

## SUPERSYMMETRY AND STRING THEORY

## Beyond the Standard Model

The past decade has witnessed some dramatic developments in the field of theoretical physics, including advancements in supersymmetry and string theory. There have also been spectacular discoveries in astrophysics and cosmology. The next few years will be an exciting time in particle physics with the start of the Large Hadron Collider at CERN.

This book is a comprehensive introduction to these recent developments, and provides the tools necessary to develop models of phenomena important in both accelerators and cosmology. It contains a review of the Standard Model, covering non-perturbative topics, and a discussion of grand unified theories and magnetic monopoles. The book focuses on three principal areas: supersymmetry, string theory, and astrophysics and cosmology. The chapters on supersymmetry introduce the basics of supersymmetry and its phenomenology, and cover dynamics, dynamical supersymmetry breaking, and electric–magnetic duality. The book then introduces general relativity and the big bang theory, and the basic issues in inflationary cosmologies. The section on string theory discusses the spectra of known string theories, and the features of their interactions. The compactification of string theories is treated extensively. The book also includes brief introductions to technicolor, large extra dimensions, and the Randall–Sundrum theory of warped spaces.

*Supersymmetry and String Theory* will enable readers to develop models for new physics, and to consider their implications for accelerator experiments. This will be of great interest to graduates and researchers in the fields of particle theory, string theory, astrophysics, and cosmology. The book contains several problems and password-protected solutions will be available to lecturers at [www.cambridge.org/9780521858410](http://www.cambridge.org/9780521858410).

MICHAEL DINE is Professor of Physics at the University of California, Santa Cruz. He is an A. P. Sloan Foundation Fellow, a Fellow of the American Physical Society, and a Guggenheim Fellow. Prior to this Professor Dine was a research associate at the Stanford Linear Accelerator Center, a long-term member of the institute for Advanced Study, and Henry Semat Professor at the City College of the City University of New York.

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“An excellent and timely introduction to a wide range of topics concerning physics beyond the standard model, by one of the most dynamic researchers in the field. Dine has a gift for explaining difficult concepts in a transparent way. The book has wonderful insights to offer beginning graduate students and experienced researchers alike.”

*Nima Arkani-Hamed, Harvard University*

“How many times did you need to find the answer to a basic question about the formalism and especially the phenomenology of general relativity, the Standard Model, its supersymmetric and grand unified extensions, and other serious models of new physics, as well as the most important experimental constraints and the realization of the key models within string theory? Dine’s book will solve most of these problems for you and give you much more, namely the state-of-the-art picture of reality as seen by a leading superstring phenomenologist.”

*Lubos Motl, Harvard University*

“This book gives a broad overview of most of the current issues in theoretical high energy physics. It introduces and discusses a wide range of topics from a pragmatic point of view. Although some of these topics are addressed in other books, this one gives a uniform and self-contained exposition of all of them. The book can be used as an excellent text in various advanced graduate courses. It is also an extremely useful reference book for researchers in the field, both for graduate students and established senior faculty. Dine’s deep insights and broad perspective make this book an essential text. I am sure it will become a classic. Many physicists expect that with the advent of the LHC a revival of model building will take place. This book is the best tool kit a modern model builder will need.”

*Nathan Seiberg, Institute for Advanced Study, Princeton*

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MICHAEL DINE

*University of California, Santa Cruz*



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This book is dedicated to Mark and Esther Dine

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## Preface

As this is being written, particle physics stands on the threshold of a new era, with the commissioning of the Large Hadron Collider (LHC) not even two years away. In writing this book, I hope to help prepare graduate students and postdoctoral researchers for what will hopefully be a period rich in new data and surprising phenomena.

The Standard Model has reigned triumphant for three decades. For just as long, theorists and experimentalists have speculated about what might lie beyond. Many of these speculations point to a particular energy scale, the teraelectronvolt (TeV) scale which will be probed for the first time at the LHC. The stimulus for these studies arises from the most mysterious – and still missing – piece of the Standard Model: the Higgs boson. Precision electroweak measurements strongly suggest that this particle is elementary (in that any structure is likely far smaller than its Compton wavelength), and that it should be in a mass range where it will be discovered at the LHC. But the existence of fundamental scalars is puzzling in quantum field theory, and strongly suggests new physics at the TeV scale. Among the most prominent proposals for this physics is a hypothetical new symmetry of nature, supersymmetry, which is the focus of much of this text. Others, such as technicolor, and large or warped extra dimensions, are also treated here.

