

Contents

<i>Preface</i>	page ix
<i>Acknowledgements</i>	xiii
I MOTIVATION	1
Introduction to Part I	2
1 The Mathieu group M_{12}	3
1.1 The combinatorial approach	3
1.2 The regular dodecahedron	7
1.3 The algebraic approach	9
1.4 Independent proofs	10
2 The Mathieu group M_{24}	15
2.1 The combinatorial approach	15
2.2 The Klein map	18
2.3 The algebraic approach	25
2.4 Independent proofs	26
Conclusions to Part I	40
II INVOLUTORY SYMMETRIC GENERATORS	43
3 The (involutory) progenitor	45
3.1 Free products of cyclic groups of order 2	45
3.2 Semi-direct products and the progenitor P	46
3.3 The Cayley graph of P over N	50
3.4 The regular graph preserved by P	54
3.5 Homomorphic images of P	54
3.6 The lemma	58
3.7 Further properties of the progenitor	59
3.8 Coxeter diagrams and Y-diagrams	62

viii	<i>Contents</i>
3.9	Introduction to MAGMA and GAP 64
3.10	Algorithm for double coset enumeration 66
3.11	Systematic approach 74
4	Classical examples 89
4.1	The group $\mathrm{PGL}_2(7)$ 89
4.2	Exceptional behaviour of S_n 97
4.3	The 11-point biplane and $\mathrm{PGL}_2(11)$ 116
4.4	The group of the 28 bitangents 121
5	Sporadic simple groups 127
5.1	The Mathieu group M_{22} 127
5.2	The Janko group J_1 137
5.3	The Higman–Sims group 147
5.4	The Hall–Janko group and the Suzuki chain 161
5.5	The Mathieu groups M_{12} and M_{24} 173
5.6	The Janko group J_3 174
5.7	The Mathieu group M_{24} as control subgroup 177
5.8	The Fischer groups 226
5.9	Transitive extensions and the O’Nan group 233
5.10	Symmetric representation of groups 235
5.11	Appendix to Chapter 5 238
III	NON-INVOLUTORY SYMMETRIC GENERATORS 247
6	The (non-involutory) progenitor 249
6.1	Monomial automorphisms 249
6.2	Monomial representations 250
6.3	Monomial action of a control subgroup 256
7	Images of the progenitors in Chapter 6 263
7.1	The Mathieu group M_{11} 263
7.2	The Mathieu group M_{23} 267
7.3	The Mathieu group M_{24} 271
7.4	Factoring out a ‘classical’ relator 273
7.5	The Suzuki chain and the Conway group 288
7.6	Systematic approach 292
7.7	Tabulated results 301
7.8	Some sporadic groups 308
	<i>References</i> 309
	<i>Index</i> 315