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Philosophical Perspectives on Infinity

This book is an exploration of philosophical questions about infinity. Graham Oppy examines how the infinite lurks everywhere, both in science and in our ordinary thoughts about the world. He also analyzes the many puzzles and paradoxes that follow in the train of the infinite. Even simple notions, such as counting, adding, and maximizing, present serious difficulties. Other topics examined include the nature of space and time, infinities in physical science, infinities in theories of probability and decision, the nature of part/whole relations, mathematical theories of the infinite, and infinite regression and principles of sufficient reason.

Graham Oppy is Associate Dean of Research in the Faculty of Arts at Monash University in Australia. He is the author of *Ontological Arguments and Belief in God* and the forthcoming *Arguments about Gods*, and he has received fellowships from the Australian Research Council. He is an associate editor of the *Australasian Journal of Philosophy* and sits on the editorial boards of *Philo*, *Religious Studies*, and *Sophia*.

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Preface

This book began life as the intended first part of a larger work with the provisional title *God and Infinity*. While it will still be used by me as a starting point for further work in the philosophy of religion, the book has grown into a final product that is more or less entirely independent of that starting point. What it does is to explore various issues about the infinite that emerge in many different areas of philosophy and whose resolution ought not to be tied to the details of those particular areas of philosophy in which particular versions of those issues arise. For those who are not interested in philosophy of religion, this is all that you need to know by way of introduction; you can now happily proceed to the book proper. However, those who are interested in philosophy of religion may like to know a little bit more about the reasons that I had for starting to work on this book. The remainder of this Preface is for you.

When I completed my book on ontological arguments (Oppy 1995c) I immediately commenced work on the next stage of the larger project announced in the preface of that earlier book: an examination of cosmological arguments for and against the existence of various deities. My plan was to follow the structure of the discussion that I provided of ontological arguments in Oppy (1995c), namely, to obtain an exhaustive taxonomy of cosmological arguments discussed in the philosophical literature and to use a thorough discussion of all of the key concepts that are used in those arguments as a basis for criticism of those arguments. Since a thorough discussion of cosmological

arguments would require a thorough discussion of *time*, *causation*, *necessity and contingency*, *infinity*, *sufficient reason*, and *contemporary cosmology* – and maybe other topics as well – it is perhaps predictable that I eventually came to realise that a comprehensive discussion of this kind is not something that I could ever carry out. While I am convinced that it is a mistake to try to discuss arguments about the existence of God without paying attention to wider philosophical debates about the concepts that are employed in those arguments, I now suspect that a frontal assault on cosmological arguments – of the magnitude that I initially envisaged – may be beyond the reach of any individual researcher.

In view of these difficulties, I decided to try a different approach. Rather than divide up discussion in philosophy of religion according to the received topics, I decided to choose one of the key concepts that figures in cosmological arguments, and then to see how that concept is treated both in wider philosophical debates and in other areas of philosophy of religion. If this approach is fruitful, then it can be repeated using some of the other key concepts that figure in cosmological arguments – and, perhaps, over the long term, something like the project that I initially conceived might eventually be completed. Of course, if we approach the subject matter in this way, then the results of discussion of particular topics are provisional: What we say about cosmological arguments from the standpoint of considerations about the infinite will not exhaust what there is to say about cosmological arguments when other considerations are taken into account (and likewise for the other topics to which we give attention). On the other hand, there are advantages to approaching the subject matter in this way: In particular, questions about the consistency of the application of the concept of infinity across a range of different subjects in philosophy of religion come clearly into view in a way that is not possible if we stick to more orthodox ways of dividing up our subject matter. Moreover, a suitably extensive examination of the use of the concept of infinity in a range of wider philosophical contexts helps to concentrate attention on the costs and benefits of particular choices that one might make in particular domains in the philosophy of religion.

According to the plan suggested by the remarks that I have just made, I decided to write a book divided into two parts. The first part – *A Primer on Infinity* – would be an attempt to discuss wider philosophical

views about infinity that, in one way or another, bear on discussions of the infinite in the context of philosophy of religion. The second part – *The Infinite in Philosophy of Religion* – would then apply the discussion of the first part to a range of topics in philosophy of religion. In particular, the second part of the book would focus on the role that the concept of infinity plays in traditional monotheistic conceptions of God and the attributes of God, and on the role that the concept of infinity plays in traditional monotheistic arguments for the existence of God.

While I would like to be able to say that this work is part of a larger project, I think that the most that I can reasonably claim is that it is clearly part of a *possible* larger project. Moreover, it is also worth noting that work on a similar plan might be conducted using the concepts of time, causation, sufficient reason, design, person, goodness, cosmology, and so forth as their foci. I see no reason to suppose that such work would not turn up interesting results concerning the consistency and broader philosophical adequacy of treatments of these topics in orthodox philosophy of religion.