Even as they await evidence for such new phenomena, physicists have become more ambitious, attacking fundamental problems of quantum gravity, and speculating on possible final formulations of the laws of nature. This ambition has been fueled by *string theory*, which seems to provide a complete framework for the quantum mechanics of gauge theory and gravity. Such a structure is necessary to give a framework to many speculations about beyond the Standard Model physics. Most models of supersymmetry breaking, theories of large extra dimensions, and warped spaces cannot be discussed in a consistent way otherwise.

It seems, then, quite likely that a twenty-first-century particle physicist will require a working knowledge of supersymmetry and string theory, and in writing this

text I hope to provide this. The first part of the text is a review of the Standard Model. It is meant to complement existing books, providing an introduction to perturbative and phenomenological aspects of the theory, but with a lengthy introduction to non-perturbative issues, especially in the strong interactions. The goal is to provide an understanding of chiral symmetry breaking, anomalies and instantons, suitable for thinking about possible strong dynamics, and about dynamical issues in supersymmetric theories. The first part also introduces grand unification and magnetic monopoles.

The second part of the book focuses on supersymmetry. In addition to global supersymmetry in superspace, there is a study of the supersymmetry currents, which are important for understanding dynamics, and also for understanding the BPS conditions which play an important role in field theory and string theory dualities. The MSSM is developed in detail, as well as the basics of supergravity and supersymmetry breaking. Several chapters deal with supersymmetry dynamics, including dynamical supersymmetry breaking, Seiberg dualities and Seiberg–Witten theory. The goal is to introduce phenomenological issues (such as dynamical supersymmetry breaking in hidden sectors and its possible consequences), and also to illustrate the control that supersymmetry provides over dynamics.

I then turn to another critical element of beyond the Standard Model physics: general relativity, cosmology and astrophysics. The chapter on general relativity is meant as a brief primer. The approach is more field theoretic than geometrical, and the uninitiated reader will learn the basics of curvature, the Einstein Lagrangian, the stress tensor and equations of motion, and will encounter the Schwarzschild solution and its features. The subsequent two chapters introduce the basic features of the FRW cosmology, and then very early universe cosmology: cosmic history, inflation, structure formation, dark matter and dark energy. Supersymmetric dark matter and axion dark matter, and mechanisms for baryogenesis, are all considered.

The third part of the book is an introduction to string theory. My hope, here, is to be reasonably comprehensive while not being excessively technical. These chapters introduce the various string theories, and quickly compute their spectra and basic features of their interactions. Heavy use is made of light cone methods. The full machinery of conformal and superconformal ghosts is described but not developed in detail, but conformal field theory techniques are used in the discussion of string interactions. Heavy use is also made of effective field theory techniques, both at weak and strong coupling. Here, the experience in the first half of the text with supersymmetry is invaluable; again supersymmetry provides a powerful tool to constrain and understand the underlying dynamics. Two lengthy chapters deal with string compactifications; one is devoted to toroidal and orbifold compactifications, which are described by essentially free strings; the other introduces the basics of Calabi–Yau compactification. Four appendices make up the final part of this book.

The emphasis in all of this discussion is on providing tools with which to consider how string theory might be related to observed phenomena. The obstacles are made clear, but promising directions are introduced and explored. I also attempt to stress how string theory can be used as a testing ground for theoretical speculations. I have not attempted a complete bibliography. The suggested reading in each chapter directs the reader to a sample of reviews and texts.

