Part I

Fundamental ideas and general formalisms
One hundred and forty-four years elapsed between the publication of Copernicus’s *De Revolutionibus*, which opened the great scientific revolution of the seventeenth century, and the publication of Newton’s *Principia*, the final synthesis that brought that revolution to a spectacularly successful end. During those 144 years, the basic grammar for understanding the physical world changed and the old picture of reality was reshaped in depth.

At the beginning of the twentieth century, General Relativity (GR) and Quantum Mechanics (QM) once again began reshaping our basic understanding of space and time and, respectively, matter, energy and causality – arguably to a no lesser extent. But we have not been able to combine these new insights into a novel coherent synthesis, yet. The twentieth-century scientific revolution opened by GR and QM is therefore still wide open. We are in the middle of an unfinished scientific revolution. Quantum Gravity is the tentative name we give to the “synthesis to be found”.

In fact, our present understanding of the physical world at the fundamental level is in a state of great confusion. The present knowledge of the elementary dynamical laws of physics is given by the application of QM to fields, namely Quantum Field Theory (QFT), by the particle-physics Standard Model (SM), and by GR. This set of fundamental theories has obtained an empirical success nearly unique in the history of science: so far there isn’t any clear evidence of observed phenomena that clearly escape or contradict this set of theories – or a minor modification of the same, such as a neutrino mass or a cosmological constant. But, the theories in this set are based on badly self-contradictory assumptions. In GR the gravitational field is assumed to be a classical deterministic dynamical field, identified with the (pseudo) Riemannian metric of spacetime: but with QM we have understood that all dynamical fields have quantum properties. The other way around, conventional

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1 Dark matter (not dark energy) might perhaps be contrary evidence.
QFT relies heavily on global Poincaré invariance and on the existence of a non-dynamical background spacetime metric: but with GR we have understood that there is no such non-dynamical background spacetime metric in nature.

In spite of their empirical success, GR and QM offer a schizophrenic and confused understanding of the physical world. The conceptual foundations of classical GR are contradicted by QM and the conceptual foundation of conventional QFT are contradicted by GR. Fundamental physics is today in a peculiar phase of deep conceptual confusion.

Some deny that such a major internal contradiction in our picture of nature exists. On the one hand, some refuse to take QM seriously. They insist that QM makes no sense, after all, and therefore the fundamental world must be essentially classical. This doesn’t put us in a better shape, as far as our understanding of the world is concerned.

Others, on the other hand, and in particular some hard-core particle physicists, do not accept the lesson of GR. They read GR as a field theory that can be consistently formulated in full on a fixed metric background, and treated within conventional QFT methods. They motivate this refusal by insisting than GR’s insight should not be taken too seriously, because GR is just a low-energy limit of a more fundamental theory. In doing so, they confuse the details of the Einstein’s equations (which might well be modified at high energy), with the new understanding of space and time brought by GR. This is coded in the background independence of the fundamental theory and expresses Einstein’s discovery that spacetime is not a fixed background, as was assumed in special relativistic physics, but rather a dynamical field.

Nowadays this fact is finally being recognized even by those who have long refused to admit that GR forces a revolution in the way to think about space and time, such as some of the leading voices in string theory. In a recent interview [1], for instance, Nobel laureate David Gross says: “[...] this revolution will likely change the way we think about space and time, maybe even eliminate them completely as a basis for our description of reality”. This is of course something that has been known since the 1930s [2] by anybody who has taken seriously the problem of the implications of GR and QM. The problem of the conceptual novelty of GR, which the string approach has tried to throw out of the door, comes back by the window.

These and others remind me of Tycho Brahe, who tried hard to conciliate Copernicus’s advances with the “irrefutable evidence” that the Earth is immovable at the center of the universe. To let the background spacetime go is perhaps as difficult as letting go the unmovable background Earth. The world may not be the way it appears in the tiny garden of our daily experience.

Today, many scientists do not hesitate to take seriously speculations such as extra dimensions, new symmetries or multiple universes, for which there isn’t a
wit of empirical evidence; but refuse to take seriously the conceptual implications of the physics of the twentieth century with the enormous body of empirical evidence supporting them. Extra dimensions, new symmetries, multiple universes and the like, still make perfectly sense in a pre-GR, pre-QM, Newtonian world, while to take GR and QM seriously together requires a genuine reshaping of our world view.