Part of my interest in philosophy of religion stems from the conviction that it must be possible to convince reasonable religious believers that traditional monotheistic *arguments* for the existence of God are worthless. Hence, not surprisingly, one of the subsidiary goals for the projected larger work is to make some contribution to the case for supposing that reasonable religious believers ought to recognise that the arguments for the existence of God provide no reason at all for reasonable nonbelievers to change their minds. However, another part of my interest in philosophy of religion stems from the conviction that it must be possible for reasonable nonbelievers to hold that there are reasonable believers, that is, from the conviction that it is not the case that all religious believers are, *ipso facto*, the subjects of certain kinds of failings of rationality. Hence, not surprisingly, another of the subsidiary goals for this projected larger work is to make a plausible case for the claim that there are conceptions of the infinite that can be integrated successfully into relatively orthodox monotheistic conceptions of the world.

Perhaps it is worth noting here that there are *prima facie* plausible arguments that support the contention that there is no conception of the infinite that can be successfully integrated into relatively orthodox monotheistic conceptions of the world.

If we are strict finitists – and thus reject all actual and potential infinities – then we are obliged to say that God is finite, and that the magnitudes of the divine attributes are finite. But what reason could there be for God to possess a given magnitude to degree N rather than to degree $N + 1$? More generally, how could a finite God be the kind of endpoint for explanation that cosmological arguments typically take God to be?

If we are potential infinitists – that is, if we reject all actual infinities but allow that some entities and magnitudes are potentially infinite – then it seems that we will be obliged to say that God is potentially infinite and that the magnitudes of the divine attributes are potentially infinite. But what kind of conception of God can sustain the claim that God is susceptible of improvement in various respects? If God possesses a magnitude to degree N even though God could possess that magnitude to degree $N + 1$, surely God just isn't the kind of endpoint for explanation that cosmological arguments typically take God to be.

If we are neither strict finitists nor potential infinitists, then it seems that we must be actual infinitists, that is, we must suppose that God is actually infinite and that the magnitudes of the divine attributes are actually infinite. But is there a conception of the infinite that can sustain the claim that God is actually infinite, and the claim that the magnitudes of the divine attributes are actually infinite without undermining the kinds of considerations to which orthodox cosmological arguments appeal in attempting to establish that God exists? Indeed, more generally, are there conceptions of the infinite that can sustain the claim that God is actually infinite, and the claim that the magnitudes of the divine attributes are actually infinite *tout court*? Moreover, if there is a conception of the infinite that can sustain the claim that God is actually infinite, can this conception of the infinite also sustain the idea of an incarnate God, and the idea that there is an afterlife in which people share the same abode as God?

Perhaps some theists will claim that we are not forced to choose among the three options outlined above. Even if these options are unattractive, why not say instead that we can say nothing positive about God – so that, while we can say that God is neither finite nor potentially infinite, we cannot say that God is actually infinite; or that we can speak only analogically about God – so that we cannot say, literally, that God is either finite, or potentially infinite, or actually infinite; or that God

so exceeds our limited comprehension that we should not expect to be able to talk sensibly about whether God is finite, or potentially infinite, or actually infinite; or . . . ?

I think that there are several reasons why one should not take any of these options. On the one hand, if we are interested in the standing of arguments for the existence of God, then we need to be able to give literal content to the claim that God exists. If there is no literal content to give to the claim that God exists, then there is no contest: Nontheists win by default. An argument for the conclusion that “well, there’s something but I can’t make any literally true claims about what it is like and, indeed, I haven’t the least understanding what it is literally like” is hopelessly crippled before it begins. And, on the other hand, if we are interested in the doxastic credentials of theism – can there be reasonable belief that God exists? – then, again, we need to be able to give literal content to the claim that God exists. Perhaps one can believe that “well, there’s something but I can’t make any literally true claims about what it is like and, indeed, I haven’t the least understanding what it is literally like,” but it is very hard to see how one could have *reasons* for holding such a belief. Moreover, of course, if one is going to take any line of this type, then one has to take the line consistently: It is not good taking this line here – in order to avoid a confronting objection – while elsewhere supposing that there are positive, literal claims about God that one is perfectly well placed to make. Any theology based on these kinds of assumptions should be very *brief*.

Since all that this brief introduction aims to do is to make it seem plausible that there is a *prima facie* interesting question to address, I shall leave further discussion of this argument to the future. When I return to consider it further, I shall be able to draw on the following examination of the role of the concept of infinity in a wide range of philosophical discussions that have no obvious connections at all to the philosophy of religion. While I do not insist that the role of the concept of infinity in these other areas determines the role that the concept can play in philosophy of religion, I do insist that one cannot ignore these other discussions when one turns to the philosophy of religion: There is a single concept of the infinite that is required to have application outside the philosophy of religion, and no concept that is inadequate to those external applications can be deemed adequate for the purposes of philosophy of religion.