What I know in field theory and string theory is the result of many wonderful colleagues. It is impossible to name all of them, but Tom Appelquist, Nima Arkani-Hamed, Tom Banks, Savas Dimopoulos, Willy Fischler, Michael Green, David Gross, Howard Haber, Jeff Harvey, Shamit Kachru, Andre Linde, Lubos Motl, Ann Nelson, Yossi Nir, Michael Peskin, Joe Polchinski, Pierre Ramond, Lisa Randall, John Schwarz, Nathan Seiberg, Eva Silverstein, Bunji Sakita, Steve Shenker, Leonard Susskind, Scott Thomas, Steven Weinberg, Frank Wilczek, Mark Wise and Edward Witten have all profoundly influenced me, and this influence is reflected in this text. Several of them offered comments on the text or provided specific advice and explanations, for which I am grateful. I particularly wish to thank Lubos Motl for reading the entire manuscript and correcting numerous errors. Needless to say, none of them are responsible for the errors which have inevitably crept into this book.

Some of the material, especially on anomalies and aspects of supersymmetry phenomenology, has been adapted from lectures given at the Theoretical Advanced Study Institute, held in Boulder, Colorado. I am grateful to K. T. Manahathapa for his help during these schools, and to World Scientific for allowing me to publish these excerpts. The lectures “Supersymmetry Phenomenology with a Broad Brush” appeared in *Fields, Strings and Duality*, ed. C. Efthimiou and B. Greene (Singapore: World Scientific, 1997); “TASI Lectures on M Theory Phenomenology” appeared in *Strings, Branes and Duality*, ed. C. Efthimiou and B. Greene (Singapore: World Scientific, 2001); and “The Strong CP Problem” in *Flavor Physics for the Millennium: TASI 2000*, ed. J. L. Rosner (Singapore: World Scientific, 2000).

I have used much of the material in this book as the basis for courses, and I am also grateful to students and postdocs (especially Patrick Fox, Assaf Shomer, Sean Echols, Jeff Jones, John Mason, Alex Morisse, Deva O’Neil, and Zheng Sun) at Santa Cruz who have patiently suffered through much of this material as it was developed. They have made important comments on the text and in the lectures, often filling in missing details. As teachers, few of us have the luxury of devoting a full year to topics such as this. My intention is that the separate supersymmetry or string parts are suitable for a one-quarter or one-semester special topics course.

Finally, I wish to thank Aviva, Jeremy, Shifrah, and Melanie for their love and support.

## A note on choice of metric

There are two popular choices for the metric of flat Minkowski space. One, often referred to as the “West Coast Metric,” is particularly convenient for particle physics applications. Here,

$$ds^2 = dt^2 - d\vec{x}^2 = \eta_{\mu\nu} dx^\mu dx^\nu \quad (0.1)$$

This has the virtue that  $p^2 = E^2 - \vec{p}^2 = m^2$ . It is the metric of many standard texts in quantum field theory. But it has the annoying feature that ordinary, space-like intervals – conventional lengths – are treated with a minus sign. So in most general relativity textbooks, as well as string theory textbooks, the “East Coast Metric” is standard:

$$ds^2 = -dt^2 + d\vec{x}^2. \quad (0.2)$$

Many physicists, especially theorists, become so wedded to one form or another that they resist – or even have difficulty – switching back and forth. This is a text, however, meant to deal both with particle physics and with general relativity and string theory. So, in the first half of the book, which deals mostly with particle physics and quantum field theory, we will use the “West Coast” convention. In the second half, dealing principally with general relativity and string theory, we will switch to the “East Coast” convention. For both the author and the readers, this may be somewhat disconcerting. While I have endeavored to avoid errors from this somewhat schizophrenic approach, some have surely slipped by. But I believe that this freedom to move back and forth between the two conventions will be both convenient and healthy. If nothing else, this is probably the first textbook in physics in which the author has deliberately used both conventions (many have done so inadvertently).

At a serious level, the researcher must always be careful in computations to be consistent. It is particularly important to be careful in borrowing formulas from



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*A note on choice of metric*

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papers and texts, and especially in downloading computer programs, to make sure one has adequate checks on such matters of signs. I will appreciate being informed of any such inconsistencies, as well as of other errors, both serious and minor, which have crept into this text.

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Even as this book was going to press, there were important developments in a number of these subjects. The website <http://scipp.ucsc.edu/~dine/book/book.html> will contain

- (1) updates,
- (2) errata,
- (3) solutions of selected problems, and
- (4) additional selected reading.