After a century of empirical successes that have equals only in Newton’s and Maxwell’s theories, it is time to take seriously GR and QM, with their full conceptual implications; to find a way of thinking the world in which what we have learned with QM and what we have learned with GR make sense together – finally bringing the twentieth-century scientific revolution to its end. This is the problem of Quantum Gravity.

1.1 Quantum spacetime

Roughly speaking, we learn from GR that spacetime is a dynamical field and we learn from QM that all dynamical field are quantized. A quantum field has a granular structure, and a probabilistic dynamics, that allows quantum superposition of different states. Therefore at small scales we might expect a “quantum spacetime” formed by “quanta of space” evolving probabilistically, and allowing “quantum superposition of spaces”. The problem of Quantum Gravity is to give a precise mathematical and physical meaning to this vague notion of “quantum spacetime”.

Some general indications about the nature of quantum spacetime, and on the problems this notion raises, can be obtained from elementary considerations. The size of quantum mechanical effects is determined by Planck’s constant $\hbar$. The strength of the gravitational force is determined by Newton’s constant $G$, and the relativistic domain is determined by the speed of light $c$. By combining these three fundamental constants we obtain the Planck length $l_P = \sqrt{\hbar G/c^3} \sim 10^{-33}$ cm. Quantum-gravitational effects are likely to be negligible at distances much larger than $l_p$, because at these scales we can neglect quantities of the order of $G$, $\hbar$ or $1/c$.

Therefore we expect the classical GR description of spacetime as a pseudo-Riemannian space to hold at scales larger than $l_p$, but to break down approaching this scale, where the full structure of quantum spacetime becomes relevant. Quantum Gravity is therefore the study of the structure of spacetime at the Planck scale.

1.1.1 Space

Many simple arguments indicate that $l_p$ may play the role of a minimal length, in the same sense in which $c$ is the maximal velocity and $\hbar$ the minimal exchanged action.
For instance, the Heisenberg principle requires that the position of an object of mass $m$ can be determined only with uncertainty $x$ satisfying $mv x > \hbar$, where $v$ is the uncertainty in the velocity; special relativity requires $v < c$; and according to GR there is a limit to the amount of mass we can concentrate in a region of size $x$, given by $x > Gm/c^2$, after which the region itself collapses into a black hole, subtracting itself from our observation. Combining these inequalities we obtain $x > \ell_P$. That is, gravity, relativity and quantum theory, taken together, appear to prevent position from being determined more precisely than the Planck scale.

A number of considerations of this kind have suggested that space might not be infinitely divisible. It may have a quantum granularity at the Planck scale, analogous to the granularity of the energy in a quantum oscillator. This granularity of space is fully realized in certain Quantum Gravity theories, such as loop Quantum Gravity, and there are hints of it also in string theory. Since this is a quantum granularity, it escapes the traditional objections to the atomic nature of space.

### 1.1.2 Time

Time is affected even more radically by the quantization of gravity. In conventional QM, time is treated as an external parameter and transition probabilities change in time. In GR there is no external time parameter. Coordinate time is a gauge variable which is not observable, and the physical variable measured by a clock is a nontrivial function of the gravitational field. Fundamental equations of Quantum Gravity might therefore not be written as evolution equations in an observable time variable. And in fact, in the quantum-gravity equation par excellence, the Wheeler–deWitt equation, there is no time variable $t$ at all.

Much has been written on the fact that the equations of nonperturbative Quantum Gravity do not contain the time variable $t$. This presentation of the “problem of time in Quantum Gravity”, however, is a bit misleading, since it mixes a problem of classical GR with a specific Quantum Gravity issue. Indeed, classical GR as well can be entirely formulated in the Hamilton–Jacobi formalism, where no time variable appears either.

In classical GR, indeed, the notion of time differs strongly from the one used in the special-relativistic context. Before special relativity, one assumed that there is a universal physical variable $t$, measured by clocks, such that all physical phenomena can be described in terms of evolution equations in the independent variable $t$. In special relativity, this notion of time is weakened. Clocks do not measure a universal time variable, but only the proper time elapsed along inertial trajectories. If we fix a Lorentz frame, nevertheless, we can still describe all physical phenomena in terms of evolution equations in the independent variable $x^0$, even though this description hides the covariance of the system.
In general relativity, when we describe the dynamics of the gravitational field (not to be confused with the dynamics of matter in a given gravitational field), there is no external time variable that can play the role of observable independent evolution variable. The field equations are written in terms of an evolution parameter, which is the time coordinate $x^0$; but this coordinate does not correspond to anything directly observable. The proper time $\tau$ along spacetime trajectories cannot be used as an independent variable either, as $\tau$ is a complicated non-local function of the gravitational field itself. Therefore, properly speaking, GR does not admit a description as a system evolving in terms of an observable time variable. This does not mean that GR lacks predictivity. Simply put, what GR predicts are relations between (partial) observables, which in general cannot be represented as the evolution of dependent variables on a preferred independent time variable.