Acknowledgments

Monash University supported the writing of this book in various ways. In 1997, the structure of the Honours Program at Monash allowed me to present a series of seminars on infinity to a group of senior students; much of the material in this book was assembled during the teaching of that course. In 2002 and again in 2003, I received generous Monash Research Fund grants that allowed me to make time to work on my manuscript. Then, in the first half of 2004, I was granted a sabbatical during which I was able to devote myself full-time to the completion of this work. I am very grateful for the support that I have received from my colleagues at Monash, from within the School of Philosophy and Bioethics, from within the Faculty of Arts more widely, and from within the university community as a whole.

Some of the material in this book was developed during the time between the completion of my book on ontological arguments and my appointment to Monash, while I was still a Research Fellow in the Philosophy Program, in the Research School for the Social Sciences (RSSH), at the Australian National University. Here, I would just like to underscore the appreciative remarks that I made in my acknowledgments in my book on ontological arguments: The Philosophy Program at RSSH is a national treasure, and it is a great privilege to have spent time working there.

There are many people with whom I have discussed material that appears in this book. While I am sure that I won't be able to recall

everyone who ought to be mentioned at this point, I can make the beginnings of a list.

I was introduced to Cantor's paradise in John Groves's first-year lectures on algebra and geometry in 1980, and to many of the topics that are discussed in the first half of this book by Allen Hazen in lectures and conversation during the period 1982–7. I am indebted to Allen, in particular, for his guidance and encouragement during those formative years (though, somehow, his enthusiasm for constructivism and intuitionism seems not to have rubbed off in quite the way that he might have hoped).

David Dowe went through the undergraduate mathematics program at Melbourne University at the same time as I did; our conversations on topics that appear in this book – as well as on a great range of other topics – have continued ever since.

I had many conversations with John Broome, Frank Jackson, Doug MacLean, Peter Menzies, and others about the two-envelope paradox and the St. Petersburg game during 1993, and some very enlightening correspondence with David Chalmers on this and related topics. More recently – towards the end of 2003 – I also had extensive correspondence with Bruce Langtry on the two-envelope paradox.

Jeremy Aarons, Steve Gardner, and Brian Weatherson attended the 1997 lectures and subjected the presented material to very careful critical scrutiny; and Alan Hájek gave a guest performance on Pascal's wager that was *the* highlight of the course. I have had many conversations with Brian and Alan on the topics discussed herein and have been greatly influenced by them in many different ways.

Some of the material discussed in this book is examined far more carefully and deeply in Daniel Nolan's doctoral dissertation. I have learned a great deal from discussions with Daniel – and Greg Restall – about most of the more technical matters that are mentioned in this book.

While I am indebted to all of my colleagues at Monash, I am particularly grateful to Dirk Baltzly, John Bigelow, Lloyd Humberstone, and Aubrey Townsend for discussion of material that is included in this volume.

There are a couple of places where I have drawn on my own previously published work. In particular, part of section 7.3 is taken from Graham Oppy, "Countable Fusion Not Yet Proven Guilty: It Could Be

the Whiteheadian Account of Space Whatdunnit,” *Analysis* 57, no. 4 (1997): 249–53, reprinted with kind permission of Blackwell Publishing. Part of section 7.8 is taken from Graham Oppy, “Humean Supervenience?” *Philosophical Studies* 101, no. 1 (2000): 75–105 (© 2000 Kluwer Academic Publishers), with kind permission of Springer Science and Business Media. Parts of Graham Oppy, “Arguing about the *Kalām* Cosmological Argument,” *Philo* 5, no. 1 (2002): 34–61, are reprinted with kind permission of *Philo*.

There are also places where I draw heavily on the work of other philosophers and mathematicians. My exposition of set theory in section 2.1 owes a lot to Drake (1974); and the exposition of material on nonstandard numbers in section 2.5 relies on Conway (1976), Nelson (1977; 1987), Boolos and Jeffrey (1980), and Lavine (1994). Section 4.2 makes much use of Grünbaum (1953), while section 4.3 is indebted to Skyrms (1983). Section 4.6 follows Earman and Norton (1993), and section 4.7 depends extensively on Earman (1995). Sections 5.1 and 5.2 are indebted to Ehrlich (1982), while section 5.4 draws on Teller (1989) and section 5.5 follows Harrison (1987). Section 6.1 largely follows Ross (1988), while section 6.3 is reliant on Rapaport (1998), and section 6.5 owes quite a bit to Dreier (2003). Section 7.2 has a considerable debt to Lewis (1991), and section 7.5 draws on Lavine (1991). Finally, sections 8.2 to 8.6 make considerable use of Lavine (1994).

I am grateful to the readers at Cambridge University Press for extensive and helpful comments and suggestions; I am also grateful for the various kinds of editorial support provided by Beatrice Rehl and Stephanie Sakson.

Last but not least, I must acknowledge the support of friends and family, without whom this book would never have been completed. In particular, Camille, Gilbert, Calvin, and Alfie have had to put up with me during the seemingly endless hours in which I have been thinking obsessively about the material that is presented in this book. While they may be ambivalent about the trade-off that has been effected, this book is for them.