equatorwards from latitudes of about 42° at a rate which causes the angular velocity at any given latitude to oscillate with a period of about 11 years. At higher latitudes the bands appear to migrate towards the poles. Sunspots, whose locations also migrate equatorwards as the sunspot cycle progresses, but from latitudes of only 30°, are found very roughly at latitudes at which the potential vorticity associated with the