This weakening of the notion of time in classical GR is rarely emphasized: after all, in classical GR we may disregard the full dynamical structure of the theory and consider only individual solutions of its equations of motion. A single solution of the GR equations of motion determines “a spacetime”, where a notion of proper time is associated to each timelike worldline.

But in the quantum context a single solution of the dynamical equation is like a single “trajectory” of a quantum particle: in quantum theory there are no physical individual trajectories: there are only transition probabilities between observable eigenvalues. Therefore in Quantum Gravity it is likely to be impossible to describe the world in terms of a spacetime, in the same sense in which the motion of a quantum electron cannot be described in terms of a single trajectory.

To make sense of the world at the Planck scale, and to find a consistent conceptual framework for GR and QM, we might have to give up the notion of time altogether, and learn ways to describe the world in atemporal terms. Time might be a useful concept only within an approximate description of the physical reality.

### 1.1.3 Conceptual issues

The key difficulty of Quantum Gravity may therefore be to find a way to understand the physical world in the absence of the familiar stage of space and time. What might be needed is to free ourselves from the prejudices associated with the habit of thinking of the world as “inhabiting space” and “evolving in time”.

Technically, this means that the quantum states of the gravitational field cannot be interpreted like the $n$-particle states of conventional QFT as living on a given spacetime. Rather, these quantum states must themselves determine and define a spacetime – in the manner in which the classical solutions of GR do.
Conceptually, the key question is whether or not it is logically possible to understand the world in the absence of fundamental notions of time and time evolution, and whether or not this is consistent with our experience of the world.

The difficulties of Quantum Gravity are indeed largely conceptual. Progress in Quantum Gravity cannot be just technical. The search for a quantum theory of gravity raises once more old questions such as: What is space? What is time? What is the meaning of “moving”? Is motion to be defined with respect to objects or with respect to space? And also: What is causality? What is the role of the observer in physics? Questions of this kind have played a central role in periods of major advances in physics. For instance, they played a central role for Einstein, Heisenberg, and Bohr; but also for Descartes, Galileo, Newton and their contemporaries, as well as for Faraday and Maxwell.

Today some physicists view this manner of posing problems as “too philosophical”. Many physicists of the second half of the twentieth century, indeed, have viewed questions of this nature as irrelevant. This view was appropriate for the problems they were facing. When the basics are clear and the issue is problem-solving within a given conceptual scheme, there is no reason to worry about foundations: a pragmatic approach is the most effective one. Today the kind of difficulties that fundamental physics faces has changed. To understand quantum spacetime, physics has to return, once more, to those foundational questions.

1.2 Where are we?

Research in Quantum Gravity developed slowly for several decades during the twentieth century, because GR had little impact on the rest of physics and the interest of many theoreticians was concentrated on the development of quantum theory and particle physics. In the past 20 years, the explosion of empirical confirmations and concrete astrophysical, cosmological and even technological applications of GR on the one hand, and the satisfactory solution of most of the particle physics puzzles in the context of the SM on the other, have led to a strong concentration of interest in Quantum Gravity, and the progress has become rapid. Quantum Gravity is viewed today by many as the big open challenge in fundamental physics.

Still, after 70 years of research in Quantum Gravity, there is no consensus, and no established theory. I think it is fair to say that there isn’t even a single complete and consistent candidate for a quantum theory of gravity.

In the course of 70 years, numerous ideas have been explored, fashions have come and gone, the discovery of the Holy Grail of Quantum Gravity has been several times announced, only to be later greeted with much scorn. Of the tentative theories studied today (strings, loops and spinfoams, non-commutative geometry, dynamical triangulations or others), each is to a large extent incomplete and none has yet received a whit of direct or indirect empirical support.
However, research in Quantum Gravity has not been meandering meaninglessly. On the contrary, a consistent logic has guided the development of the research, from the early formulation of the problem and of the major research programs in the 1950s to the present. The implementation of these programs has been laborious, but has been achieved. Difficulties have appeared, and solutions have been proposed, which, after much difficulty, have lead to the realization, at least partial, of the initial hopes.

It was suggested in the early 1970s that GR could perhaps be seen as the low energy limit of a Poincaré invariant QFT without uncontrollable divergences [3]; and today, 30 years later, a theory likely to have these properties – perturbative string theory – is known. It was also suggested in the early 1970s that non-renormalizability might not be fatal for quantum GR [4; 5] and that the Planck scale could cut divergences off nonperturbatively by inducing a quantum discrete structure of space; and today we know that this is in fact the case – ultraviolet finiteness is realized precisely in this manner in canonical loop Quantum Gravity and in some spinfoam models. In 1957 Charles Misner indicated that in the canonical framework one should be able to compute quantum eigenvalues of geometrical quantities [6]; and in 1995, 37 years later, eigenvalues of area and volume were computed – within loop quantum gravity [7; 8]. Much remains to be understood and some of the current developments might lead nowhere. But looking at the entire development of the subject, it is difficult to deny that there has been substantial progress.

In fact, at least two major research programs can today claim to have, if not a complete candidate theory of Quantum Gravity, at least a large piece of it: string theory (in its perturbative and still incomplete nonperturbative versions) and loop quantum gravity (in its canonical as well as covariant – spinfoam – versions) are both incomplete theories, full of defects – in general, strongly emphasized within the opposite camp – and without any empirical support, but they are both remarkably rich and coherent theoretical frameworks, that might not be far from the solution of the puzzle.

Within these frameworks, classical and long intractable, physical, astrophysical and cosmological Quantum Gravity problems can finally be concretely treated. Among these: black hole’s entropy and fate, the physics of the big-bang singularity and the way it has affected the currently observable universe, and many others. Tentative predictions are being developed, and the attention to the concrete possibility of testing these predictions with observations that could probe the Planck scale is very alive. All this was unthinkable only a few years ago.

The two approaches differ profoundly in their hypotheses, achievements, specific results, and in the conceptual frame they propose. The issues they raise concern the foundations of the physical picture of the world, and the debate between the two approaches involves conceptual, methodological and philosophical issues.
In addition, a number of other ideas, possibly alternative, possibly complementary to the two best developed theories and to one another, are being explored. These include noncommutative geometry, dynamical triangulations, effective theories, causal sets and many others.

The possibility that none of the currently explored hypotheses will eventually turn out to be viable, or, simply, none will turn out to be the way chosen by Nature, is very concrete, and should be clearly kept in mind. But the rapid and multi-front progress of the past few years raises hopes. Major well-posed open questions in theoretical physics (Copernicus or Ptolemy? Galileo’s parabolas or Kepler’s ellipses? How to describe electricity and magnetism? Does Maxwell theory pick a preferred reference frame? How to do the Quantum Mechanics of interacting fields...?) have rarely been solved in a few years. But they have rarely resisted more than a few decades. Quantum Gravity – the problem of describing the quantum properties of spacetime – is one of these major problems, and it is reasonably well defined: is there a coherent theoretical framework consistent with quantum theory and with General Relativity? It is a problem which is on the table since the 1930s, but it is only in the past couple of decades that the efforts of the theoretical physics community have concentrated on it.

Maybe the solution is not far away. In any case, we are not at the end of the road of physics, we are half-way through the woods along a major scientific revolution.

Bibliographical note

For details on the history of Quantum Gravity see the historical appendix in [9]; and, for early history see [10; 11] and [12; 13]. For orientation on current research on Quantum Gravity, see the review papers [14; 15; 16; 17]. As a general introduction to Quantum Gravity ideas, see the old classic reviews, which are rich in ideas and present different points of view, such as John Wheeler 1967 [18], Steven Weinberg 1979 [5], Stephen Hawking 1979 and 1980 [19; 20], Karel Kuchar 1980 [21], and Chris Isham’s magisterial syntheses [22; 23; 24]. On string theory, classic textbooks are Green, Schwarz and Witten, and Polchinski [25; 26]. On loop quantum gravity, including the spinfoam formalism, see [9; 27; 28], or the older papers [29; 30]. On spinfoams see also [31]. On noncommutative geometry see [32] and on dynamical triangulations see [33]. For a discussion of the difficulties of string theory and a comparison of the results of strings and loops, see [34], written in the form of a dialogue, and [35]. On the more philosophical challenges raised by Quantum Gravity, see [36]. Smolin’s popular book [37] provides a readable introduction to Quantum Gravity. The expression “half way through the woods” to characterize the present state of fundamental theoretical physics is taken from [38; 39]. My own view on Quantum Gravity is developed in detail in [9].
References


